FOR THE INCREASE AND DIFFVSION OF KNOWLEDGE AMONG MEN

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SMITHSONIAN INSTITVTION WASHINGTON 1846

SMITHSONIAN

MISCELLANEOUS COLLECTIONS

VOL. 62

"EVERY MAN IS A VALUABLE MEMBER OF SOCIETY WHO, BY HIS OBSERVATIONS, RESEARCHES, AND EXPERIMENTS, PROCURES KNOWLEDGE FOR MEN"—SMITHSON

(PUBLICATION 2716)

CITY OF WASHINGTON
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ADVERTISEMENT

The present series, entitled "Smithsonian Miscellaneous Collections," is intended to embrace all the octavo publications of the Institution, except the Annual Report. Its scope is not limited, and the volumes thus far issued relate to nearly every branch of science. Among these various subjects zoology, bibliography, geology, mineralogy, and anthropology have predominated.

The Institution also publishes a quarto series entitled "Smithsonian Contributions to Knowledge." It consists of memoirs based on extended original investigations, which have resulted in important additions to knowledge.

CHARLES D. WALCOTT,
Secretary of the Smithsonian Institution.

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SMITHSONIAN MISCELLANEOUS COLLECTIONS
VOLUME 62, NUMBER 1

Hodgkins Fund

ADVISORY COMMITTEE ON THE LANGLEY AERODYNAMICAL LABORATORY

(Publication 2227)

CITY OF WASHINGTON
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Hodgkins Fund

ADVISORY COMMITTEE ON THE LANGLEY AERODYNAMICAL LABORATORY

OFFICIAL STATUS

Authorization.—On May 1, 1913, the Regents of the Smithsonian Institution, approving a general scheme submitted by Secretary Walcott, authorized the Secretary, with the approval of the Executive Committee, to reopen the Langley Aerodynamical Laboratory; to appoint an Advisory Committee; to add, as means are provided, other laboratories and agencies; to group them into a bureau organization; and to secure the cooperation with them of the Government and other agencies.

Functions.—The Committee is to advise as to the organization and work of the Langley Aerodynamical Laboratory and of the bureau organization when adopted, and the coordination of their activities with the kindred labors of other establishments, Governmental and private; it is to plan for such theoretical and experimental investigations, tests and reports, as may serve to increase the safety and effectiveness of aerial locomotion for the purposes of commerce, national defense, and the welfare of man. But neither the Committee nor the Smithsonian Institution will promote patented devices, furnish capital to inventors, or manufacture commercially, or give regular courses of instruction for aeronautical pilots or engineers.

The organization, under regulations to be established and fees to be fixed by the Secretary, approved by the Smithsonian Executive Committee, may exercise its functions for the military and civil departments of the Government of the United States, and also for any individual, firm, association, or corporation within the United States; provided, however, that such department, individual, firm, association, or corporation shall defray the cost of all material used and of all services of persons employed in the exercise of such functions.

With the approval of the Secretary of the Institution, the Committee is to collect aeronautical information, such part of the same as may be valuable to the Government, or the public, to be issued in bulletins and other publications.

Membership and Privileges.—The Advisory Committee is to be composed of the Director of the Langley Aerodynamical Laboratory, when appointed, and one member to be designated by the Secretary of War, one by the Secretary of the Navy, one by the Secretary of Agriculture, and one by the Secretary of Commerce, together with such other persons, to be designated by the Secretary of the Smithsonian Institution, as may be acquainted with the needs of aeronautics, the total membership of such Committee not to exceed fourteen.

The members of the Advisory Committee, as such, are to serve without compensation, but will have refunded the necessary expenses incurred by them in going to Washington to attend the meetings of the Committee and returning therefrom, and while attending the meetings.

Approval of the President.—On May 9, 1913, the President of the United States, by request of the Secretary of the Smithsonian Institution, approved the designation of representatives of the above-named Departments to serve on the Advisory Committee.

ORGANIZATION

Officers.—The Advisory Committee, as constituted at its organization meeting, convened by Secretary Walcott at the Smithsonian Institution, May 23, 1913, comprises a Chairman, a Recorder, and twelve additional members, all of whom are to serve for one year. The officers are to be elected annually on or about May 6, and the members for the ensuing year are to be appointed prior to the date of such election.

The Chairman has general supervision of the work of the Advisory Committee, presides at its meetings, receives the reports of the Subcommittees, and makes an annual report to the Secretary of the Smithsonian Institution. Said report must include an account of the work done for any Department of the Government, individual, firm, association, or corporation, and the amounts paid by them to defray the cost of material and services, as hereinbefore mentioned.

The Recorder keeps the minutes of the meetings of the Committee, and assists the Chairman in conducting correspondence and preparing reports pertaining to the business of the Committee.

Subcommittees.—The Chairman, with the approval of the Advisory Committee, may appoint standing and special Subcommittees to perform such functions as may be assigned to them.

The standing Subcommittees may have assigned to them investigations and tests of a permanent character, which they may prosecute

from year to year, and on which they are to make quarterly reports to the Chairman followed by an annual report. Each Subcommittee comprises a Chairman who must be a member of the Advisory Committee, and others, chosen by him from that Committee or elsewhere.

AGENCIES, RESOURCES, AND FACILITIES

Smithsonian Institution.—The Advisory Committee has been provided by the Smithsonian Institution with suitable office headquarters, an administrative and accounting system, library and publication facilities, lecture and assembly rooms, and museum space for aeronautic models. The Langley Aerodynamical Laboratory has an income provided for it not to exceed ten thousand dollars the first year (of which five thousand dollars has been allotted), and five thousand annually for five years.

U. S. Bureau of Standards.—For the exact determination of aerophysical constants, the calibration of instruments, the testing of aeronautic engines, propellers and materials of construction, the Committee has the cooperation of the U. S. Bureau of Standards, from which the Secretary of Commerce has designated one representative.

This Bureau has a complete equipment for studying the mechanics of materials and structural forms used in air-craft; for standardizing the physical instruments—thermometers, barographs, pressure gauges, etc.—used in air navigation; and for testing the power, efficiency, etc., of aeronautical motors in a current of air representing the natural conditions of flight.

In these general branches the technical staff of the Bureau is prepared to undertake such theoretical and experimental investigations as may come before the Advisory Committee on behalf of either the Government or private individuals or organizations.

U. S. Weather Bureau.—For studies of and reports on every phase of aeronautic meteorology, besides the usual forecasting, the Committee has the cooperation of the U. S. Weather Bureau, from which the Secretary of Agriculture has designated one representative.

This Bureau has an extensive library of works on or allied to aeronautics, an instrument division for every type of apparatus for studying the state of the atmosphere, a whirling table of thirty foot radius for standardizing anemometers, a complete kite equipment with power reel, and a sounding balloon equipment with electrolytic hydrogen plant, all of which are available for scientific investigations. For special forecasts, anticipating field tests or cross country voyages, the general service of the Bureau may be called upon.

War and Navy Departments.—These Departments, while especially interested in aeronautics for national defense, can be of service in advancing the general science. Each has an aeronautical library; each has an official representative in foreign countries who reports periodically on every important phase of the art, whether civil or military; each has an assignment of officers who design, test, and operate air craft, and who determine largely the scope and character of their development; each has its aeronautic station equipped with machines in actual service throughout the year. Besides various aviation establishments, the War Department has a balloon plant at Fort Myer, Va., and at Omaha, Neb.; the Navy has its marine Model Basin, useful for special experiments in aeronautics, its extensive shops at the Washington Navy Yard, available for the alteration or repair of air craft, or the manufacture of improved military types, and at Fort Myer, three lofty open-work steel towers suitable for studies in meteorology or aerodynamics in the natural wind. Furthermore, the Navy Department has detailed an officer for special research in aeronautics at one of the principal Engineering Schools.

Because of their fundamental interest in aeronautics, each of these Departments has two representatives on the Advisory Committee, and each will be able to place at the service of the Committee one or more skilled aviators and aeroplanes for systematic experimentation.

PRESENT NEEDS

In presenting the needs of the organization, it is well to remark that the Smithsonian Institution possesses the unique character of being a private organization having Governmental functions and prerogatives. It can receive appropriations directly from Congress; it can be the recipient or the custodian of private funds for the increase and diffusion of knowledge; it can deposit such private funds with the United States Treasury, or place them otherwise, as may be required by the donor. Likewise, it can be the recipient or custodian of material objects representing any province of nature, or any branch of human knowledge or art. This unique character allows the public to anticipate or supplement the cooperation of Congress in promoting the aerodromical (aeronautical) work of the Institution.

Endowment Funds.—Persons approving the purpose of the organization and desiring its continuity and permanence can not do better than to provide for it a steady income, either for general or for specific use. Individual endowment funds bearing the name of the giver or other person, and presented to the Smithsonian Institution,

or placed in its custody at the disposal of the Committee, may be recommended; also collective funds bearing the name of a society, organization, or section of the country, whether in the interest of scientific progress or of national defense.

Temporary Funds.—For the prompt achievement of definite results, funds may well be offered for immediate application, both of principal and interest; as, for example, for the erection of laboratories or other buildings; for the purchase of experimental air craft, or apparatus, instruments, etc.

Most needed is an expansion of the Langley Aerodynamical Laboratory providing a large and a small wind tunnel, ampler shops, and instrument and model rooms. Adjacent to this, or forming a part of it, may well be the headquarters of the Committee, with the collections of aeronautic publications and exhibits, and with designing rooms where plans for air craft may be matured by fabricators in consultation with the technical staff. This new building, if placed on the Smithsonian grounds, should be of good architecture and cost not less than $100,000.

Of immediate importance is an air-craft field laboratory, adjacent to ample flying space of land and water, and adapted to assembling, adjusting and repairing several full scale land and water aeroplanes; and subjecting them to indoor tests and measurements, as of stress, strain, factor of safety, center of gravity, moment of inertia, working condition, etc. One such plant suitably located would serve all Governmental and civilian requirements for the present. A suitable site is the public land in Potomac Park in the vicinity of the Smithsonian Institution. Here might be held air-craft competitions under the auspices of the Government.

Prizes and Awards.—As a stimulus to the highest aeronautic achievement, or as an honorable recognition thereof, suitable prizes or awards might advantageously be offered. Provision should be made for liberal cash prizes for competitive tests of motors, propellers, etc., in a purely scientific way not trenching upon the province of aero clubs.

Fellowships.—For the prosecution of special aeronautic investigations in cooperation with the advisory Committee, educational institutions and scientific or engineering organizations should be provided with research fellowships whose incumbents may have the counsel of the Committee and the advantage of its equipments.

Until adequate appropriations have been made by the Government, the activities of the organization and Committee will have to be sustained largely by private resources.

SMITHSONIAN MISCELLANEOUS COLLECTIONS
VOLUME 62, NUMBER 2

Hodgkins Fund

ADVISORY COMMITTEE ON THE LANGLEY AERODYNAMICAL LABORATORY

HYDROMECHANIC EXPERIMENTS WITH FLYING BOAT HULLS

(WITH SIX PLATES)

BY
H. C. RICHARDSON, NAVAL CONSTRUCTOR, U. S. NAVY
Chairman of Sub-Committee on Hydromechanic Experiments in Relation to Aeronautics

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ADVISORY COMMITTEE ON THE LANGLEY AERODYNAMICAL LABORATORY

HYDROMECHANIC EXPERIMENTS WITH FLYING BOAT HULLS

By H. C. RICHARDSON, Naval Constructor, U. S. Navy
CHAIRMAN OF SUB-COMMITTEE ON HYDROMECHANICS IN RELATION TO AERONAUTICS

(With Six Plates)

During the latter half of 1913 the following work of interest was carried on at the Model Basin at the Washington Navy Yard.

This work comprised an investigation of the forms of hulls of flying boats in order to determine (1) their resistance at "displacements corresponding to speeds," on the water, and (2) their resistances "submerged," as a means of approximating their total head resistances in air and of determining an approximate "coefficient of fineness of form."

As a result a form of hull has been derived which appears to have decided advantages over those already in use in the Navy, so far as resistance on the surface and in the air is concerned. Such a hull slightly modified to overcome structural difficulties is now being tried on a new navy machine.

Plates 1, 2, and 3 contain plots of the results of the experiments, while plates 4, 5 and 6, illustrate the models used.

PLATE 1

This plate contains plots of model runs for the following models, 1591-3, 1592-1, 1592-5, 1593-1, 1602-1, and 1617-1.

At the foot of the sheet are plots of net resistance, and of derived e. h. p. for the full size computed on the assumption that the total resistance of the full size model at the corresponding speed is proportional to the displacement. As the models were all 1/9 full size the corresponding speeds for the full size are $\sqrt{9}=3$ times those for the models, and the resistances $9^3=729$ times those of the models.

This assumption is interesting as a means of comparison but the e. h. p.'s so determined are inaccurate and exceed the true values, because the frictional resistance which is an important component varies with the speed to some power less than the square, approximately the 1.85 power. It is probable that the maximum error is less than 40% and the minimum as high as 20% in excess of the actual e. h. p., but the discrepancies are not considered to invalidate their value for comparative purposes. At the top of the plate the change of level curves are plotted.

The resistance curves were determined by towing the models at "displacements corresponding to speeds," with the models at a set "trim" but free to rise and fall under the influence of "suction" or "planing," and the change of level curves show how much the planing effect changes the draft at each condition.

All models were towed under conditions representing a full load of 2,200 lbs. and the assumption that the get-away occurs at 45 m. p. h. All were of the ventilated step type. An inspection of the resistance and trim curves will show the following general features:

a. At low speeds, suction is present.

b. This is succeeded by a condition in which the models run hard.

c. Which is succeeded by a condition at which the model begins to plane.

d. And just before the planing is established the slope of the curve lessens rapidly.

e. And when planing is established the resistance falls off sharply with one exception.

f. Just preceding the "get-away" there is a tendency for the resistance to remain at an appreciable value, which

g. Falls to nothing sharply at the last.

Model 1591-3 was designed to obviate the defects of the flat scow bow type, and introduces the V bottom for the purpose of parting rather than pushing the water aside. The ventilated step was located so as to be slightly to the rear of the center of gravity. This model 1591-3 was derived from 1591-1, which was a true V type. 1591-1 ran very well except for a remarkable sheet of spray at a speed corresponding to 12 m. p. h. This sheet of spray is shown in plate 4, fig. 2. Due to this spray it was considered necessary to modify the model and this was done by making the V sections "full," the principal effect of the change was to augment the sheet of spray, so the opposite tack was next taken, that of making the V sections hollow in wake of the position from which the sheet of spray originated,

and 1591-3 was thus derived. The result was that the spray was held down, the planing effect increased and the resistance reduced, an all round improvement.

Model 1592-1 was made from the lines of the Navy Flying Boat C-1.

Model 1593-1 was made from the lines of the Navy Flying Boat D-1.

Model number 1602-1 was derived from 1591-3, but the beam was increased from 30" to 34", otherwise the bottom was the same, at the same time an attempt was made to improve the form by changing the form of the front hood, and by sloping the upper deck abaft the position of the planes. The results of these changes are apparent in the submerged runs.

Model 1592-5 was derived from 1592-1 by adding a shallow V bottom just forward of the step of 1592-1.

Model 1617-1 was designed on the general lines of the E-1, combination type (Owl, type). The object of this experiment was principally to determine whether the shorter form was disadvantageous from an air resistance point of view.

An inspection of the performance of 1591-3, shows that from a resistance point of view it excels all but 1592-5, and an inspection of the change of level curves will show this to be intimately associated with the valuable " planing " qualities of 1591-3.

1592-1 behaves very similarly but the resistance is higher, while 1592-5 behaves very similarly but the resistance is lower than either of the two preceding and the change of level curves clearly demonstrate that the improvement is due principally to the improved " planing " effect, and it is also due to the better flow induced by parting rather than pushing the water aside, due to the V-shape at the step.

1602-1 behaves much the same, but the broader beam appears to increase the resistance slightly except where the planing effect reaches its maximum:

1593-1 behaves well at low speeds, but the resistance grows to a maximum at a much higher speed than any of the other models and falls off steadily but much less sharply than any of the others. The hump for the resistance curves occurs at about 27 m. p. h. for this model; at about 21 m. p. h. for 1591-3, 1592-1 and 1602-1, and at 19.5 m. p. h. for 1592-5. The sustained hard running of this model is clearly due to its failure to " plane " as is evidenced by the change of level curve.

1617-2 has high resistance at speeds corresponding to 24 m. p. h., but in general behaves similarly to the other central step models.

Comparison of the model results with the actual performance of full sized machines, show a fair analogy exists, confirming satisfactorily the behavior of the models. Certain experiments indicate that up to about 15 or 20 m. p. h. the aeroplane controls have very little influence on the change of trim of the full size machines, and thus practically require the full size machines to follow the trim imposed by the flow of the water about the hulls, and the models were set to closely approximate the "natural trim." Once planing is attained, or the same thing once "suction" is broken, the controls become effective and may be used to modify the trim. In the case of D-1, however, this condition is not reached till about 39 m. p. h. These experiments have also shown that the "planing" effect is very sensible to improvement if the angle of the bottom is increased, and as this can be brought about once planing is attained this shows a further advantage for those models which plane early.

The conclusions drawn from these and previous experiments are as follows:

a. The step should be close to the position of the center of gravity, to eliminate a nosing tendency, to facilitate change of trim while planing, to avoid change of balance when getting away or landing.

b. Hollow V sections keep the spray down, cut the water more easily and cleanly, plane better, and greatly reduce shock on landing or when ploughing through broken water, and practically eliminate the necessity of shock absorbers.

c. A shallow step is sufficient, but ventilation is essential to facilitate the breaking of suction effects.

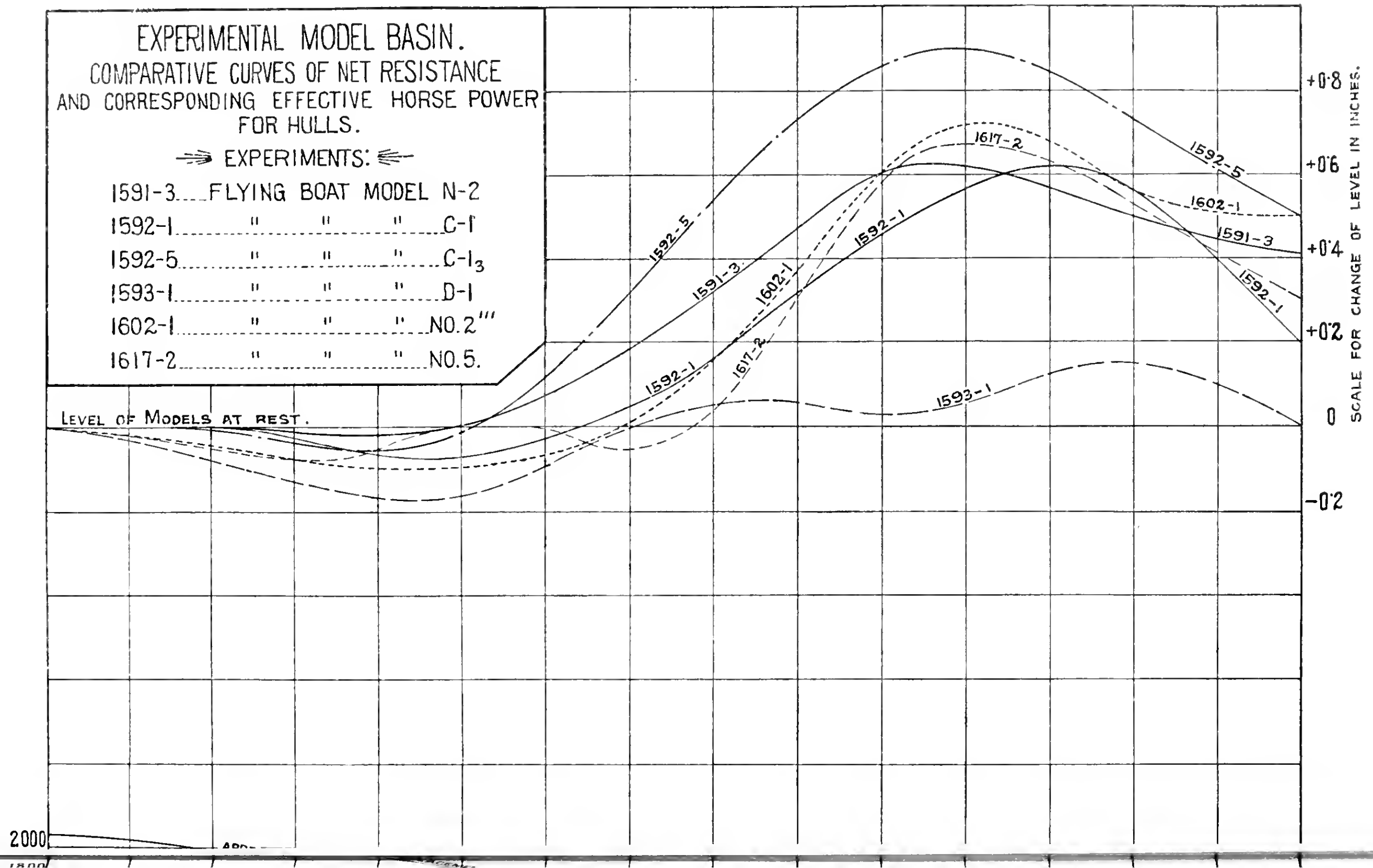
d. The bottom forward of the step should be inclined to the axis of the machine, but

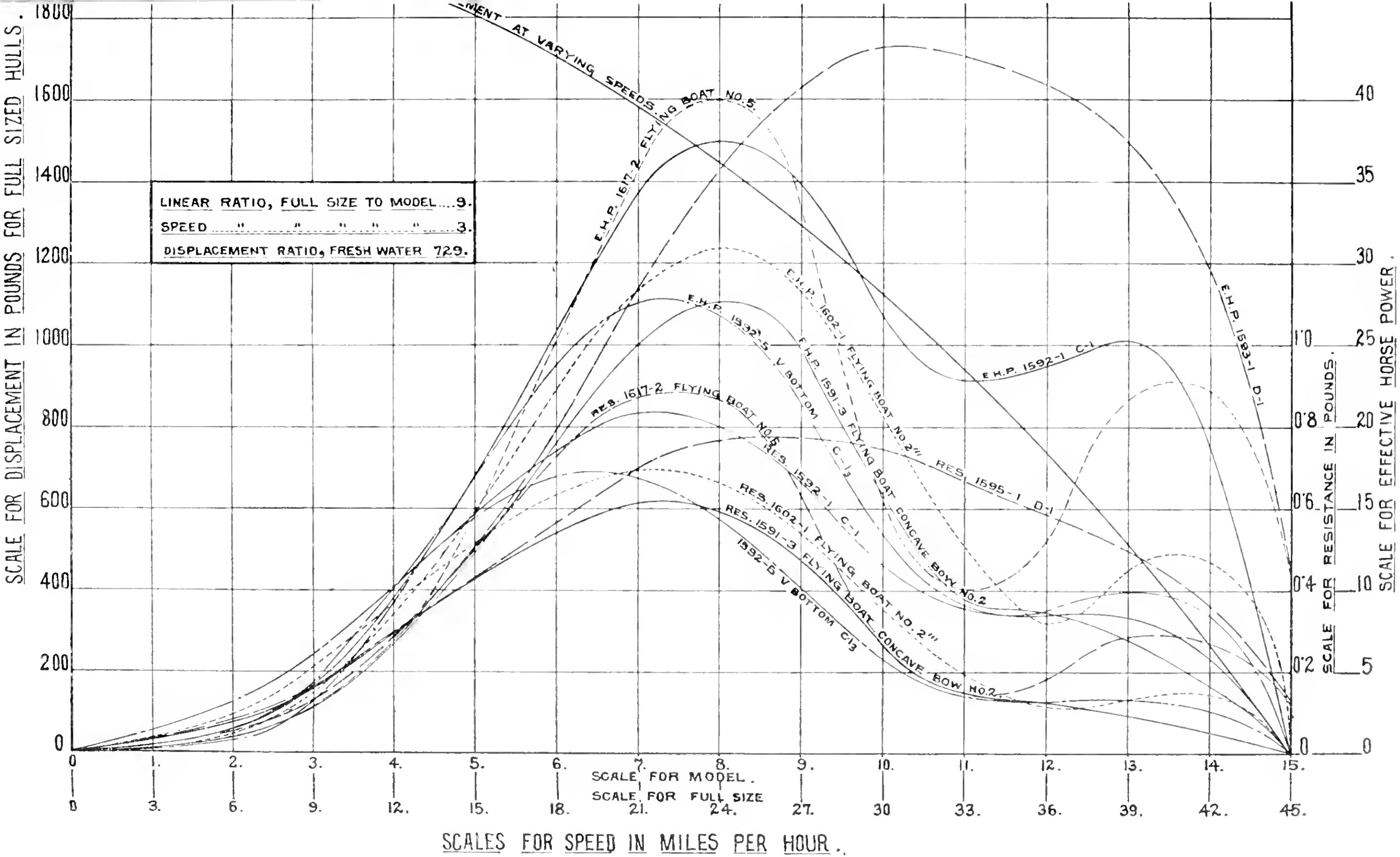
e. The inclination must not be so great as to cause planing before the controls are effective, and this is particularly necessary when running before the wind. If the planing of the hull is too pronounced, the machine rises to the surface with but very little control available to maintain balance, and when running before the wind this is more apt to occur due to the higher water speed necessary before the machine can take the air.

f. The bottom abaft the step should rise strongly as this favors a steepening of the planing bow before suction is eliminated, and gets the tail well clear when planing begins.

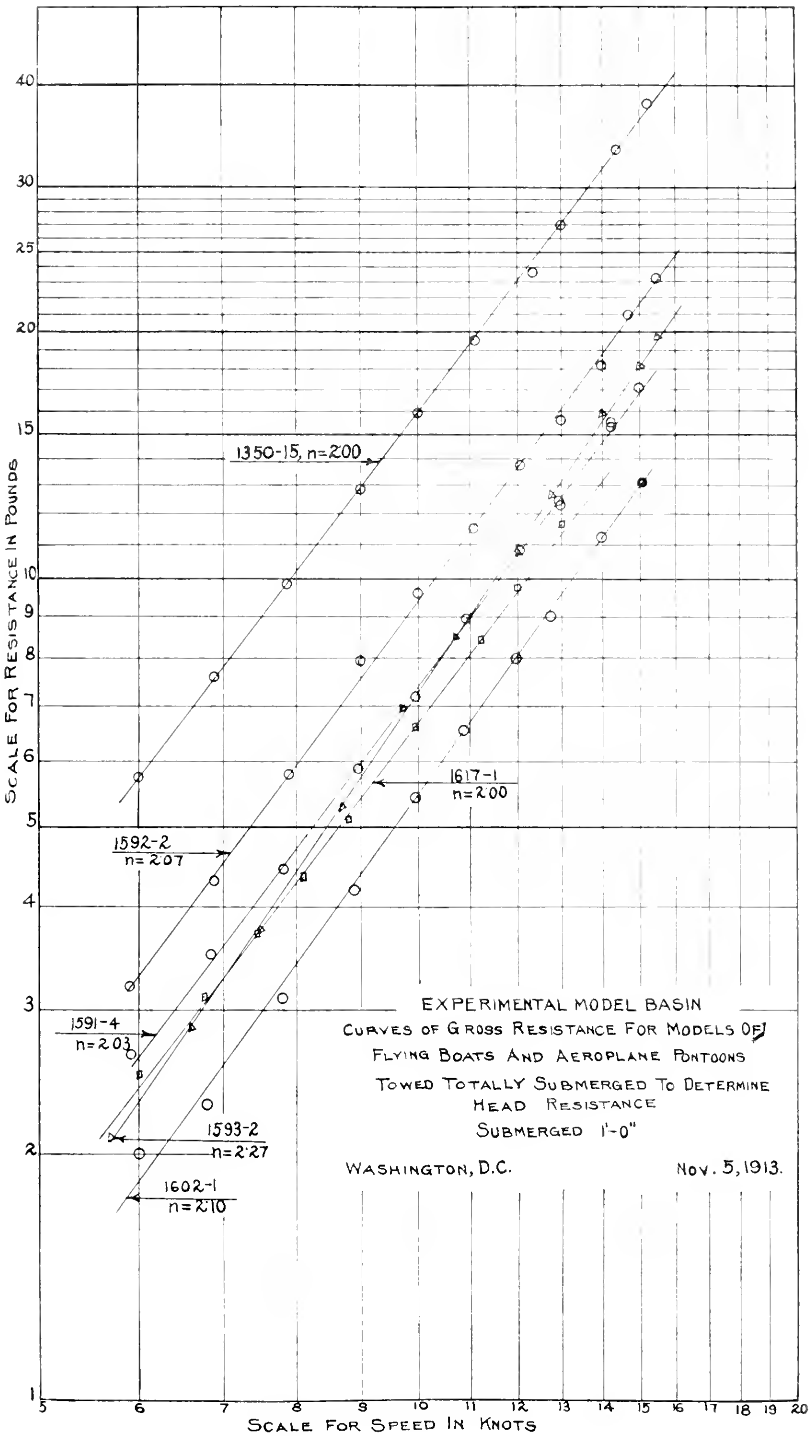
PLATES 2 AND 3

Plate 2 shows the logarithmic plots of the resistance of the preceding models towed submerged at speeds up to 15 knots; and also for model 1350-15, a quarter sized model of the original Curtiss


EXPERIMENTAL MODEL BASIN.
COMPARATIVE CURVES OF NET RESISTANCE
AND CORRESPONDING EFFECTIVE HORSE POWER
FOR HULLS.
EXPERIMENTS:
1591-3 FLYING BOAT MODEL N-2
1592-1 " " " C-1'
1592-5 " " " C-1₃
1593-1 " " " D-1
1602-1 " " " NO. 2'''
1617-2 " " " NO. 5.
LEVEL OF MODELS AT REST.
SCALE FOR CHANGE OF LEVEL IN INCHES.
+0.8
+0.6
+0.4
+0.2
0
-0.2
1592-5
1617-2
1602-1
1591-3
1592-1
1593-1
2000



LINEAR RATIO, FULL SIZE TO MODEL ... 9.
SPEED " " " " " ... 3.
DISPLACEMENT RATIO, FRESH WATER 729.
...MENT AT VARYING SPEEDS
E.H.P. 1617-2 FLYING BOAT NO.5
E.H.P. 1602-1 FLYING BOAT NO.2'''
E.H.P. 1592-5 V BOTTOM
E.H.P. 1591-3 FLYING BOAT CONCAVE BOW NO.2
E.H.P. 1592-1 C-1
E.H.P. 1593-1 D-1
RES. 1617-2 FLYING BOAT NO.5
RES. 1592-1 C-1
RES. 1595-1 D-1
RES. 1602-1 FLYING BOAT NO.2'''
RES. 1591-3 FLYING BOAT CONCAVE BOW NO.2
1592-5 V BOTTOM C-1/3
SCALE FOR DISPLACEMENT IN POUNDS FOR FULL SIZED HULLS.
SCALE FOR RESISTANCE IN POUNDS.
SCALE FOR EFFECTIVE HORSE POWER.
SCALE FOR MODEL.
SCALE FOR FULL SIZE
SCALES FOR SPEED IN MILES PER HOUR.


EXPERIMENTAL MODEL BASIN
Curves of Gross Resistance for Models of Flying Boats and Aeroplane Pontoons Towed Totally Submerged to Determine Head Resistance
Submerged 1'-0"
Washington, D.C.
Nov. 5, 1913.
1350-15, n=2.00
1617-1 n=2.00
1592-2 n=2.07
1591-4 n=2.03
1593-2 n=2.27
1602-1 n=2.10
Scale for Resistance in Pounds
Scale for Speed in Knots
1
2
3
4
5
6
7
8
9
10
15
20
25
30
40
5
6
7
8
9
10
11
12
13
14
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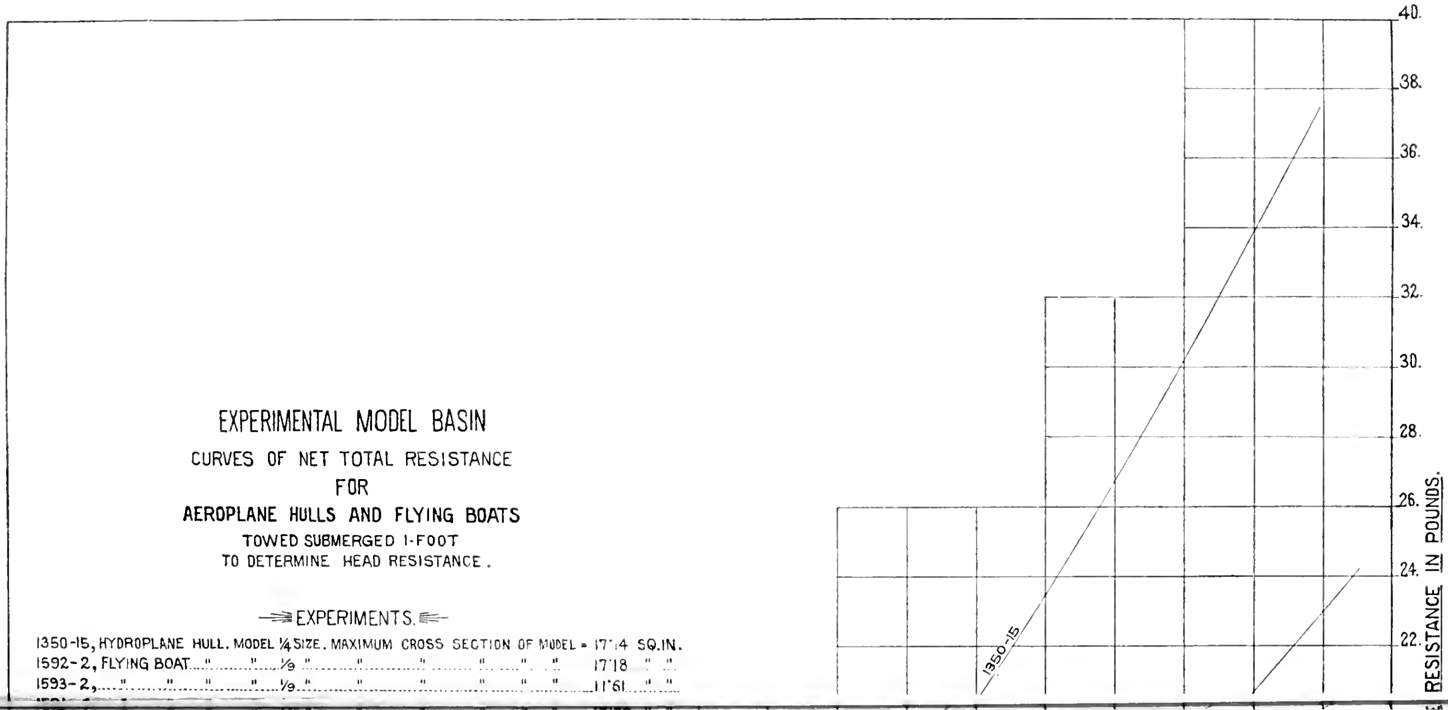
16
17
18
19
20



EXPERIMENTAL MODEL BASIN

CURVES OF NET TOTAL RESISTANCE

FOR

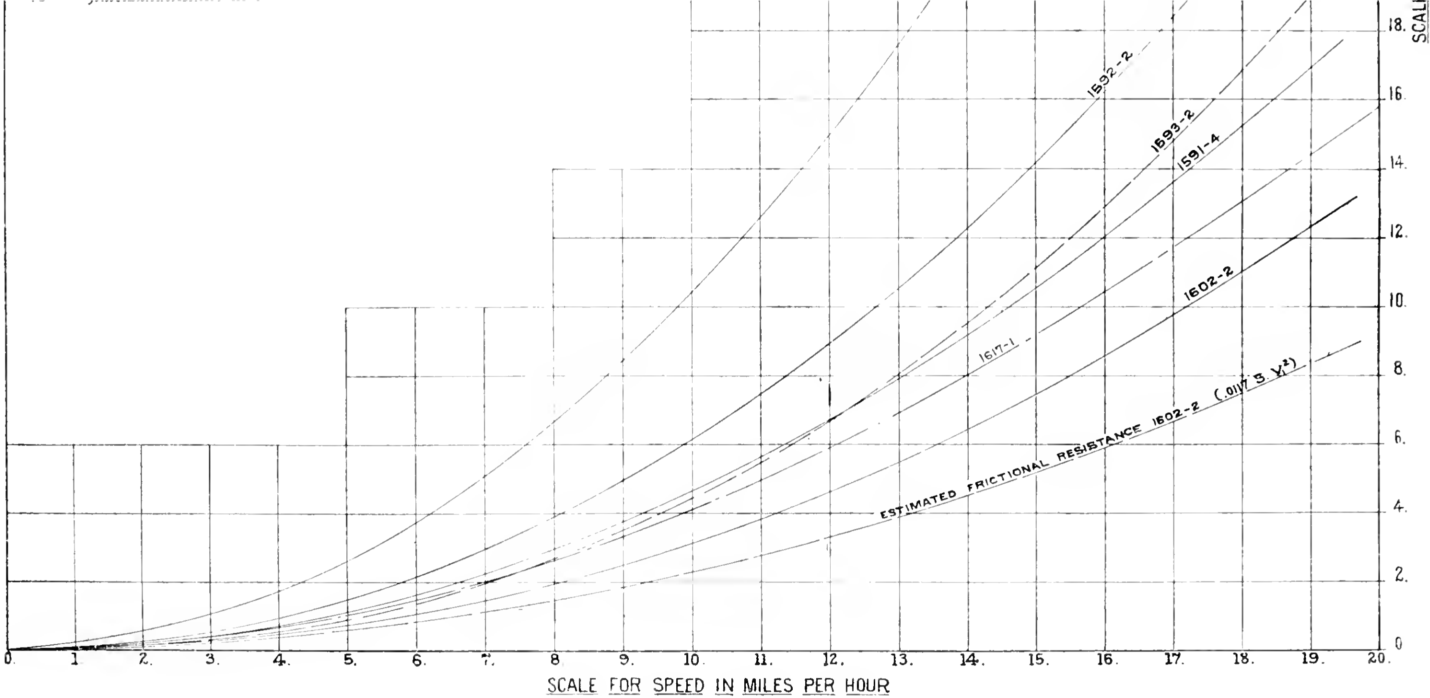
AEROPLANE HULLS AND FLYING BOATS

TOWED SUBMERGED 1-FOOT

TO DETERMINE HEAD RESISTANCE.

EXPERIMENTS.

1350-15, HYDROPLANE HULL. MODEL ¼ SIZE. MAXIMUM CROSS SECTION OF MODEL = 17·14 SQ.IN.
1592-2, FLYING BOAT " " 1/9 " " " " " 17·18 " "
1593-2, " " " " 1/9 " " " " " 11·61 " "


1592-2
1693-2
1591-4
1617-1
1602-2
ESTIMATED FRICTIONAL RESISTANCE 1602-2 (.0117 S. V²)
SCALE FOR SPEED IN MILES PER HOUR
SCAL

pontoon. It will be seen that a straight line on the logarithmic plots very closely represents the locus of the observed points and thus indicate that the resistances of the models closely approximate the law of the square of the speed. As is well known, any equation of the form $Y = a\ x^n$ will plot as a straight line on a logarithmic plot, and the slope of the curve transferred to the origin passes through the margin at the upper end at a point corresponding to the exponent n. The exponents are given in line 12 of Table 1, which shows the value of the exponent n in the equation of the lines plotted on Plate 3 from the equation $R \propto V^n$ the points being taken direct from the straight line plots.

TABLE I

Table 1 shows the computation of the head resistance in water for each of these models, in detail, the final results appearing in lines 19, 20 and 21, giving results by three different methods of computation.

TABLE II

Line 22 gives the head resistance computed by analysis of the total resistance of the model into frictional resistance and residual resistance, and then augmenting these values to the "full size" values in accordance with Froude's method.

Line 23 gives the head resistance in the same manner as line 22, except the residual resistance is determined on the basis of relative areas and relative speeds, being proportioned to the latter in accordance with the exponent determined for the variation of residual resistance of the model, instead of using the law of the square.

Line 24 is an *approximation*, assuming the resistance to be directly proportional to the cube of the linear ratio at "corresponding speeds."

Line 25 is computed on the basis of the Lord Rayleigh method, which has been found reasonably satisfactory for the comparison of dirigible models in England. In England ebonite models 1 inch in diameter were used in water, and in air gold beater's skin models 3 feet in diameter.

The National Physical Laboratory formula, which is based on the Lord Rayleigh method, is

$$H = \kappa \rho \nu^{.14} L^{1.86} V^{1.86}$$

in which κ is a constant of form to be derived by experiment, ρ is the density of the medium in which the experiment is carried on, ν is the kinetic viscosity, L is the length in feet, and V is the velocity in feet per second.

This method has been introduced at the suggestion of Naval Con-

TABLE I.

Computation of Head Resistance of Aeroplane Hulls, from Resistances of Models Towed Submerged in Model Basin.

Model	A-1	C-1	D-1	N-1	N-2	N-3
1. Number	1350-15	1592-2	1593-2	1591-4	1602-2	1617-2
2. Linear ratio, full size to model .... K	4	9	9	9	9	9
3. Wetted surface of model in sq. ft. .... s	5.95	2.440	2.488	2.688	2.618	2.144
4. Wetted surface, full size, in sq. ft. .... $K^2s=S$	95.25	197.5	201.5	218.0	212.0	173.6
5. Maximum section of model in sq. ft. .... $=a$	.118	.110	.081	.103	.116	.146
6. Maximum section, full size, in sq. ft. .... $K^2a=A$	1.91	9.65	6.55	8.3	9.4	11.86
7. Speed of model, m. p. h. .... $=v$	20	20	20	20	20	20
8. Speed of model, k. p. h. .... $=v'$	17.38	17.38	17.38	17.38	17.38	17.38
9. Corresponding speed, full size, m. p. h. .... $\sqrt{K}v=V$	40	60	60	60	60	60
10. Corresponding speed, full size, k. p. h. .... $\sqrt{K}v'=V'$	34.76	52.14	52.14	52.14	52.14	52.14
11. Total resistance of model at v m. p. h. .... $=r_t$	41.25	25.4	21.2	18.65	13.65	15.8
12. Exponent of v with which r_t varies .... n	2.0	2.07	2.27	2.03	2.1	1.96
13. Frictional resistance of model $.0117sv'^2$ .... $=r_f$	21.0	8.63	8.8	9.54	9.25	7.58
14. Residual resistance of model .... $r_t-r_f=r_r$	20.25	16.77	12.4	9.11	4.4	8.22
15. Exponent of v with which r_r varies .... n'	2.0	2.06	2.48	2.00	2.28	1.91
16. Residual resistance of full size at V m. p. h. .... $r_r\times K^3=R_r$	1,295	12,250	9,050	6,630	3,210	5,992
17. Residual resistance of full size at V m. p. h., $r_r\times\left(\frac{A}{a}\right)\times\left(\frac{V^{n'}}{v}\right)=R'_r$	1,296	13,060	15,490	6,570	5,183	5,430
18. Frictional resistance at V m.p.h., full size ... $.01\times S\times V'^{1.85}=R_f$	672	2,970	3,025	3,270	3,180	1,506
19. Total resistance at V m. p. h., full size .... $R_r\times R_f=R_t$	1,967	15,220	12,075	9,900	6,390	7,498
20. Total resistance at V m. p. h., full size .... $R'_r\times R_f=R'_t$	1,968	16,030	18,515	9,840	8,363	6,936
21. Total resistance at V m. p. h., full size .... $K^3r_t=R''_t$	2,640	18,500	15,450	13,600	9,950	11,518

TABLE II.

	Model	A-1	C-1	D-1	N-1	N-2	N-3
1.	Number	1350–15	1592–2	1593–2	1591–4	1602–2	1617–2
19.	Total resistance in water, Table I ..... $R_r+R_t=R_t$	1,967	15.220	12.075	9.900	6.390	7.498
20.	Total resistance in water, Table I ..... $R'_r+R_t+R'_t$	1,968	16.030	18.515	9.840	8.363	6.936
21.	Total resistance in water, Table I ..... $K^2r_t=R''_t$	2,640	18.500	15.450	13.600	9.950	11.518
22.	Resistance in air, full size, at V m. p. h. ..... $R_t/813=H_1$#	2.42	18.72	14.85	12.18	7.86	9.22
23.	Resistance in air, full size, at V m. p. h. ..... $R'_t/813=H_2$#	2.42	19.72	22.77	12.15	10.29	8.53
24.	Resistance in air, full size, at V m. p. h. ..... $R''_t813=H_3$#	3.24	22.75	19.00	16.73	12.24	14.17
25.	Resistance in air, full size ..... $K\rho v.^{14}V^{1.86}L^{1.86}$(Rayleigh)$=H_4$#	3.48	20.56	17.16	15.09	11.05	12.78
26.	Resistance in air of normal plane of equal maximum section, $.0033AV^2=H_5$#.	10.10	114.5	77.7	86.5	111.6	141.0
27.	Ratio in per cent ..... $H_4/H_3=$	107.2	90.3	90.3	90.00	90.3	90.25
28.	Fineness coefficient, assuming H_4 as best value ..... $H_4/H_5=F$	.34	.179	.221	.156	.0991	.0906
29.	Excess resistance at 60 m. p. h., over 1602–2 ..... $=E$	*	9.5#	9.5#	4.04#	......	1.74#
30.	($K\rho v.^{14}$) from model tests, "form factor" ..... $=F'$	.0000258	.0000364	.0000284	.0000239	.0000183	.0000261

* Model of radically different form from others.

structor J. C. Hunsaker, in commenting on the proofs of this paper as originally written. He further suggested the comparison with line 24, which is of peculiar interest as shown by the percentage relation of these values. Investigation of the relation of the two methods as per line 27, shows that if we confine the use of the "approximate" method to the "corresponding speed," as per the Law of Comparison, the values as determined by Lord Rayleigh's method should be 90.25% of the values attained by the approximate method, for models one-ninth the full size, and 107% for models one-quarter the full size, so that the approximate method which is much simpler can be used with a fair degree of accuracy if we put it in the form—

$$H_6 = .00176K^{2.79}r_t$$

$$\text{This assumes } \frac{\rho_1}{\rho_2} = .00123 \text{ and } \frac{v_1}{v_2} = 13.$$

Line 26 gives the head resistance of a plane of the maximum section according to Eiffel's coefficient for flat plates normal to the wind.

Line 28 gives a fineness coefficient based on the comparison of lines 25 and 26.

Line 30 gives the value of $\kappa\rho v^{.14}$ for each of the boat models. These "form factors" are of interest when compared with the values of the same coefficients for models of dirigibles in which the form is unrestricted by requirements such as enter into the flying boat problem. Thus, to make the comparison more ready, Table III is compiled:

TABLE III.

Model	Type	$\kappa\rho v^{.14}$ (Air)
N. P. L.	Dirigible	.0000152
Beta	do.	.0000164
Gamma	do.	.0000165
B. F. 36	do.	.0000142
Lebaudy	do.	**.0000124**
B. F. 32	do.	.0000140
A-1	Pontoon	.0000258
C-1	Flying boat	.0000364
D-1	do.	.0000298
N-2	do.	**.0000183**
N-3	Owl	.0000261

It thus appears that the N-2 form, while superior in the air to the other flying boat forms, may still be improved, and if the efficiency of the Lebaudy form could be approached its head resistance might be reduced to 68% of the present value.

However, when we come to consider that the total head resistance of the N-2 model is only about 11# in air at 60 m. p. h., and consider

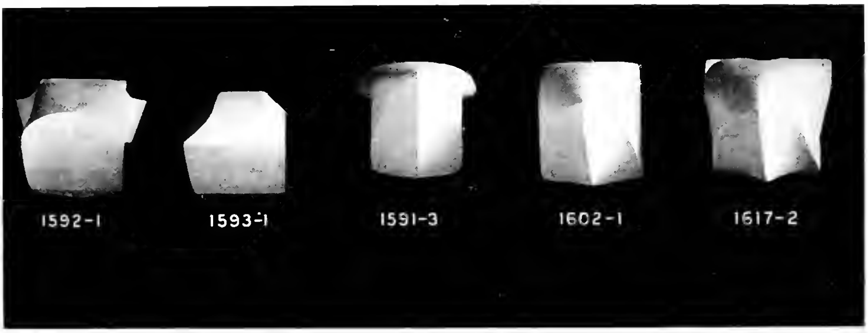


FIG 1.—BOW END OF MODELS

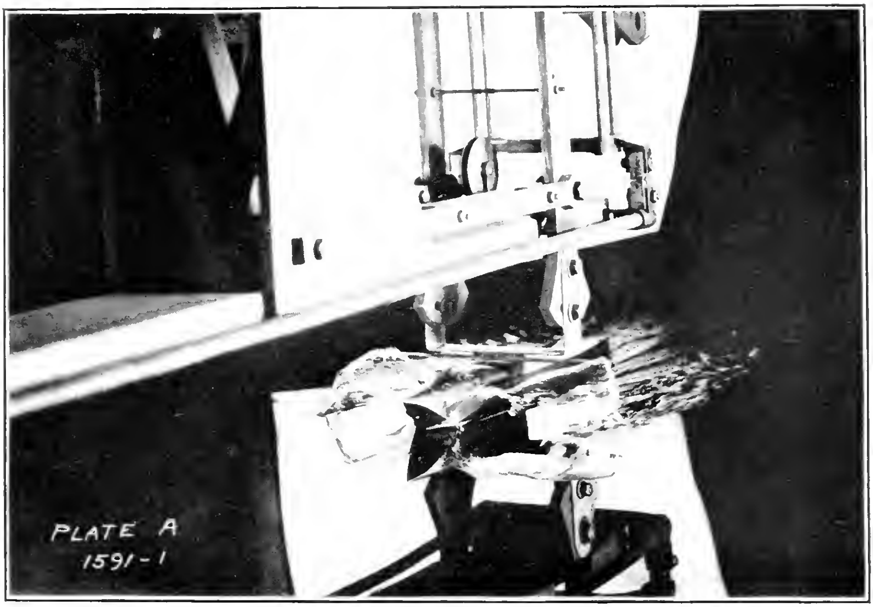
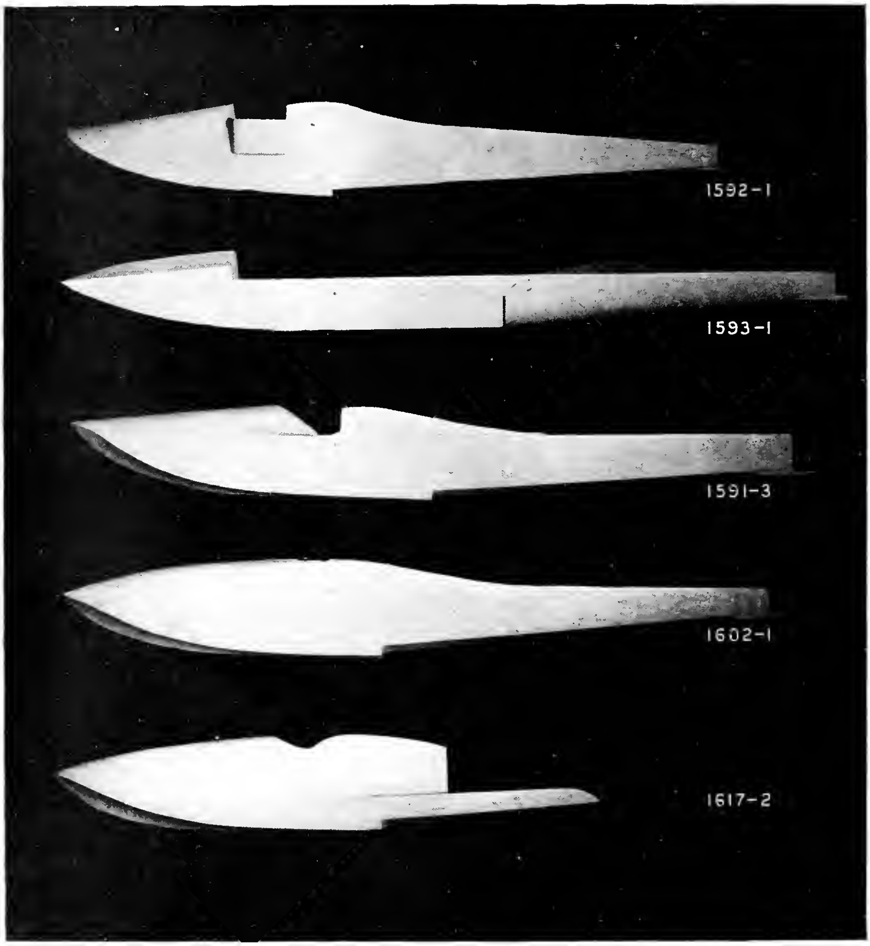
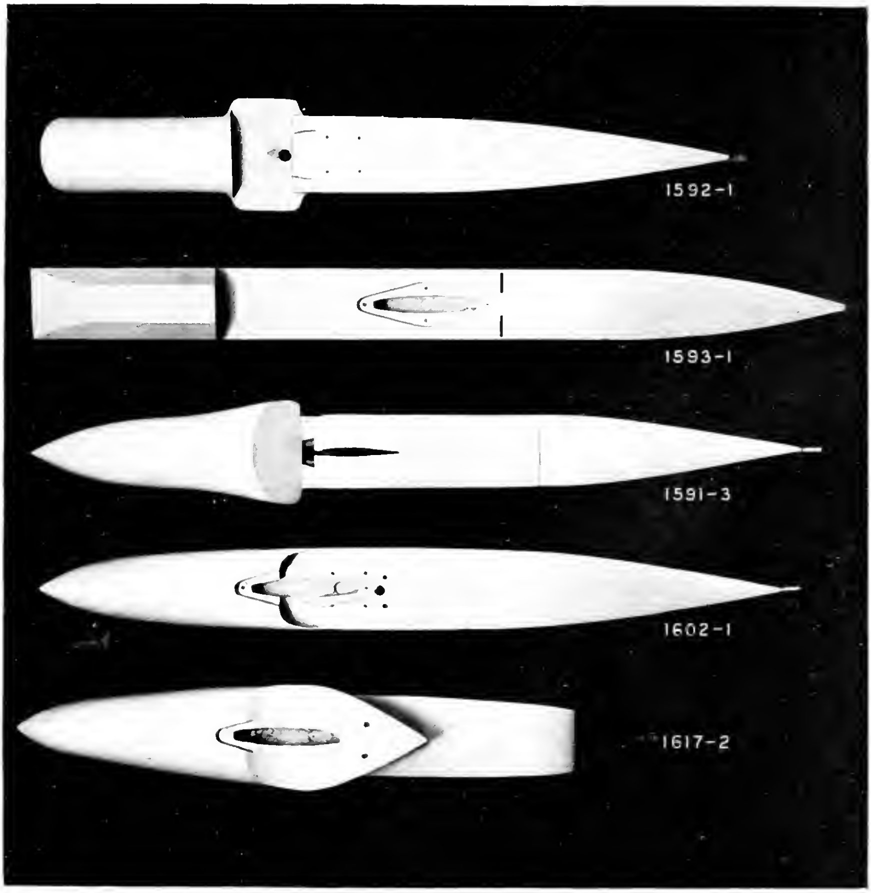


FIG. 2.—SHEET OF SPRAY MADE BY MODEL AT SPEED OF 5.5 M. P. H.



SIDE VIEW OF MODELS



VIEW OF MODELS FROM ABOVE

the difficulty of construction involved, particularly if the surface running qualities are to be retained, we see that the present forms are reasonably satisfactory. While the possible saving of 3.5# head resistance is worth considering, it must only be considered if its attainment does not involve increased weight, cost or difficulty of construction to such a degree as to outweigh the small gain possible. Such savings increase in importance in proportion to the square of the speed desired. It thus appears that increased efficiency must be aimed at in those members of the structure which offend to a greater degree than the hull, namely, the multiplicity of the truss members; and the exposed power plant, especially the water cooled power plant.

The peculiar form of Model 1617-1 is due to an attempt to utilize the advantage of the flying boat arrangement of bottom and step, together with a good shape stream line hood in place of the ordinary pontoon with the hydro-aeroplane type of machine.

It is interesting to note that the coefficient of fineness of this model is less than that for Model 1602-2 which indicates that per unit of area of maximum section the resistance of this form is slightly less than that of the 1602 model. An inspection of line 29 will show that the probable reason for this is due to the very low value of the frictional component of the resistance of this model. However, when the comparison is based on $\kappa\rho v^{14}$, the form factor used by Lord Rayleigh, this form is much coarser than the 1602 model.

An independent experiment is worthy of note at this time. An experiment was made to determine the existence and amount of "nosing" torque on model 1350, at various angles of incidence. Unfortunately the apparatus carried away before the experiments were completed, but it was found that there is a "nosing" torque of about 90 ft. lbs. when the deck of the pontoon is parallel with the line of flight. To this torque should be added that due to the head resistance which is approximately 5.42 lbs. $\times$ 5 ft. $=$ 26.10 ft. lbs. or a total torque due to the pontoon tending to make the machine head down of about 116 ft. lbs. This with the c. p. of the diving rudder 15 ft. abaft the c. g. would require the diving rudder to carry a negative load of about 7.75 lbs. if the machine were "balanced" for all other effects, at 60 m. p. h.

Additional experiments on submerged models are contemplated with a view to determining the stream line flow about the models as a means of arriving at improvement of form, and other experiments to determine the effects of the cockpit openings, sponsons, etc., and a more complete series for determining torque at different angles.

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REPORT ON EUROPEAN AERONAUTICAL LABORATORIES

(With Eleven Plates)

BY
A. F. ZAHM, Ph. D.

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REPORT ON EUROPEAN AERONAUTICAL LABORATORIES

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(With Eleven Plates)

GENERALITIES

Places visited.—During August and September, 1913, in company with Jerome C. Hunsaker, Assistant Naval Constructor, U. S. N., I visited the principal aeronautical laboratories near London, Paris, and Göttingen, to study, in the interest of the Smithsonian Institution, "the latest developments in instruments, methods, and resources used and contemplated for the prosecution of scientific aeronautical investigations." Incidentally we visited many of the best aerodromes (flying fields) and air craft factories in the neighborhood of those cities, and took copious notes of our observations. We also visited many aeronautical libraries, book-stores, and aero clubs, in order to prepare a comprehensive list of the best and latest publications on aerial navigation and its immediately kindred subjects. In each of the countries, England, France and Germany, we spent about two weeks. We were made welcome at all the places visited, and thus established personal relations which should be valuable in future negotiations with the aeronautical constructors and investigators in those countries. But these incidental visits and studies, though they may prove serviceable, do not seem germane to the present report. Neither does it seem advisable to take more than passing notice of the aeronautical laboratories themselves in those manifold details which have been already published in large and comprehensive reports now accessible in the Smithsonian Library.

Organization, resources, and scope of laboratories.—The laboratories examined by us are in particular (1) the aeronautic research and test establishments of the British government near London; (2) the Institut Aerotechnique de St. Cyr, and the Laboratoire Aerodynamique Eiffel, both near Paris; (3) the Göttingen Modelver-

suchsanstalt at Göttingen; and (4) the newly organized laboratory adjoining the flying field at Johannisthal near Berlin, known as the "Deutsche Versuchsanstalt für Luftfahrt zu Adlershof."

These establishments resemble each other in some important features, but differ in others. All are devoted to both academic and engineering investigations. All are directed by highly trained scientific and technical men. The directors are not merely executives; they are the technical heads—scientists or engineers specifically qualified by superior training in aeronautical engineering and its immediately cognate branches—who initiate the researches, and assist their technical staffs in devising apparatus, interpreting results, and making systematic reports.

The establishments differ in their organization, resources, and equipments, and, to a considerable extent, in the scope and character of their investigations. Of the five institutions mentioned, the one in England and the one at Göttingen are now supported largely by governmental appropriations; and the other three are maintained by private capital, allotted as required, or accruing from fees or endowment funds. Again, the laboratories near London, at St. Cyr, and at Adlershof are practically unlimited in the scope of their researches, while Eiffel's and the Göttingen laboratory have confined their activities substantially to wind-tunnel experiments.

The aeronautical researches of the British government are in charge of the British Advisory Committee for Aeronautics, a self-governing civilian organization which was appointed by the Prime Minister of England to work out theoretical and experimental problems in aeronautics for the army and navy, and comprises twelve to fourteen expert men, under the presidency of Lord Rayleigh. This committee initiates and directs investigations and tests at the Royal Air Craft Factory, at the National Physical Laboratory, at the Meteorological Office, at Vickers Sons and Maxim's, etc. It expends, in performing its regular functions, a sum exceeding the income of any private aeronautical laboratory, and received directly from the government treasury.

The committee is primarily occupied with work for the government, but also performs researches and tests for private individuals, for suitable fees, but without guaranteeing secrecy as to the results. The work of the committee is manifold and comprehensive. Whirling-table measurements, wind-tunnel measurements, testing of engines, propellers, woods, metals, fabrics, varnishes, hydromechanic studies, meteorological observations, mathematical investigations in

fluid dynamics, the theory of gyroscopes, aeroplane and dirigible design—whatever studies will promote the art of air craft construction and navigation may be prosecuted by this committee. A detailed program and the results of actual investigations have been published in the annual report of this committee.

M. Eiffel has paid from his personal fortune all the expenses of his plant and elaborate researches, though it is understood that he may sometimes charge nominal fees for investigations made for private individuals who wish exclusive rights to the data and results obtained. The general director of the laboratory is Eiffel himself—who initiates the researches and publishes the results. He has in immediate charge two able engineers, MM. Rith and Lapresle, aided by three trained observers who are skilled draughtsmen. Two mechanics and one janitor complete the personnel. The work of the laboratory is all indoors, and is confined to researches in aerodynamics alone, or more specifically to wind-tunnel measurements and reports thereon.

The institute at St. Cyr was founded by Deutsch de La Meurth, who gave $100,000 for the original plant and has provided $3000 per year, during his life, for maintenance. It was presented by him to the University of Paris, and is now under the general direction of the professor of physics, M. Maurain, aided by a technical staff and a large advisory council of eminent engineers, scientists and officers of the university, officers of the French government and members of various clubs and aeronautical organizations. The staff comprises the director in charge and his assistant, together with such students, two or three at a time, as may come as temporary volunteers from the University of Paris.

The institute conducts large-scale experiments in the open field as well as indoor researches, makes investigations for general publication or for private interests, on payment of suitable fees, and permits private persons to conduct researches in the laboratory. The scope of the work is practically unlimited, as is the case in the English aeronautical laboratories. A special feature of the institute is its three-quarter mile long track with electric cars for tests on large screws, large models and full-size aeroplanes.

The Göttingen aerodynamical laboratory was begun as a private enterprise, but is now to be enlarged and maintained in part by financial aid of the Kaiser Foundation. The original building, with its wind-tunnel, was erected in 1908 after the plans of its director, Prof. Prandtl, of the University of Göttingen, at a cost of 20,000 marks,

supplied by the Motorluftschiff-Studiengesellschaft. Its available income is said to be $7,000 a year. The enterprise was inaugurated on a small scale because of the uncertainty, at that date, as to the practical value of such an establishment. The work of this laboratory, as in Eiffel's, has been practically limited to wind-tunnel experiments, though Prof. Prandtl has written some valuable theoretical investigations, and is reported to be undertaking large-scale experiments in the open air by use of a car on a level track, as at St. Cyr.

The Deutsche Versuchsanstalt für Luftfahrt zu Adlershof has been recently founded by the Verein Deutscher Ingeniure. The laboratory adjoins the great Flugplatz, with its two square kilometer flying field surrounded by numerous air craft factories, scores of hangars, an aero club house and a grand stand. Major Von Tschudi, a retired German officer, is general manager of the organization, which operates the flying field in the interest of all aero manufacturers and experimentalists, whether civilian or governmental. Dr. Eng. F. Bendeman is director of the laboratory and has ten assistants, comprising, among others, Dr. Fuhrman, who was formerly assistant in the Göttingen laboratory. I have not ascertained the financial resources of the laboratory; but a prelude to its present operations was a competition, involving some three score German aeronautical motors, for the Kaiser Prize and additional contributions from the country at large, aggregating in all 125,000 marks. It is understood that the laboratory is liberally supported, is unlimited in the scope of its work, and will conduct both indoor researches and field experiments similar to those at St. Cyr.

After this general view, a technical account of the foregoing aeronautical establishments may be useful.

BUILDINGS, EQUIPMENT AND OPERATION

ENGLISH AERONAUTICAL LABORATORIES

Aeronautical laboratories used by the British government.—Of the various aerotechnical plants supervised or used by the British Advisory Committee for Aeronautics, we visited the one at the National Physical Laboratory, at Teddington, and the one at the Royal Air Craft Factory, at Farnborough; but not the meteorological stations, nor the plants of private concerns working for the committee, such as Vickers Sons and Maxim.

The National Physical Laboratory, which corresponds to the U. S. Bureau of Standards, is under the directorship of Dr. R. T. Glazebrook, F. R. S., Chairman of the Advisory Committee for

Aeronautics; its engineering department is directed by Dr. T. E. Stanton; and the subdivision of this assigned to aeronautic investigation is in general charge of Mr. L. Bairstow.

The part of the National Physical Laboratory devoted exclusively to aeronautics comprises the whirling-table house, the large wind-tunnel house and the small wind-tunnel house with its liberal space for minor apparatus. The parts available for aeronautics, but not exclusively devoted thereto, are the general grounds, the large marine model tank, the ample shops for wood and metal working, the store rooms, the offices and library, the heating and lighting system, etc.

The whirling-table house, a separate building, is a corrugated iron shed some 80 feet square, having an earth floor, and at its center a motor driven vertical shaft which supports a trussed horizontal arm and causes it to whirl at any desired speed, its outer extremity describing a circle about 60 feet in diameter, and carrying in steady flight any model that has to be tested. The most important use of this whirling tabel hitherto made seems to have been to prove what was demonstrated and published in America by myself about one decade previously, viz., that a suitably designed pressure-tube anemometer is competent to measure the velocity of a uniform air current accurately to one per cent, or less,[1] and needs no calibration when its readings are interpreted in accordance with Bernouili's theorem. The whirling arm has also been used to test model screw propellers, but is not necessary for this work, and is much less convenient for the purpose than the wind-tunnel, as used by Eiffel, for example. It is therefore questionable whether the Smithsonian Institution will require a whirling table for its aerodynamic studies.

The large wind-tunnel house, a wing of the engineering laboratory, is a concrete structure 100 feet long, 40 wide and 30 high, having a wooden horizontal wind-tunnel placed equidistant from the side walls, and midway between floor and ceiling, and supported between concrete columns reaching from floor to ceiling.

The large tunnel is some 80 feet long and 7 feet square in cross-section from its mouth to its middle; it expands considerably through the rest of its length. Its larger extremity abuts against the end wall of the room, while its mouth stops well short of the opposite wall. At mid-tunnel, just aft the enlargement, is placed a low-pitch wooden screw actuated by a 30-horse electric motor, and designed to give a current of 60 feet a second in the fore part of the

[1] The calibration made at the British laboratory is reported to be reliable to one-tenth of one per cent.

tunnel. The screw sucks the air of the closed room through the mouth of the tunnel, which is somewhat flaring, thence through a metal honeycomb, into the experimental part of the tunnel where the models are placed for study. Thence the air flows into the expanded half of the tunnel, passing first through the screw, then laterally outward through innumerable holes in the tunnel wall, thence in uniform circulation through the unobstructed room till it curves again easily into the mouth at the opposite end. The air stream so produced is, where it emerges from the honeycomb, uniform in velocity at all parts of a section, at least to a fraction of one per cent, if due care be taken. The expanded and perforated part of the tunnel is said to be the final outcome of long months of trial and study by the technical staff, and has enabled them to produce the steadiest aerodynamic current in the world; thus removing one of the greatest difficulties in the accurate determination of the flow and pressure of air about wind models. The current velocity is reported to be uniform to one-half per cent both in time and in space.

The complete structure of the tunnel need not be delineated here, as it may be had better from the general plans and detailed working drawings which the director of the laboratory has kindly offered to furnish the Smithsonian Institution. It may be explained, however, that the "honeycomb," just within the flaring mouth of the tunnel, consists of crossed metal sheets forming, post-office-box-like, a tubular partition of many cells through which the air entering the tunnel is straightened and deprived of eddies. It may also be observed that a glass door is placed on the side of the tunnel, through which one may take observations, or enter to adjust the models to be tested.

The cost of the seven-foot wind-tunnel is given as about $2,000, and of its wind balance about $2,000. This, with an expenditure of $12,500 for the building, makes a total of $16,500 for the plant.

The velocity of the air flow in the unchecked current, near the model held inside the tunnel, is computed from the observed pressure difference between the inside and outside of the tunnel wall. The accuracy of this method was experimentally proved by me in 1902 at the request of the Navy Department, and, together with a mathematical proof, was set forth in the *Physical Review* the following year. It was there shown that the speed of air rushing steadily through a horizontal cylindrical tube from the quiet atmosphere of the room into a chamber at low pressure is, for ordinary transportation speeds, given truly to a fraction of one per cent by the

formula $V = \sqrt{\frac{2g}{\rho}(p_o - p)}$, $p_o - p$ being the pressure difference between the room and chamber, V the speed[1] of inrush, and ρ the nearly constant density. The method has since been adopted at Eiffel's laboratory and at the National Physical Laboratory.[2] This for the speed of flow; the direction may be shown by fine silk threads moored in the current, or by floating particles, fine streams of smoke, etc. In passing it may be mentioned that the direction of flow in the unchecked current is parallel to the tunnel walls truly to a fraction of one degree.

The pressure difference in question is found by connecting the interior of the tunnel wall by means of external nipple and hose, to one branch of a U tube manometer whose other branch opens into the quiet air of the room; then observing the difference of level of the liquid in the two arms. Manometers are made in many forms, according to the accuracy desired. The English one, known as the "Chattock tilting gauge," made public in 1903, measures barometric pressure differences truly to one five hundred thousandth of an atmosphere. My gauge, made in 1902 on a different principle, was graduated to millionths of an atmosphere and for the most accurate measurements of static pressure differences was always read to tenths of a graduation. At Eiffel's Laboratory, and at various other places, a less accurate, but somewhat simpler, manometer gauge is used. It consists merely of an inclined alcohol tube suitably mounted beside a graduated scale. The latter instrument, a long known type of gauge, I would recommend for its convenience; but where great precision is required the English gauge or mine would perhaps serve better.

[1] More strictly speaking, V is the *increase* of velocity of the air as it flows from the room into the tunnel; but as the air starts from near rest, the increase of velocity is practically the whole velocity of inflow. A considerable error may ensue if V be taken as the true speed of inflow for the case of a tunnel of goodly section as compared with that of the room. Thus for the new English tunnel the cross-section is 7 x 7 feet in a room whose section is 30 x 40 feet. Hence the average speed of flow through the room is 4 per cent of the speed through the tunnel. Hence something like 4 per cent must be added to the speed computed from the true static pressure difference in question.

[2] At the National Physical Laboratory, the velocity along the axis of the tunnel as computed from the pressure difference inside and outside the tunnel wall is corrected by use of a small calibration constant obtained by placing a Pitot tube in the center of this tunnel before the place where the models are tested.

Such gauges are equally useful for measuring the difference between some standard pressure and the actual pressure at various points on the surface of a model, or elsewhere. Thus one arm of the U tube may be connected with the internal surface of the tunnel while the other arm is connected successively to various points on the surface of a model. The difference between the standard wall pressure and that at each point of the model surface may then be plotted giving a diagram of surface pressure distribution all over the model.

The wind balance.—Besides the pressure distribution and resultant pressure on models, it is desirable to determine also the total wind force, which is composed of both the pressure and the friction of the air. To this end the English experimentalists use a bell-crank balance which is a modification of the type devised and used in my laboratory, and now employed also by Eiffel for the accurate measurement of small wind forces. The English balance consists of two horizontal weighing arms, one parallel to the tunnel, the other perpendicular, attached to a round vertical tube, or arm, supported at its center on a conical pivot just beneath the tunnel floor. The vertical arm of the bell-crank balance has its upper half extending through the tunnel floor up to the center of the current and is duly shielded by a stream-line encasing sheath, while its lower half extends downward from the pivot and dips into a pail of oil intended to damp the oscillations. The upper half supports at its extremity the wind model and near the pivot has a graduated joint, so that it can be rotated about its own axis, and thus orient the model as desired. Sliding weights on the two weighing arms are made to measure the components of wind force parallel to the flow and perpendicular thereto. If the wind force tends to rotate the balance about the vertical axis, this tendency, or wind moment on the model, is determined by observing what horizontal restraining force must be applied to one of the horizontal arms to prevent such turning. Thus the balance may be used to measure lift, drift and center of pressure. There are numerous ingenious and important details, such as those for studying stability coefficients, which can best be obtained from the British Aeronautical Committee's technical report for 1913, or from the working drawings which the laboratory has furnished the Smithsonian Institution. Though this aerodynamic balance is accurate and moderately convenient, I am of the opinion that several new types can be devised which shall be equally precise and probably more expeditions, requiring less adjustment at each

change of model. Such new types I have had in contemplation since first devising the bell-crank aerodynamic balance, in 1902.

The small wind-tunnel house, a wing of the engineering building, is of structural iron and covers rather more space than the room just described. Its chief apparatus is a four-foot wind channel for testing small models. Other apparatus in this room are an engine testing plant, now dismantled; a horizontal water channel, described in the Advisory Committee's report for 1912-13; and a small vertical tube down which tobacco smoke, formed at its top, can be sucked by an up-draft in a parallel pipe beside it having a burning gas jet in the bottom to maintain a heated column, the purpose of the descending air mingled with the smoke being to delineate the flow about models immersed therein and visible through the glass sides of the tube.

The small wind-tunnel is the working prototype of the seven-foot tunnel already described. Made of one inch lumber, it measures some 40 feet in length and is supported more than 6 feet above the floor by heavy angle iron trestle work which also forms the framing for the wooden tunnel wall. The first half of the tunnel measures 4 feet square; the second half, joined to it by an expanding metal cone, measures 6 feet square, is thickly perforated with inch square holes in its sides, and has its farther end abutting against the brick wall of the room. In the expanding cone at mid-tunnel is a low-pitch four-blade wooden screw driven by a steel shaft proceeding from a ten horse-power electric motor on a wall bracket at the large closed end of the tunnel, and capable of maintaining an air current of 40 feet per second in the four-foot tunnel. The character of the air flow and the instruments used are practically the same for the small as for the large tunnel. Some $20,000 was expended in developing and constructing this small tunnel and its appurtenances.

The small water channel, some 4 inches square in cross-section, has been used to exhibit the stream-line flow about models of ships' hulls, aeroplane posts, inclined wing forms, etc. By photographing the stream, duly dotted with tiny particles of foreign matter, clear pictures of the stream-lines and eddies have been obtained. These serve to show what forms are likely to encounter least resistance in moving through a fluid. But it can hardly be supposed that the phenomena of flow about a model slightly submerged in a shallow stream of water are identical with those for deep submergence in the atmosphere, unless for very slow speeds.

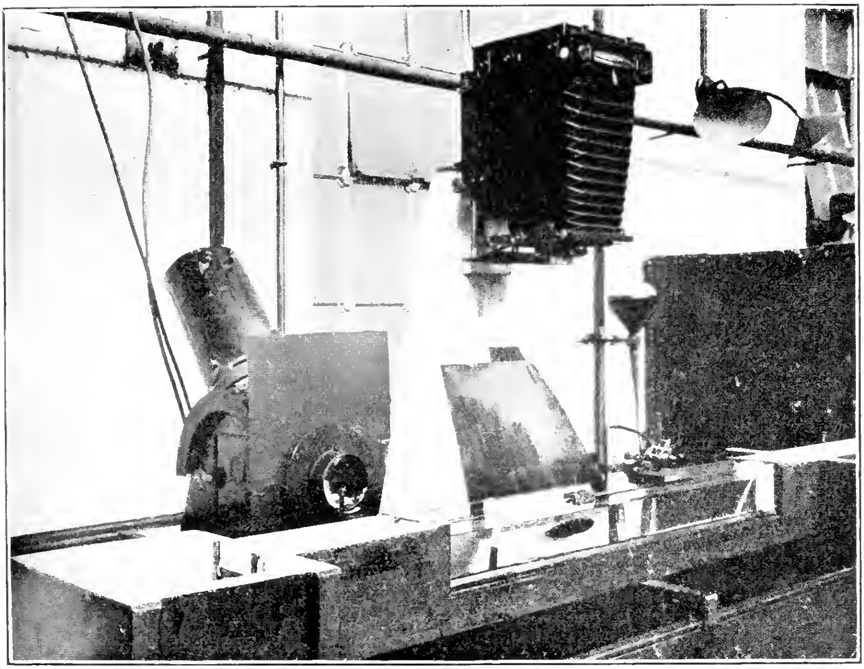
Wind towers.—On the ground to the west of the National Physical Laboratory buildings, two wind towers, each 60 feet high and provided with rotating platforms 20 feet long, are used to determine the flow and pressure of free air about large-scale models. The first results of such determinations were published by Dr. Stanton in the proceedings of the Institution of Civil Engineers for 1907, and later studies may be found in the reports of the Advisory Committee. The Smithsonian Institution can doubtless obtain a like service from the three tall radio towers in its neighborhood.

The Royal Air Craft Factory, under the direction of Mr. Mervin O'Gorman, member of the Advisory Committee, is adjacent to the headquarters and flying grounds of the Military Wing,[1] at South Farnborough. Its work is coordinated with the aeronautical researches of the National Physical Laboratory, and is professedly concerned with the scientific improvement of air craft construction, though in reality directed at times to the manufacture, on a large scale, of aeroplanes, propellers and parts of dirigibles. The factory construction and research are in charge of a civilian staff which cooperates with the Advisory Committee for Aeronautics in performing aerotechnical work for the naval and military branches of the aerial service. The close coordination of the Air Craft Factory with the Flying Corps and with the Advisory Committee is an obvious advantage to the progress of aerotechnics, which might be still further enhanced if all the experimental plants were in one locality as proposed for the United States.

Apparently no very sharp line separates the aerotechnical work of the Royal Air Craft Factory from that of the National Physical Laboratory. Both have a whirling table; both have an engine testing plant; both have studied the materials of construction; both design instruments. But this overlapping is not excessive. Broadly speaking, the laboratory investigates models; the factory full-scale air craft, their parts and appurtenances.

The factory investigates, develops, manufactures, and tests air craft. It is a mammoth plant, covering many acres and comprising half a dozen large buildings. It is said to expend half a million

[1] It may be noted that the entire military aerial service of England is known as "The Royal Flying Corps," and is under general supervision of the Air Committee, itself a subcommittee of the Committee on Imperial Defense. The Flying Corps comprises at present four branches: The Central Flying School; the Naval Wing; the Military Wing; the Reserve. The Advisory Committee for Aeronautics is an independent body, appointed by the Prime Minister, and receiving its appropriations directly from the Lord of the Treasury.



WATER CHANNEL AT NATIONAL PHYSICAL LABORATORY, ENGLAND

dollars per year and to employ 700 men, 400 of them working on aeroplanes. It has facilities for producing daily one complete aeroplane, excepting the engine, which at present is bought elsewhere. Its air craft are systematically tested on the great flying field nearby, bearing instruments which reveal their complete working in practical maneuver. One instrument alone, called the "ripograph," records simultaneously the angles of pitch, roll and yaw, the speed through air, the altitude, the three control movements and the time. The stress in the wires, the propeller thrust and the pressure distribution on the wings and other surfaces may likewise be recorded.[1] The establishment does in fact the work planned in the United States for both the field laboratory and the experimental air craft factory. But the Royal Air Craft Factory lacks some of the facilities planned for our plant, such as an expanse of water for testing naval aeroplanes, and the immediate accessibility of allied laboratories, workshops and other resources.

The result of the full-scale experiments has been to disclose the defects of the leading types of aeroplanes, and to indicate means of betterment. Substantial improvement has been made in the efficiency, stability, factor of safety and range of speed of the aeroplanes specially studied at the factory. The final outcome has been to produce a stable, efficient and safe biplane having a range of speed of 40 to 80 miles an hour. It is expected shortly that a standard control will be adopted after the best types have been given a comparative test. The type at present most in favor is the Deperdussin control, which rotates a wheel for warping, shoves it for elevating, and uses a foot lever for steering. Such practical full-scale work cannot be too strongly recommended for the Smithsonian Institution, especially if the army and navy will, as already intimated, furnish for such tests their typical air craft and their experienced pilots.

Reports.—The scope and character of the activities of both the factory and the laboratory can best be gathered from the annual reports of the Advisory Committee. These set forth all the work initiated by the committee, including, besides reports of experiments, all the theoretical researches, and all the summaries made by its members, in any form, whether of elaborate memoirs, translations or abstracts. The catholic character of these reports is praiseworthy, but their literary form and editing could well be improved. In this latter respect Eiffel's reports form more elegant models.

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[1] All these measuring and recording devices can be purchased from the Cambridge Scientific Instrument Co.

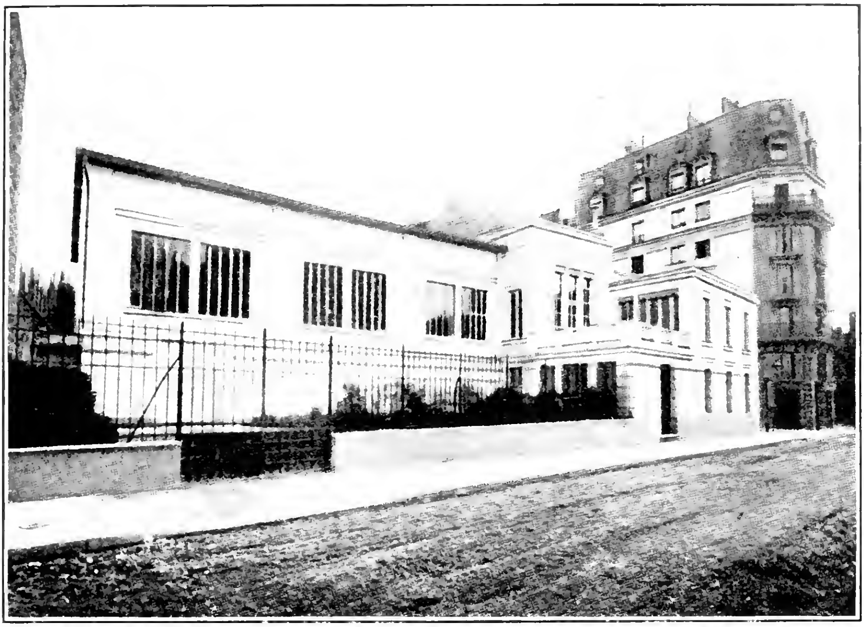
Other English aeronautical laboratories worth mentioning are those of the Northampton Polytechnic Institute, London, and of the East London College. For want of time I did not investigate these; but as their resources are very moderate and their reports have been irregular and meager, it is doubtful whether they contain any equipment materially worth adding to what has been hitherto described.

FRENCH AERONAUTICAL LABORATORIES

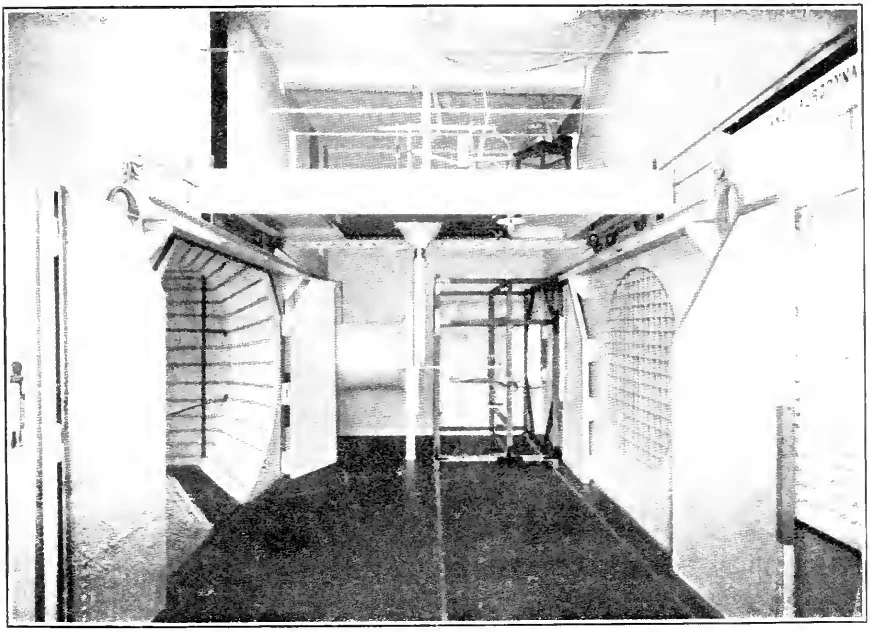
The Laboratoire Aerodynamique Eiffel consists of a single building with offices, a wind tunnel and various appurtenances, there being no workshops in the establishment. The wind-tunnel room measures, in round numbers, 40 by 100 feet, by 30 feet high; the three office rooms and garden cover about half as much additional space. Two wind-tunnels, a large and a small one, placed side by side, occupy the center of the room. They are placed well above the floor, to admit of a more nearly symmetrical flow of air. Considerable furniture—shelves, drawers, etc.—is placed about the walls; but the body of the room is kept somewhat free of obstructions to secure a less disturbed circulation.

Each tunnel comprises three main parts: the short bell-mouth intake, the model chamber, the long bell-mouth exit. The air from the room traverses the intake through honey-combs placed at either end of the bell-like form; then passes at its maximum speed in uniform rectilinear current across the model chamber; then flows in gently expanding stream and with diminishing speed onward to the larger end of the exit, where it encounters the fan which drives it with replenished energy into the open room. The model chamber is thus seen to be an enlargement of the tunnel proper, spacious enough to accommodate observers, and so sealed from the surrounding room as to have the same barometric pressure as the inflowing current at its narrowest section.

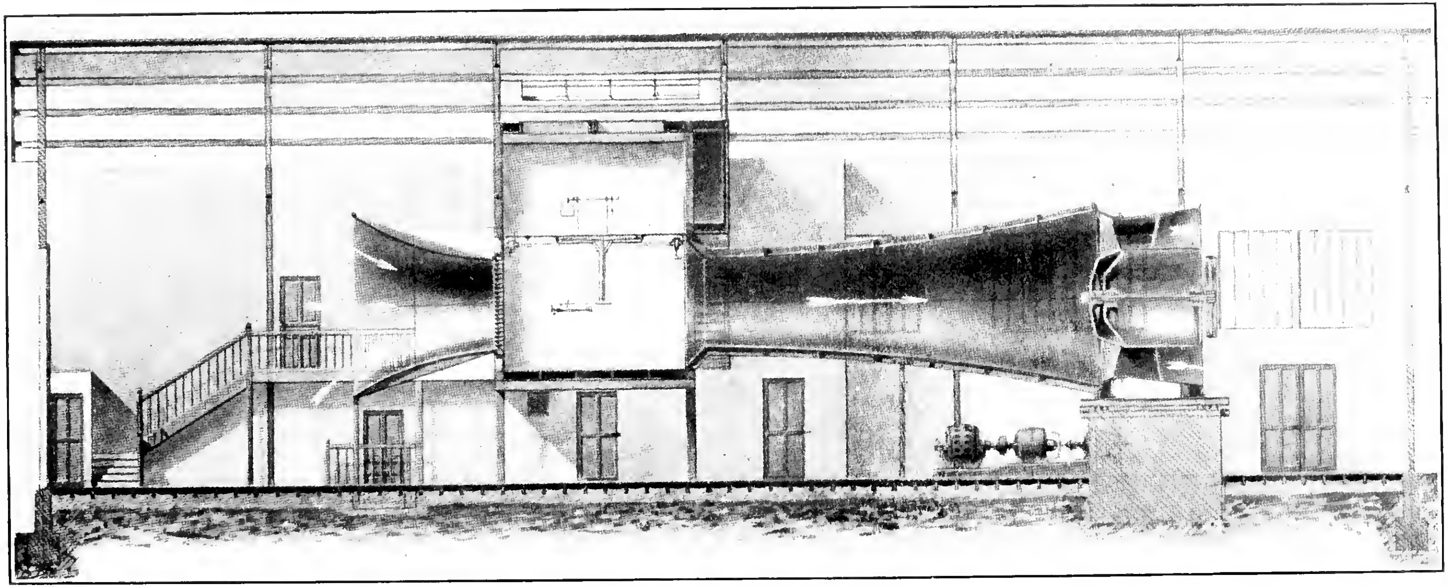
This type of tunnel, adopted by Eiffel after mature experience, has been patented by him as having features of considerable value. He prizes particularly the vacuum chamber for the observers, and for the freer flow of air about the models, uninfluenced by constraining walls. He also prizes the expanding exit, or "diffusor," for slowing the air as it approaches the fan and exhausts into the room, thus realizing great economy of power in maintaining the circulation. It is doubtful, however, whether any of the main features of Eiffel's tunnel are patentable in America. The bell-mouth entrance and exit have been known here many years. The vacuum chamber



EIFFEL AERODYNAMICAL LABORATORY



EXPERIMENT ROOM, EIFFEL AERODYNAMICAL LABORATORY



LONGITUDINAL SECTION OF THE LARGE WIND TUNNEL, EIFFEL AERODYNAMICAL LABORATORY

was employed by Mr. Mattullath and myself in our wind-tunnel constructed in 1901; was disclosed to many others then; and shortly thereafter was described in public prints.

The true function of the "diffusor," or expanding exit, seems to be to prevent turbulence, and thus to promote economy of flow, rather than to increase the pressure of the stream before it reaches the fan, as taught by Eiffel. In other words, the economy of circulation can be achieved by placing the screw at a narrower part of the exit cone, if the pitch of the blades be properly adapted to the stream at that section. But Eiffel's present arrangement presents structural advantages.

The circulation in the large tunnel is maintained by a Rateau screw suction ventilator with helicoidal blades. The screw is driven by a fifty-horse electric motor, which is found sufficient to maintain a constant flow at any desired speed up to 32 meters per second, or say up to 70 miles an hour. This is a notable result, since the air stream at its swiftest section measures two meters in diameter. Though the motor takes its current from the public mains, it requires little adjustment of the rheostat to maintain steady speed for the time of an observation, though it may vary in longer periods.

The air velocity in Eiffel's tunnel seems to be satisfactory while used for engineering studies rather than for exact researches in physics. The velocity at all points of a cross-section is uniform in magnitude to within two per cent, and varies but little in direction. A fine silk thread, however, moored in the current, plays a trifle to and fro in both the horizontal and the vertical direction. The current velocity also fluctuates in time, say 1 to 2 per cent.

The velocity is determined, as in the English and other laboratories, from the pressure difference between the vacuum chamber and the large room enclosing the tunnel. This pressure difference is measured with a Shultze manometer, or inclined tube containing alcohol and provided with a graduated scale. In ordinary practice the end of the alcohol column plays several per cent above and below a mean reading, but can easily be located on the scale to within 4 per cent by a capable observer. This means that the velocity can be determined truly to within two per cent.

For convenience, in the determination of the wind effect on the various kinds of models, Eiffel places his measuring instruments on a platform, or bridge, spanning the vacuum room, and supported on either side by wheels resting on iron rails secured to the walls, so as to be moved aside when desired. Sometimes also the models are

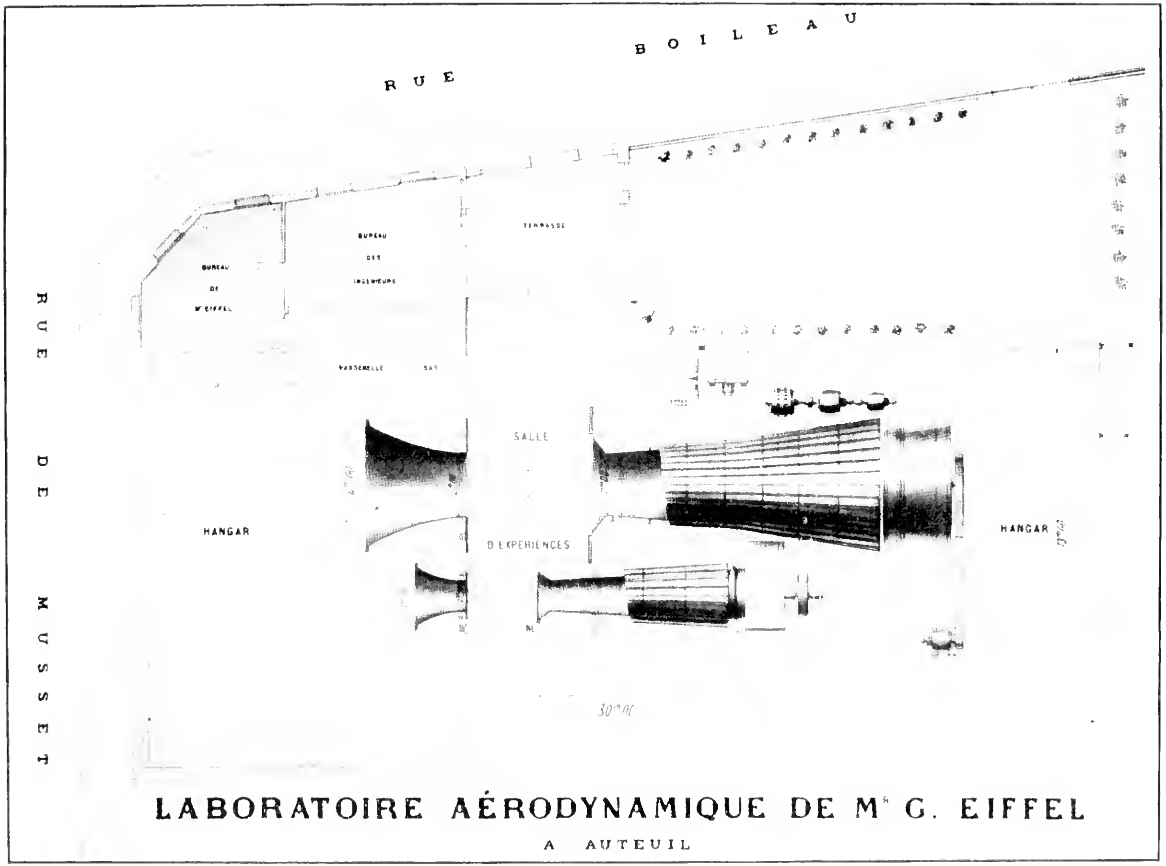
supported on a frame which can be wheeled along the floor. Thus apparatus can be adjusted outside the tunnel, quickly run into place, and again removed without dismantling. This is a unique advantage of Eiffel's arrangement. The main apparatus so employed are the aerodynamic balances, the propeller tester, and the instruments respectively for finding the distribution of pressure and the magnitude and line of action of the total wind force.

Of the *two balances* the simple bell-crank one for the precise measurement of smaller forces has been sufficiently explained as to principle in describing the English laboratory. The large aerodynamic balance, invented by Eiffel himself for determining the lift and drift of the whole wind force, and its line of action, is elaborate in theory, structure, and practical operation, and is well explained in Eiffel's book, "The Resistance of the Air and Aviation." It is not sensitive enough for measuring the smaller forces on inclined planes and on small models.

The propeller tester is elegantly simple in design and operation. A vertical electric motor, mounted on the bridge above the tunnel, and having its shaft extended down through a wind shield to the center of the air stream, there engages, through bevel gearing, with the horizontal shaft of the model propeller. The shafting of the armature and the propeller are encased in a sheathing which also contains the bearings, and transmits the propeller thrust and torque to the base of the motor. The motor, in turn, is so mounted on pivots and hydraulic gauges as to measure the thrust and torque without material displacement At the same time the motor speed is indicated by a tachometer attached to the upper end of the armature shaft. The wattmeter method, however, has lately replaced the direct method of measuring propeller torque.

The apparatus for measuring the distribution of air pressure over the surface of models has long been used by others, and in principle is like that employed in the English laboratory, and hitherto described in this report. The instrument for finding directly the line of the resultant air force, or "center of pressure," on a model surface is also an old contrivance, and need not be explained here. It is fully described in Eiffel's book.[1]

[1] It may be noted, however, that Eiffel's and the English method of allowing a model to rotate about a vertical axis by supporting it on a step bearing is not very delicate, even when a jewel step is used. A more accurate way is to suspend the body from a wire, or float it on a liquid. The writer, in 1901, discarded the jewel pivot and supported his models on a fine steel wire, an oil damper being provided to deaden oscillations. With a float no damper is needed.



PLAN OF WIND-TUNNELS

The Institut Aerotechnique de l'Université de Paris is described in sufficient detail as to its material plant and operation in its prospectus, and in the following article published in the *Engineering Magazine* for October, 1911:

The area of the site occupied is about eighteen acres. The buildings comprise a central hall, surrounded on three sides by workshops, stores, laboratories, and a power house. In the central hall will be installed experimental apparatus devoted to the study of aerial phenomena, which will include a large fan, six feet six inches in diameter, and an aerodynamic balance, whereby the pressure of a jet of air on surfaces of various shapes will be determined. There will also be an air chamber supplied by another fan wherein it will be possible to measure the strength, the center of pressure, the components, and the resultant of the reaction of a current of air at any speed up to 65 feet per second. A tunnel similar to that used by Colonel Renard will also be erected for studying the stability of models. An arrangement for measuring the friction of air on surfaces of various natures when the air is moving at all velocities, an electric dynamometer for measuring the torque of propellers fixed in position, apparatus for studying helicopter screws, and a test bench for trials on the output, endurance, and fuel consumption of aeronautical motors will also be installed. A closed chamber is to be erected, wherein the resistance of helical screws at speeds far in excess of those normally arranged for, and almost at the rupturing speed, will be investigated.

In the chemical laboratory the study of light gases, suitable for balloon work, will be carried on, and questions relating to their manufacture, purification, properties, etc., will be investigated. The chemical features of various envelope materials, the changes which occur in them under the influence of heat, light, and humidity, the properties and features of the various varnishes applied to render the material airtight and to preserve it, and similar subjects will also be studied. In the physical laboratory the instruments used in aeronautical work, the accuracy of their indications, their reliability and the modifications which are called for in their design to meet aeronautical conditions, will be investigated, while the densities and coefficients of expansion of light gases, and the best means of storing and transporting them will also receive attention.

A photographer's department has been provided next to the physical laboratory. In the workshops it will be possible to manufacture and repair all the experimental appliances required by the institution. A part of one wing is reserved for the installation of machines designed specially to test the materials employed in the construction of aircraft. In the power house, situated at the west end of the building, are two vertical compound steam engines coupled directly to dynamos supplying power and light to the entire institute.

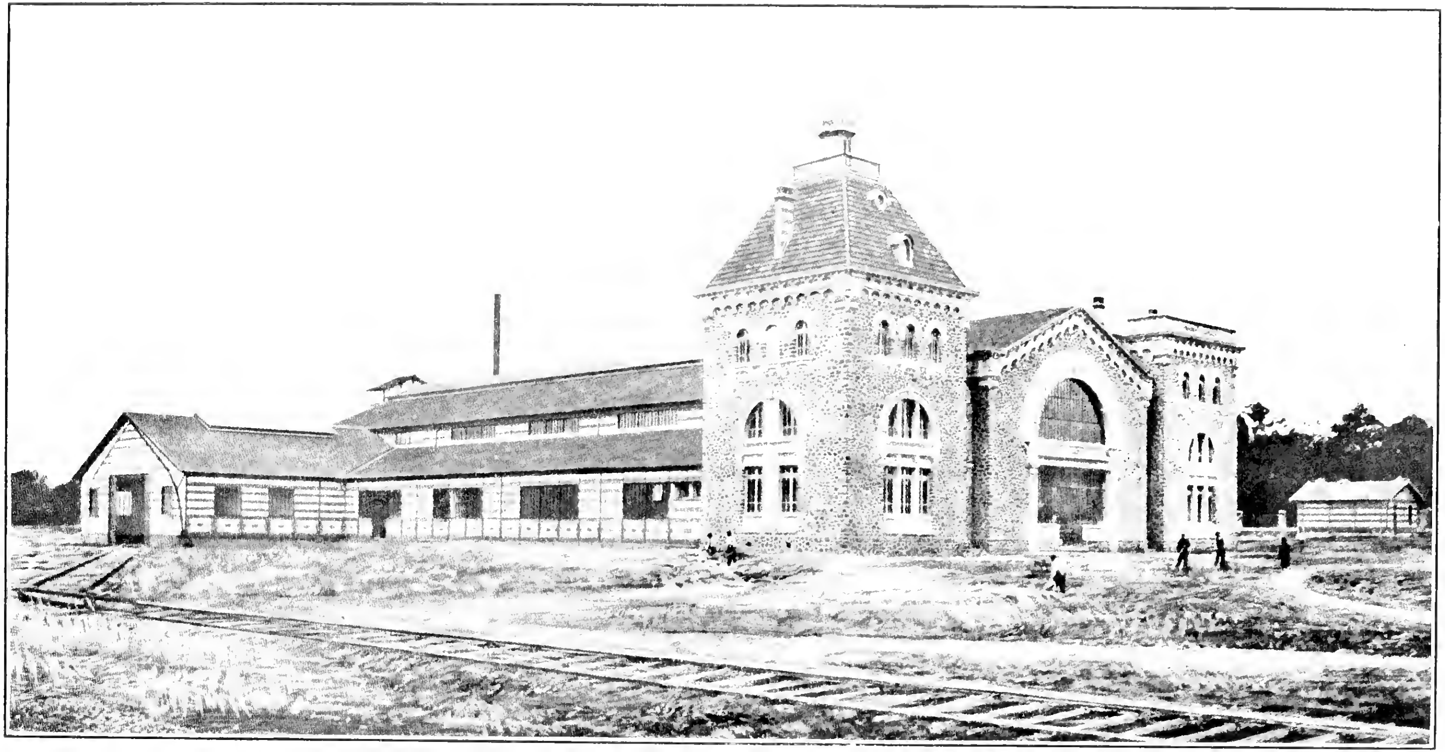
One of the most interesting features of the institute is the provision made for certain large-scale experiments with planes and propellers. To this end a long, narrow strip of ground is laid out with a normal gauge railway about seven-eighths of a mile in length. The rails are laid on oak sleepers, and are bonded in pairs by the aluminothermic process. The line is level over its entire length, with the exception of an incline at each end. At the starting point the line for a length of about 235 feet is given a slope of 1 to 100 to

facilitate the starting of the vehicles. At the terminus a slope of half this amount, but extending over about 400 feet, is provided to facilitate the arrest and return of the carriage. On each side of the line and extending along its full length is laid an electrical conductor, whereby current is fed to the motor of the carriage. The return circuit is made by way of the rails. For the last 300 feet or so of the track an additional pair of rails is laid down alongside the running rails. On these additional rails, slippers carried by the vehicle bear so that over this distance, or at least a portion of it, the carriage skates instead of rolling. This facilitates stopping, and in addition furnishes a safety device in case of emergency.

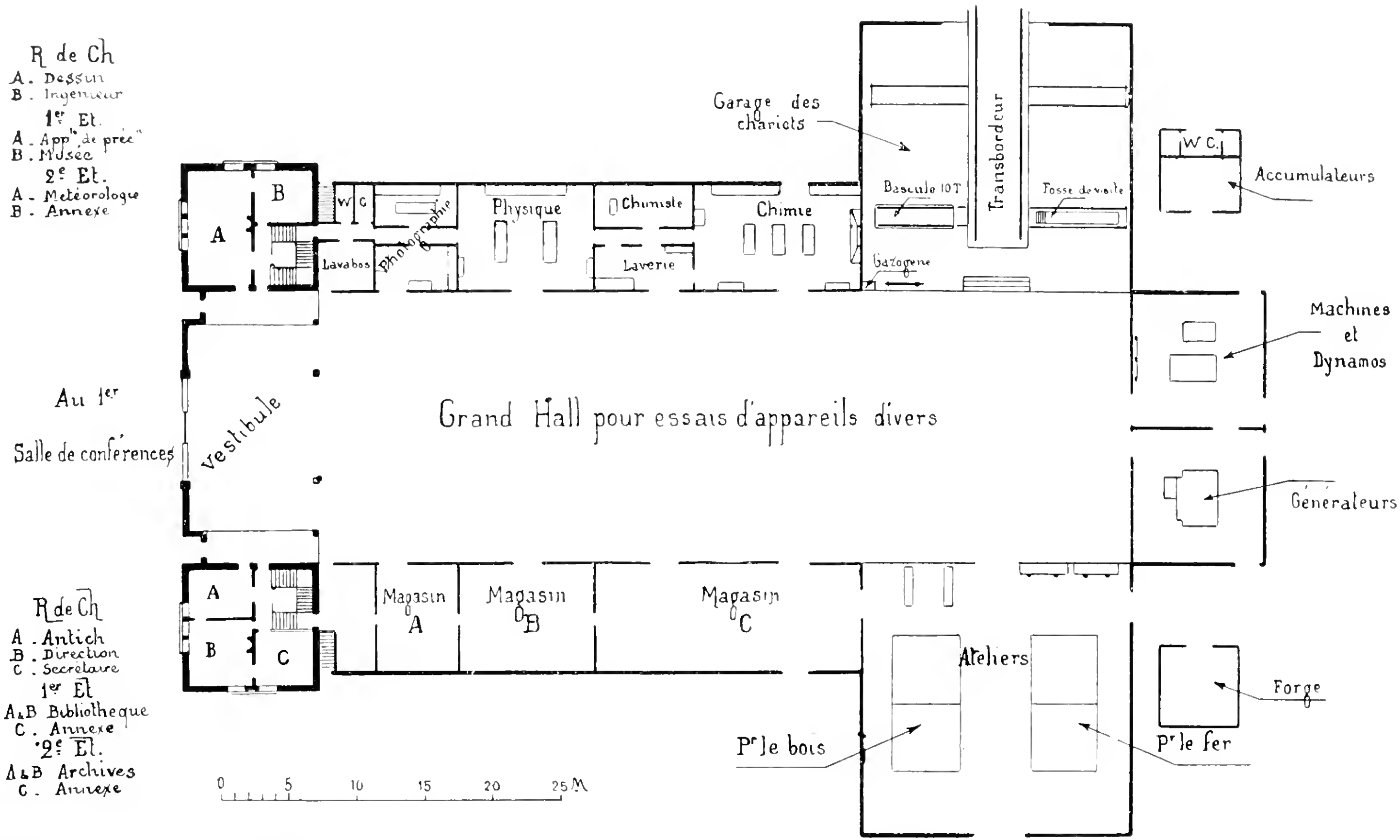
It is intended ultimately to have four electric carriages to work on the line described above. One has already been constructed, and has been used for a number of experiments. The employment of four carriages has been adopted in view of the fact that each series of experiments requires a different equipment of the carriage and different registering apparatus. If only one were used the time lost in dismantling and remounting it with each series of experiments would be very considerable. It is essential also that each vehicle should be specially designed to meet the conditions of the particular class of experiment for which it is intended. According to present intentions the first carriage will be used to measure the horizontal and vertical components and the resultant of the air pressure on surfaces of sustentation, whether plane or curved, simple or compound. The determination of the direction of the resultant, the center of pressure, its displacement when the angle of incidence is changed and the "angle of attack" will also be undertaken with this carriage. The second and third vehicles are intended for experiments on propellers or tractors, one being used for the large screws employed for dirigible balloons and the other for the smaller aeroplane screws. The tractive effort, the power absorbed and the mechanical efficiency of each type of propeller will be determined at all speeds. A further important subject of study with these two carriages will be the effect of the translational motion on the output and efficiency of the propellers. A comparison will be instituted between the efficiency, etc., of a propeller when rotating on a fixed axis and when moving with the same speed of rotation, but with various different speeds of translation. The fourth carriage will be specially equipped for measuring the resistance or "drift" of the various parts of a flying machine.

The weight of the first carriage is about three and three-fourths tons, excluding the motor, and a little less than five tons with the motor. The body of the carriage is built up of steel plates stiffened with angle irons and measures twenty feet in length and six feet six inches between the longitudinal members of the frame. Current is supplied to the motor by means of two pairs of sliding contacts carried in the side of the truck. The movement of the carriage is controlled from a lookout-post commanding the whole line.

All the carriages will be furnished with appropriate measuring instruments. A chronograph will register the number of revolutions of the wheels in a given time, from which the speed will be deduced. In addition there will be a direct speed recorder registering the value of ds/dt at each instant of the travel. A recording watt meter will register the power furnished to the motor either on a time or a distance basis. One or more recording dynamometers will also be carried whereby the particular data being determined will be measured. The efficiency of the whole plant at all speeds, the frictional



THE AEROTECHNIC INSTITUTE OF SAINT-CYR



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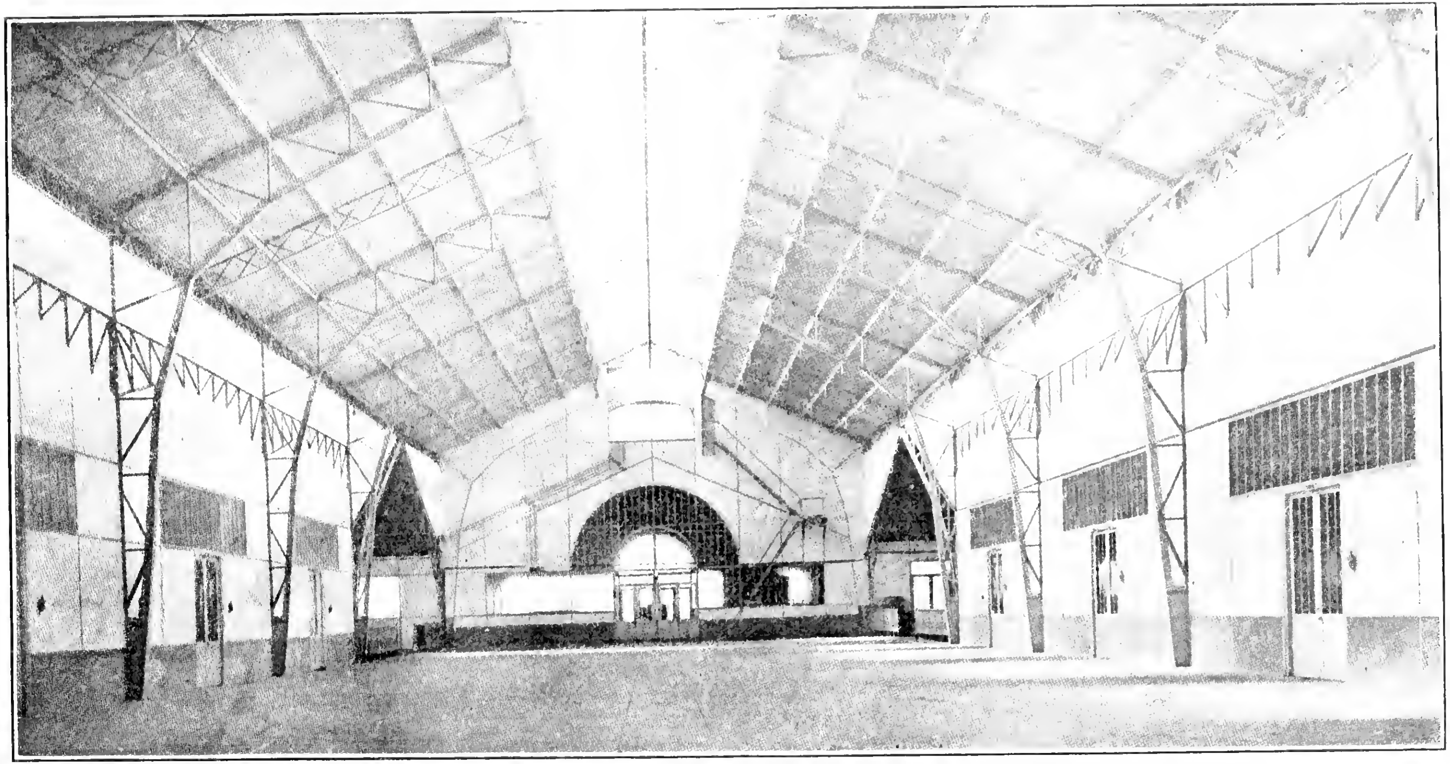

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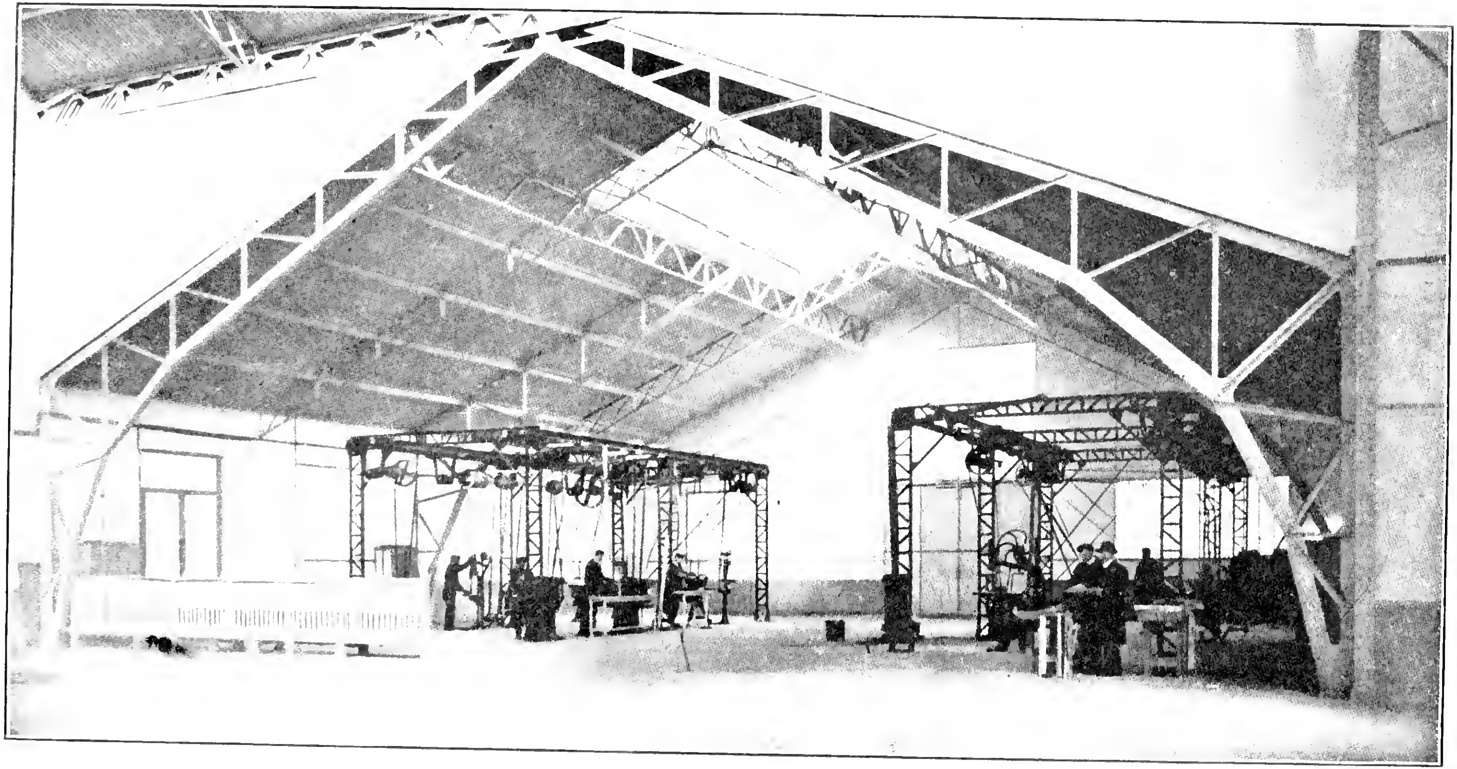

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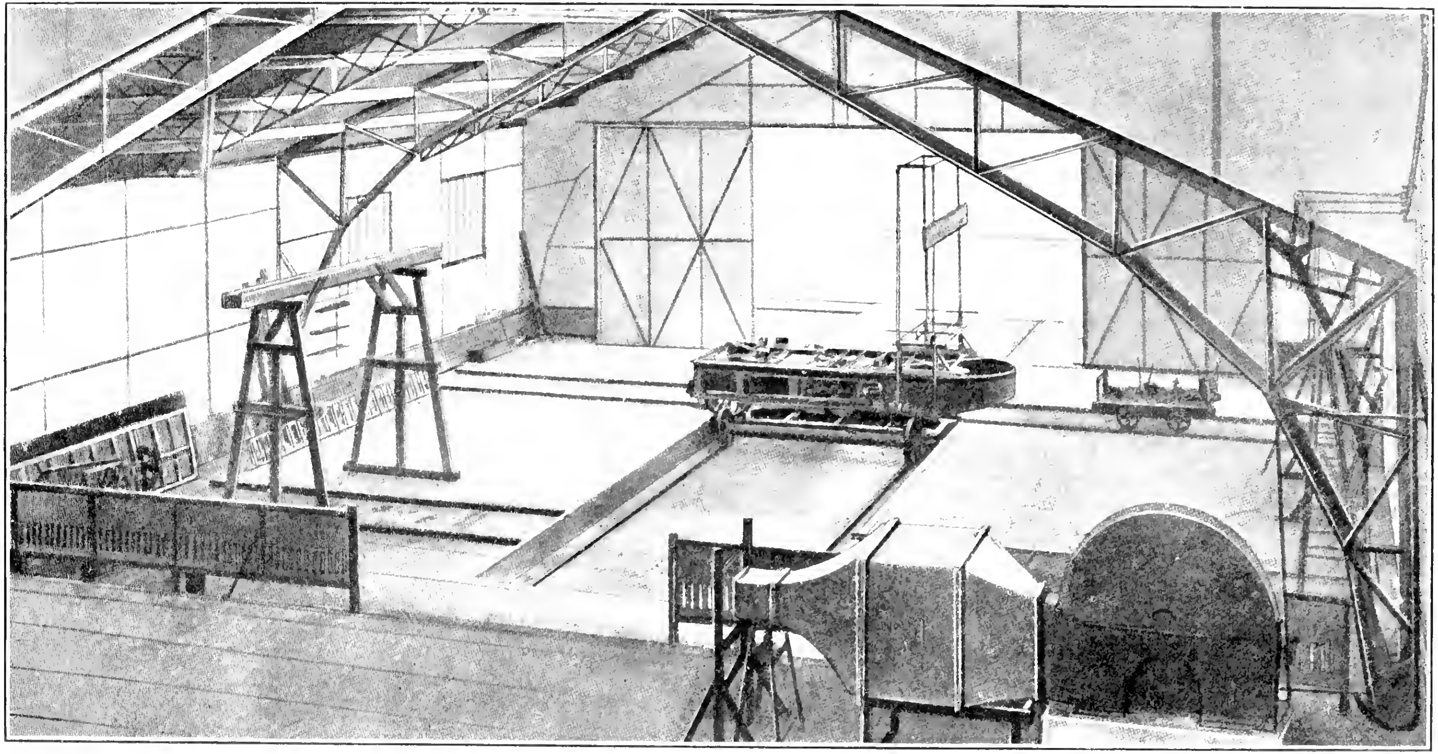
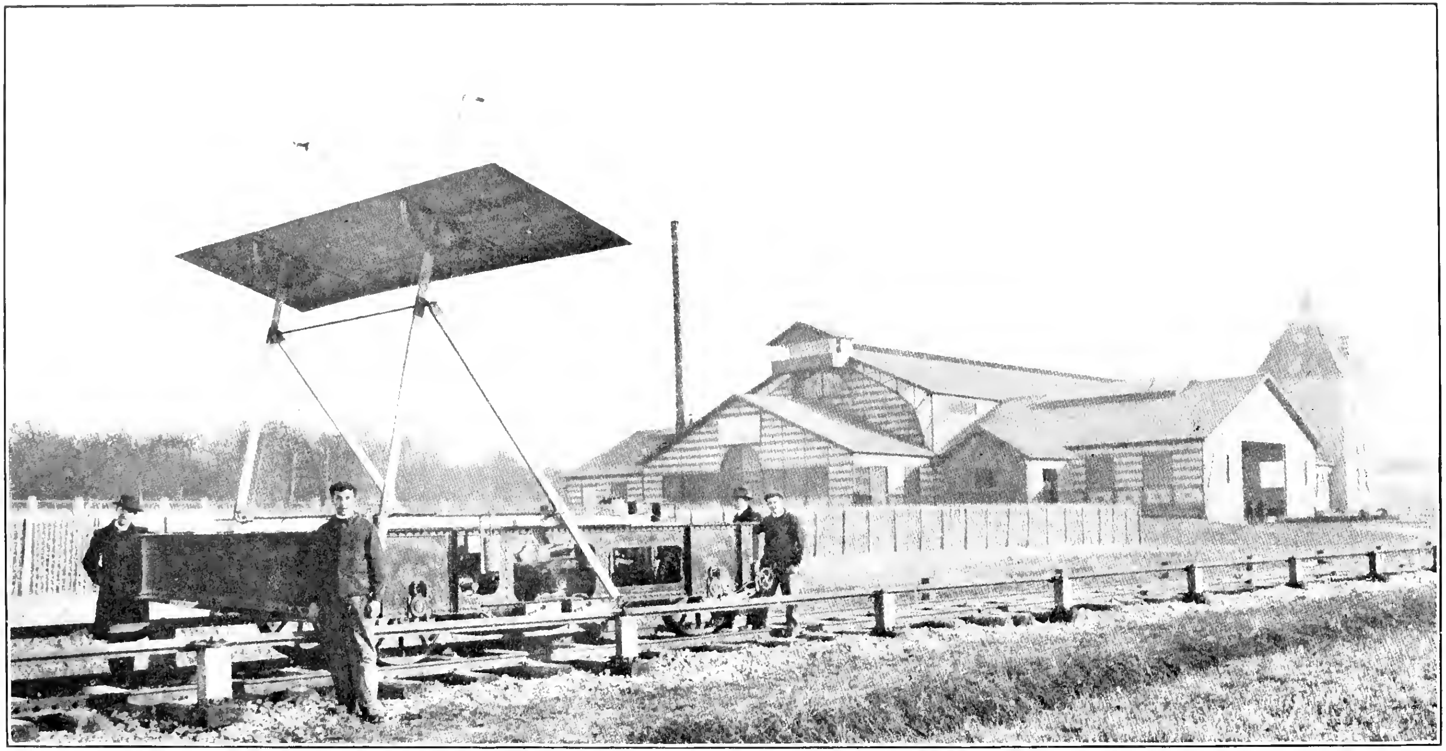


AEROTECHNIC INSTITUTE OF SAINT-CYR. GARAGE OF ELECTRIC PLATFORMS



AEROTECHNIC INSTITUTE OF SAINT-CYR. VIEW OF FIRST PLATFORM EQUIPPED FOR A TRIAL

resistance of the driving and recording gear, the resistance to rolling of the carriage and the air resistance of its elements, will all be determined once for all, so that the power actually absorbed by the surfaces or screws under test may be readily determinable.

Full-scale measurements.—We saw a full-scale Bleriot monoplane mounted on one of the electric carriages in such manner that its lift, drift and moment, or center of pressure, could be determined at one time, as it speeds across the field. The speed through the air is measured by means of a pressure-tube anemometer whose pressure collector is a Venturi tube, and has to be calibrated, since its readings are larger than those of a standard instrument such as used by Eiffel, Prandtl and others. The relative importance of such large scale experiments as compared with model tests, or full scale flights with instruments mounted on the aeroplane, has yet to be determined. If of new type, the full-scale machine may be tested more safely on a car. The measurements of lift here are said to be in error about 5 per cent; the drift measurements are much less accurate.

A roundhouse, which measures 120 feet in diameter, shelters a whirling table, the extremity of whose whirling arm describes a circle 300 feet in circumference, and carries the models subject to aerodynamic study. This can be used in any weather, while the electric road can be used only at special times, and most effectively only during fair and calm weather. The whirling table, however, does not seem to be so popular in the leading aerotechnical laboratories as the wind-tunnel and large field track. It is not an indispensable part of an aeronautical laboratory, except where studies in circular motion are to be made.

Ancillary buildings have been erected on the grounds near the main laboratory, one for the director immediately in charge, another for the caretaker, who is also a workman assisting in the experiments.

The reports of the investigations are published in the *Bulletin de l'Institut Aerotechnique de l'Universite de Paris.* The annual issues for 1912 and 1913 are in the Smithsonian library.

Other French aeronautical laboratories, operating on a smaller scale, are worth mentioning, though unvisited by me for want of time.

The military establishment at Chalais-Meudon, in charge of the Engineer Corps, and under direction of Commandant Dorand, resembles the English Royal Air Craft Factory, in developing experimental air craft and making full scale tests; but it does not manufacture air craft on such a large scale, and does not compete with commercial firms in building for the government, but rather stimulates and helps them to do their best work.

The Conservatoire National des Arts and Metiers, corresponding to our Bureau of Standards, does some aeronautical work in calibrating instruments, testing materials and motors, and furnishes a "mouline Renard"—a standardized revolving bar with paddles at either end—for attachment to a motor to determine its power at various speeds of rotation.

By the use of automobiles on a smooth road Chauviere has tested screw propellers mounted above the vehicle and advancing at natural working speed, and the Duc de Guiche has measured the lift, drift and pressure distribution on aerofoils of considerable size. The accuracy of the automobile method has, however, still to be proved satisfactory. The Chauviere propeller experiments are now made at St. Cyr Institut; but the researches of the Duc de Guiche still continue, and are reviewed from time to time in aeronautical literature. The earlier reports comprise two volumes published by Hachette, Paris.

GERMAN AERONAUTICAL LABORATORIES

The Göttingen aerodynamical laboratory, apart from the constructional and executive departments, is a one-story brick building, in size about 30 by 40 feet, comprising a wind-tunnel and two rooms, one for desk work, the other for instrumental observations. It stands alone, in a remote little meadow on the outskirts of the city, about fifteen minutes walk from Prof. Prandtl's university headquarters. It is very cheaply constructed, lighted by electricity, and heated by a little stove in one office.

The wind-tunnel consists of a continuous closed channel, two meters square in cross-section, running round the four walls of the main room. Through this tunnel the air is forced in a steady closed circulation by a screw ventilator two meters in diameter, belt driven from a thirty-horse electric motor placed in a little off room. As the blast from the blower is too fast along the tunnel walls, it is accelerated at the center of the stream by use of sheet metal fixtures placed in it near the screw, which also help to eliminate whirls. The air stream next passes through a honeycomb (fig. 1), made of 400 equal sheet metal cells, each about 4 inches square and 20 long, the sheet metal being in two thicknesses, or two ply, so that either cell can be constricted at will by spreading the cell wall inwardly. Actually, many of the cell walls were so constricted. In fact, the honeycomb looked badly distorted as if much time had been spent in adjusting the cells so that each should deliver

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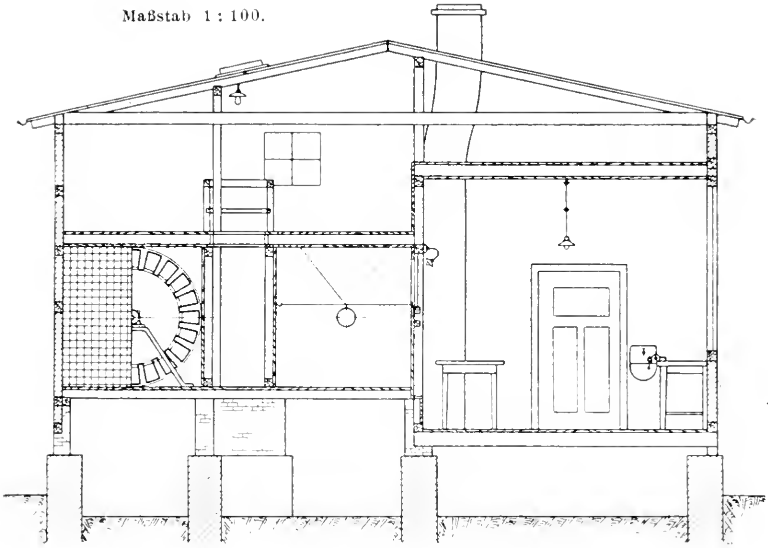

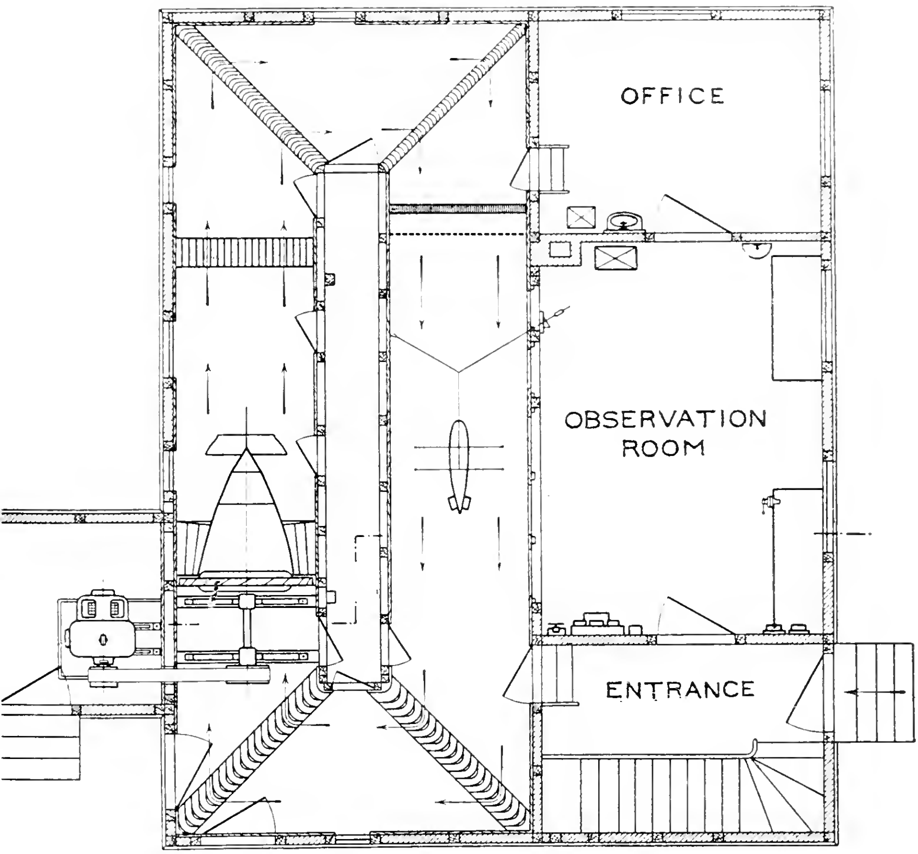

GOETTINGEN AERODYNAMICAL LABORATORY

the same amount of air. The adjustment once made assures, we were told, an air stream uniform in velocity at all points of a cross-section and at all speeds. One would think that a considerable change of speed would require a new adjustment of the cells to maintain uniformity. Emerging from the first honeycomb, the air passes through vertical sheet metal guide blades, each a double sheet and of turbin blade form, which turn the stream 90°, without eddies; thence through similar blades giving 90° more turn; thence through a much finer honeycomb to remove minor eddies. This last comb, placed just before the test part of the tunnel where the models are inserted, is made of sheet metal strips 10 centimeters wide reaching from floor to ceiling of the tunnel, and held in position by their

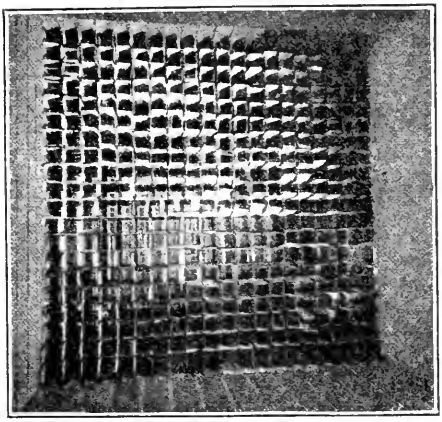

FIG. 1.—Prandtl's Honeycomb in Wind-tunnel

mutual pressure, comprising among them 90,000 cells. The stream of air issuing from the last honeycomb is said to be uniform, and has a speed ranging up to 10 meters per second.

The measuring instruments employed are numerous; but as several of them resemble the ones already described, they need not be noticed. One favorite method used by Prandtl to measure the resistance of a model, say of balloon form, is to suspend it in the current by fine wires, and hold it against stream by horizontal mooring wires whose tension is measured in the adjoining room by means of a bell crank and sliding weight. Very accurate measurements can be made without the mooring wires, if the weight and displacement of the model along stream be observed, as in my experiments of 1902. This method, as extended by Mr. Mattullath, has been

adopted at Göttingen to measure the resistance of hulls, etc., held obliquely to the current. Prandtl's differential pressure gauge, consisting of inverted cups suspended from opposite arms of a balance,

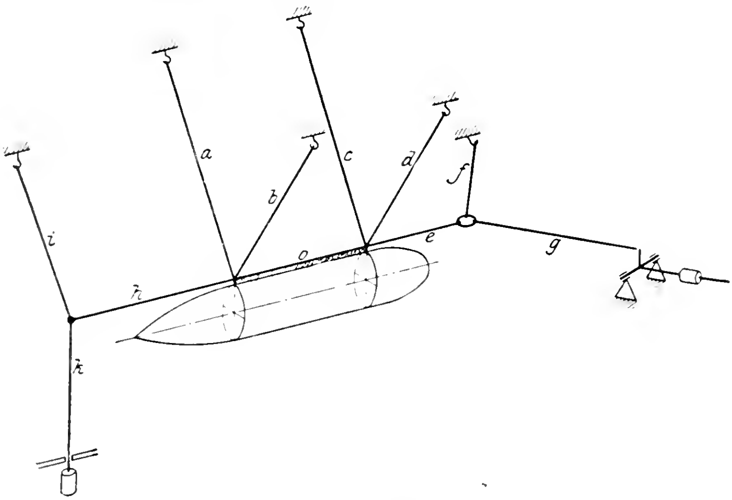

FIG. 2.—Prandtl's Suspension for Measuring Head Resistance

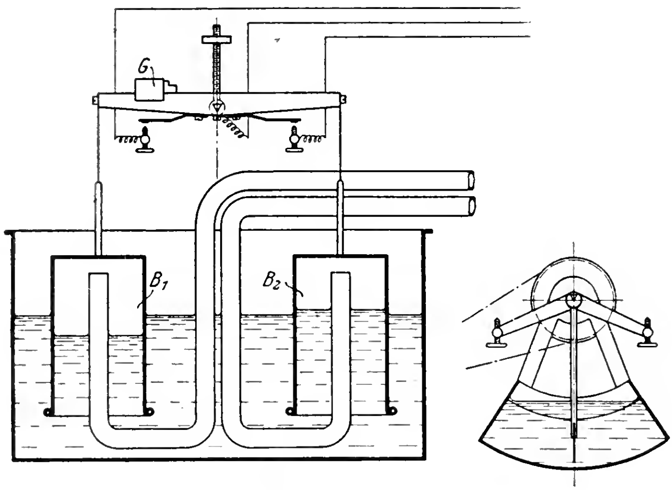

FIG. 3.—Prandtl's Manometric Balance

and dipping into a liquid, is like the one devised and used by me early in 1902, and found capable of measuring differential pressures truly to one millionth of an atmosphere, or less. This gauge was

described in the *Physical Review* for December, 1903, half a decade before Prandtl's experiments.

The pressure distribution over model screw propellers having perforated hollow blades was measured by transmission through a hollow shaft to a pressure gauge. The screws were made of copper electrically deposited on wax models, and were then emptied of the wax by heating. To show the direction of air flow past the blades,

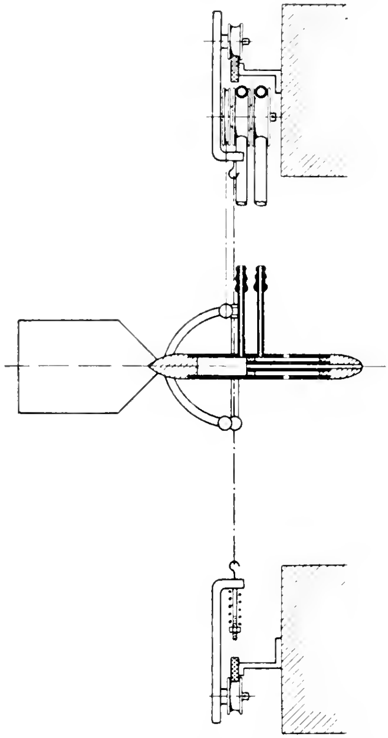

Fig. 4.—Prandtl's Pressure-tube Anemometer

sulphureted hydrogen was allowed to exude from perforations in their surfaces, and thus to stain them. The staining streaks extend fore and aft and very slightly outward radially along the screw blades.

The results of the experiments in the Göttingen laboratory have been published in various German periodicals, and in part translated and republished in *Engineering*, London, for 1911 and 1912, all of which are on file in the Smithsonian library. Particularly interesting are Prandtl's determination of pressure distribution on models of

balloon hulls designed in accordance with hydrodynamic theory; also his measurements of the resultant wind force on oblique hulls and wing forms by the method devised and used by H. Mattullath in 1902; also the resistance of wires and ropes, etc. Prandtl found in fair shapes a large difference between total resistance and the pressural resistance, and ascribed the difference to skin-friction; but this he did not attempt to measure directly.

The Deutsche Versuchsanstalt für Luftfahrt zu Adlershof comprises one main building used for offices and full-scale aeroplane testing; one used for construction; and five small houses each con-

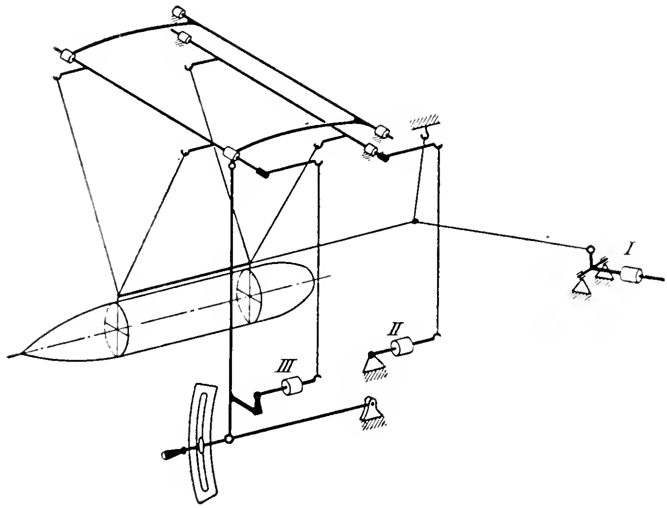

FIG. 5.—Prandtl's Suspension for Measuring Side Force

taining an engine testing apparatus. In addition to this plant, it is intended to fly full-scale machines with measuring instruments, and to mount large apparatus on an aerodynamic car pushed by a locomotive on a railway.

The laboratory of the main building is a large square room with a tower in its center 100 feet high, on top of which wind observations may be made, and inside of which suspension cords run down to support an aeroplane just above the floor, to determine its moment of inertia. In a corner of the room an aeroplane inverted and weighted with sand, as in Langley's method, was under test for stress and strain of its wing framing. In another corner was an apparatus for measuring the force applied to the controls of an

aeroplane by a pilot in practical flight. This instrument may help to determine the most suitable mechanism for a standard control.

The shop and the engine testing houses contain nothing that need be reported. The engine torque and thrust were measured by ordinary mechanical methods, and no special apparatus was used to furnish a stream of cooling air, as in the British laboratory.

Other German aeronautical laboratories worth passing mention are: the testing department of the Zeppelin Airship Co.; the aerodynamical laboratory used by Prof. Reissner of the technical high school at Achen; the laboratory in charge of Major v. Parseval in the high school at Berlin; the experimental plant of Prof. Dr. Fr. Ahlborn, at Hamburg. The Zeppelin laboratory is not, under any consideration, open to visitors from abroad; and as to the others just mentioned, I had time only for a brief visit to Ahlborn's place. Ahlborn's experiments have been confined mainly to determinations of flow about models in a tank of water. The results are well portrayed in numerous excellent photographs and publications, the best of which are in the Smithsonian Institution. His apparatus and photographs and those at the National Physical Laboratory in England are, for hydromechanical studies, the most instructive that have yet come to my notice, except perhaps the more restricted ones of Hele-Shaw. For stream-line delineation in air, however, the classical apparatus and methods of Marey have not yet been surpassed, though more precise instruments of this nature are much to be desired.

CONCLUSION

In closing I would gratefully acknowledge the courteous assistance extended to Lieut. Hunsaker and myself, by the American Embassies at London, Paris and Berlin, by the aero clubs in those cities, and by the directors of the laboratories and factories we visited. We are especially indebted to Mr. G. F. Campbell Wood, foreign representative of the Aero Club of America, at Paris, and to Major G. J. F. Von Tschudi, of the German Army, Director of the Flug und Sportplatz at Johannisthal, for many kind services which enlarged our opportunities and facilitated our work in France and Germany.

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REPORTS ON WIND TUNNEL EXPERIMENTS IN AERODYNAMICS

(With Five Plates)

BY
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REPORTS ON WIND TUNNEL EXPERIMENTS IN AERODYNAMICS

I. THE WIND TUNNEL OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY

By J. C. HUNSAKER, Assistant Naval Constructor, U. S. Navy
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An aeroplane or airship in flight has six degrees of freedom, three of translation and three of rotation, and any study of its behavior must be based on the determination of three forces—vertical, transverse, and longitudinal—as well as couples about the three axes in space. Full scale experiments to investigate the aerodynamical characteristics of a proposed design naturally become mechanically difficult to arrange. The experimental work is much simplified if tests be made on small models as in naval architecture, and a further simplification is made by holding the model stationary in an artificial current of air instead of towing the model at high speed through still air to simulate actual flying conditions.

The use of a wind tunnel depends on the assumption that it is immaterial whether the model be moved through still air or held stationary in a current of air of the same velocity. The principle of relative velocity is fundamental, and the experimental discrepancies between the results of tests conducted by the two methods may be ascribed on the one hand to the effect of the moving carriage on the flow of air about the model and to the effect of gusty air, and on the other hand to unsteadiness of flow in some wind tunnels.

The wind tunnel method requires primarily a current of air which is steady in velocity both in time and across a section of the tunnel. The production of a steady flow of air at high velocity is a delicate problem, and can only be obtained by a long process of experimentation. A study was made of the principal aerodynamical laboratories of Europe from which these conclusions were reached: (1) That the

wind tunnel method permits a leisurely study of the forces and couples produced by the wind on a model; (2) that the staff of the National Physical Laboratory, Teddington, England, have developed a wind tunnel of remarkable steadiness of flow and an aerodynamical balance well adapted to measure with precision the forces and couples on a model in any position; and (3) that the results of model tests made at the above laboratory are applicable to full scale aircraft.

Consequently it was decided to reproduce in Boston the four-foot wind tunnel of the National Physical Laboratory, together with the aerodynamical balance and instruments for velocity measurement. Dr. R. T. Glazebrook, F. R. S., director of the National Physical Laboratory, most generously presented us with detail plans of the complete installation, including the patterns from which the aerodynamical balance was made. Due to this encouragement and assistance we have been able to set up an aerodynamical laboratory with confidence in obtaining a steady flow of air of known velocity. The time saved us by Dr. Glazebrook, which must have been spent in original development, is difficult to estimate.

The staff of the National Physical Laboratory have developed several forms of wind tunnel in the past few years. In 1912-13 Mr. Bairstow and his assistants conducted an elaborate investigation into the steadiness of wind channels as affected by the design both of the channel and the building by which it is enclosed.[1] The conclusions reached may be summarized as follows:

(1) The suction side of a fan is fairly free from turbulence.

(2) A fan made by a low pitch four-bladed propeller gives a steadier flow than the ordinary propeller fan used in ventilation, and a much steadier flow than fans of the Sirocco or centrifugal type.[2]

(3) A wind tunnel should be completely housed to avoid effect of outside wind gusts.

(4) Air from the propeller should be discharged into a large perforated box or diffuser to damp out the turbulent wake and return the air at low velocity to the room.

(5) The room through which air is returned from the diffuser to the suction end of the tunnel should be at least 20 times the sectional area of the tunnel.

(6) The room should be clear of large objects.

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[1] Technical Report of the Advisory Committee for Aeronautics, London, 1912-13. Report No. 67.

[2] It is of interest to note that Mr. Eiffel has used a helicoidal blower in his new wind tunnel.

The wind tunnel of the Institute of Technology was built in accordance with the English plans, with the exception of several changes of an engineering nature introduced with a view to a more economical use of power and an increase of the maximum wind speed from 34 to 40 miles per hour.

Upon completion of the tunnel an investigation of the steadiness of flow and the precision of measurements was made in which it appeared that the equipment had lost none of its excellence in its reproduction in the United States.

As will be shown below, the current is steady both in time and across a cross-section within about 1 per cent in velocity. Measurements of velocity by means of the calibrated Pitot tube presented by the National Physical Laboratory are precise to one-half of 1 per cent. Force and couple measurements on the balance are precise to one-half of 1 per cent for ordinary magnitudes. Calculated coefficients which involve several measurements of force, moment, velocity, angle, area, and distance, as well as one or more assumptions, can be considered as precise to within 2 per cent. It is believed that it is not practicable to increase the precision of the observations to such an extent that the possible cumulative error shall be materially less than the above.

Description of Wind Tunnel

A shed 20 by 25 by 66 feet houses the wind tunnel proper, 16 square feet in section, and some 53 feet in length (pl. 1). Air is drawn through an entrance nozzle and through the square tunnel by a four-bladed propeller, driven by a 10 H. P. motor. Models under test are mounted in the center of the square trunk on the vertical arm of the balance to be described later.

The air entering the mouth passes through a honeycomb made up of a nest of 3-inch metal conduit pipes 2 feet 6 inches in length. This honeycomb has an important effect in straightening the flow and preventing swirl.

Passing through the square trunk and past the model, the air is drawn past a star-shaped longitudinal baffle into an expanding cone. In this the plans of the National Physical Laboratory were departed from by expanding in a length of 11 feet to a cylinder of 7 feet diameter. This cone expands to 6 feet in the English tunnel. M. Eiffel affirms that the working of a fan is much improved by expanding the suction pipe in such a manner as to reduce the velocity and so raise the static pressure of the air. Since the fan must discharge into the

room, the pressure difference that the fan must maintain is thus reduced. Also with a larger fan the velocity of discharge is reduced, and the turbulence of the wake kept down.

The propeller works in a sheet metal cylinder 7 feet in diameter, and discharges into the large perforated diffuser. The panels of the latter are gratings and may be interchanged fore and aft. The gratings are made of 1½-inch stock with holes 1½ by 1½ inches. Each hole is then a square nozzle one diameter long. The end of the diffuser is formed by a blank wall. The race from the propeller is stopped by this wall and the air forced out through the holes of the diffuser. Its velocity is then turned through 90 degrees. The area of the diffuser holes is several times the sectional area of the tunnel, and the holes are so distributed that the outflow of air is fairly uniform and of low velocity (pl. 2, fig. 1).

A four-bladed black walnut propeller (pl. 2, fig. 2) was designed on the Drzwiecki system and has proved very satisfactory. In order to keep down turbulence a very low pitch with broad blades had to be used. To gain efficiency such blades must be made thin. It then became of considerable difficulty to insure proper strength for 900 R. P. M. as well as freedom from oscillation.

The blade sections were considered as model aeroplane wings and their effect integrated graphically over the blade. The blade was given an angle of incidence of 3 degrees to the relative wind at every point for 600 R. P. M. and 25 miles per hour. The pitch is thus variable radially.

To prevent torsional oscillations, the blade sections were arranged so that the centers of pressure all lie on a straight line, drawn radially on the face of the blade. This artifice seems to have prevented the howling at high speeds commonly found with thin blades. The propeller has a clearance of ½ inch in the metal cylinder.

The propeller is driven by a "silent" chain from a 10 H. P. interpole motor beneath it. The propeller and motor are mounted on a bracket fixed to a concrete block and are independent of the alignment of the tunnel. Vibration of the motor and propeller cannot be transmitted to the tunnel as there is no connection.

The English plans for power contemplate a steady, direct current voltage. Such is not available here. A 15 H. P. induction motor is connected to the mains of the Cambridge Electric Light Company. This motor then turns at a speed proportional to the frequency of the supply current for a given load. Fluctuations of voltage are without sensible effect, and the frequency may be taken as practically constant.

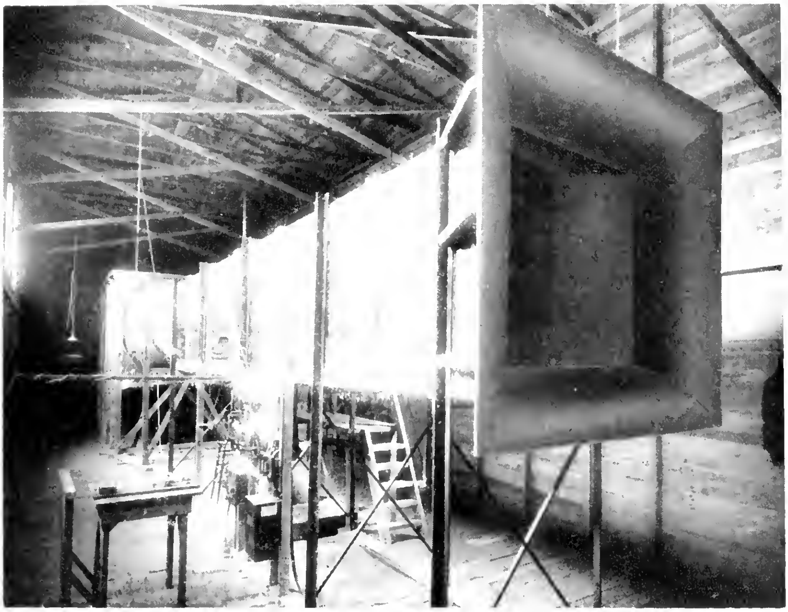


Fig. 1. SUCTION END OF WIND TUNNEL

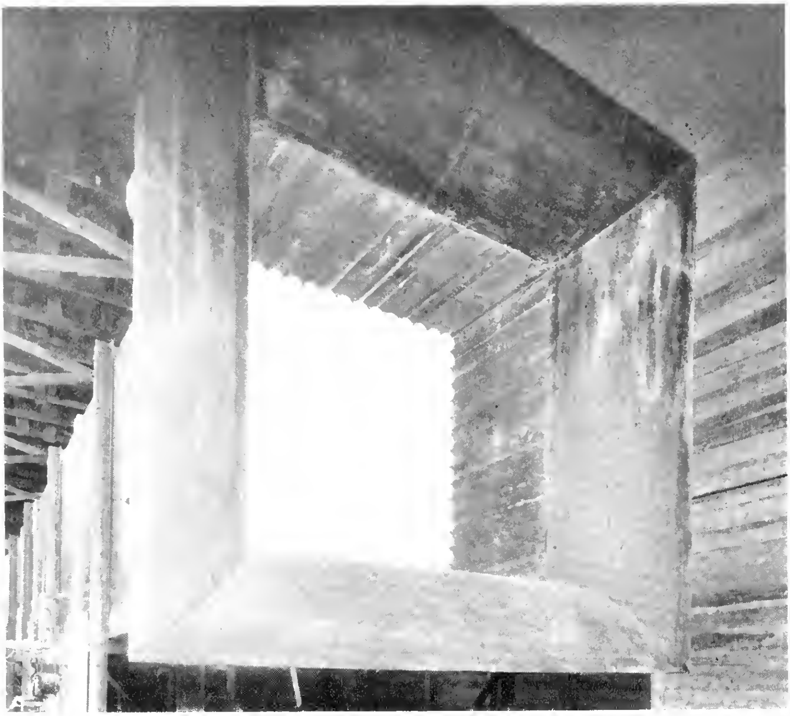


Fig. 2. ENTRANCE NOZZLE, SHOWING END OF HONEYCOMB

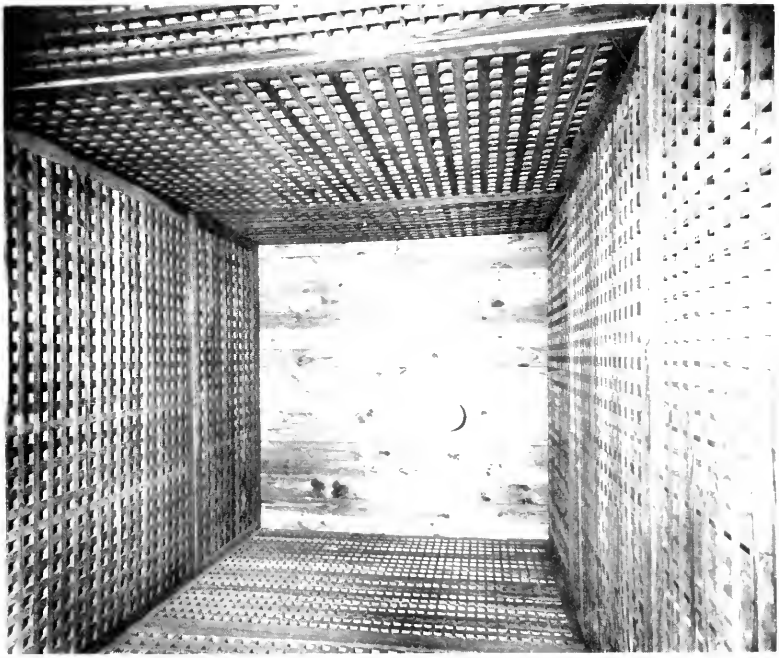


Fig. 1. INTERIOR OF DIFFUSOR, LOOKING FROM PROPELLER

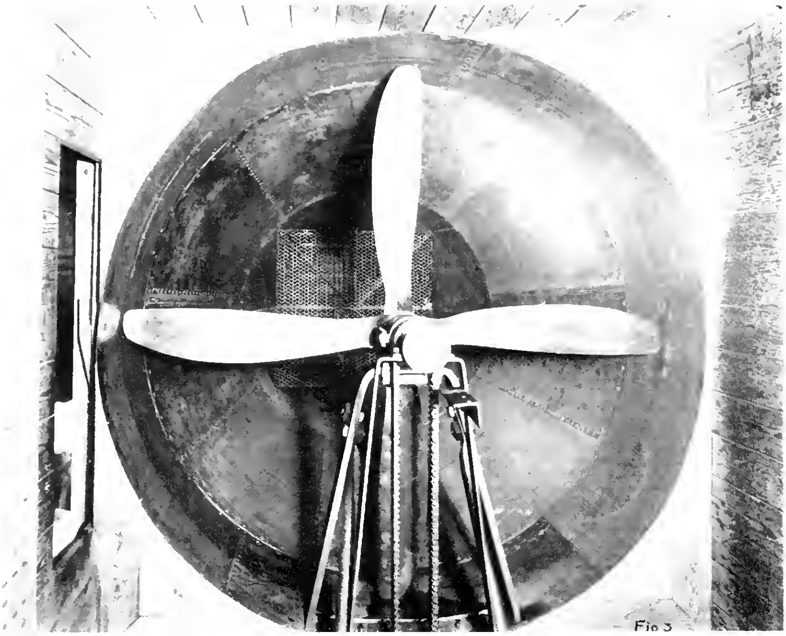


Fig. 2. PROPELLER OF WIND TUNNEL, LOOKING UP STREAM

The induction motor is directly connected to a 12 H. P. direct current generator, which is turned at constant speed and which generates, therefore, a constant direct current voltage for given load.

By change of the generator field rheostat and motor field rheostat the propeller speed can be regulated to hold any wind velocity from 4 to 40 miles per hour. The control is very sensitive. Left to itself, the speed of the wind in the tunnel will vary by 2 per cent in 2 or 3 minutes. This variation is so slow that by manipulation of the rheostats the flow can be kept constant within $\frac{1}{2}$ per cent. The cause of the surging of the air is not understood, but is probably due to hunting of the governor of the prime mover in the Cambridge power house causing changes in frequency too small to be apparent. The gustiness of outdoor winds seems to have no effect, although the building is not air-tight.

Aerodynamical Balance

The aerodynamical balance (pl. 3) was constructed by the Cambridge Scientific Instrument Company, England, to the plans and patterns of the National Physical Laboratory. The balance is described in detail by Mr. L. Bairstow in the Technical Report of the Advisory Committee for Aeronautics, London, 1912-13. For details of operation and the precision of measurements reference may be made to the original article.

In general, the balance consists of three arms mutually at right angles representing the axes of coordinates in space about and along which couples and forces are to be measured. The model is mounted on the upper end of the vertical arm which projects through an oil seal in the bottom of the tunnel.

The entire balance rests on a steel point, bearing in a steel cone. The point is supported on a cast-iron standard secured to a concrete pillar, which in turn rests on a large concrete slab. The balance is then quite free from vibration of the floor, building, or tunnel.

The balance is normally free to rock about its pivot in any direction. When wind blows against the model, the components of the force exerted are measured by determining what weights must be hung on the two horizontal arms to hold the model in position. Likewise the balance is free to rotate about a vertical axis through the pivot. The moment producing this rotation is balanced by a calibrated wire with graduated torsion head.

Force in the vertical axis is measured by means of a fourth arm. The model for this measurement is mounted on a vertical rod which

slides freely on rollers inside the main vertical arm of the balance. The lower end of this rod rests on one end of a horizontal arm having a knife edge and sliding weight.

For special work on moments, the interior vertical rod is replaced by another having a small bell crank device on its head which converts a moment about the center of the model into a vertical force to be measured as above (pl. 4).

In this way provision is made for the precise measurement of the three forces and the three couples which the wind may impress on any model held in any unsymmetrical position to the wind.

The balance is fitted with suitable oil dash pots to damp oscillations, and devices for limiting the degrees of freedom to simplify tests in which only one or two quantities are to be measured. The balance can be adjusted to tilt for 1/10,000 pound force on the model. In general, the precision of measurements is not so good as the sensitivity, and in the end is limited by the steadiness of the wind and the skill of the observer.

The weights and dimensions of the balance were verified by the National Physical Laboratory, where also the torsion wires were calibrated.

For ordinary forces, weighings may be considered correct to 0.5 per cent. Naturally for very small forces, such as the rolling moment caused by a small angle of yaw, the measurements cannot be so precise.

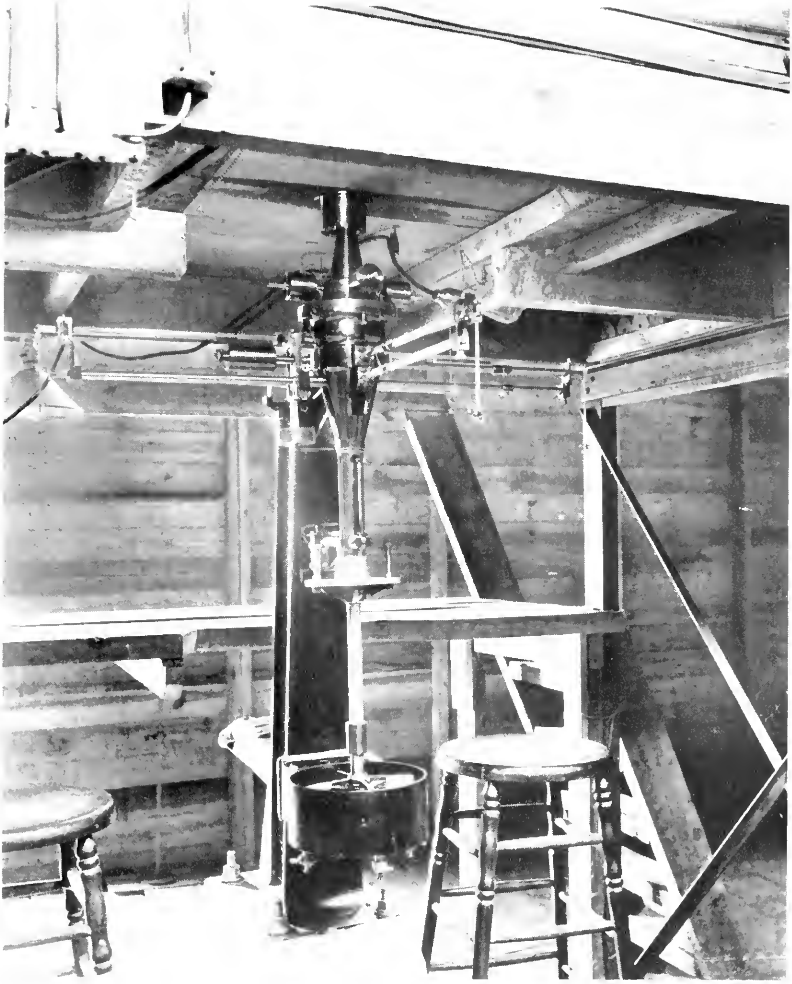
Alignment of Tunnel

The axis of the wind tunnel was desired to be horizontal from the honeycomb to the baffle plates in front of the propeller. To accomplish this an engineer's level was mounted on a platform, built on the floor of the house, opposite the mouth of the tunnel, and sighted on the intersection of diagonal threads placed at 6-foot intervals. By this means the distance of the center line of the tunnel above or below the horizontal line could be estimated to one-eighth of 1 inch.

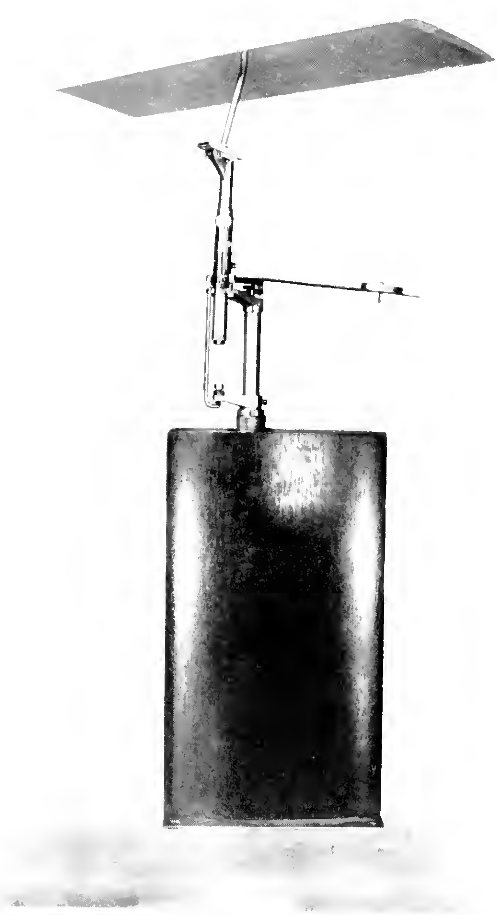
The tunnel beling low in the center, it was raised by wedges until the reference marks coincided with the horizontal. This was attained to within one-eighth of 1 inch in 6 feet of tunnel length. The tunnel may, therefore, be said to have its axis horizontal to within one-tenth degree.

Alignment of Vertical Axes of Balance

A concrete foundation having been built for the balance, the latter was set in its approximate position. Three wedges were then



AERODYNAMICAL BALANCE



MODEL WING MOUNTED IN WIND TUNNEL ON MOMENT DEVICE OF BALANCE

inserted under the base plate of the balance standard, and the whole balance raised to its proper height. It was now necessary to rectify the vertical axis of the balance.

To bring the axis of the balance more nearly vertical by more sensitive means the following method was employed: The small torsion wire, used in aerodynamical measurements with the balance, was set in place. The lower pivot of the balance was engaged in its bearing, leaving the balance free to rotate about its vertical axis, but constrained from tipping laterally.

The torsion wire was adjusted by means of the micrometer head until the cross-hair on the fixed telescope coincided with that on the mirror attached to the balance proper.

A weight of 0.4 pound was placed on one balance arm. The micrometer head was again turned until the cross-hairs were coincident. By setting up on the holding-down bolts, the balance axis was adjusted until placing a weight on either of the arms required no further rotation of the torsion head to maintain coincidence of the cross-hairs. In such case the axis of the balance is vertical. The final adjustment admits of a possible error of less than 1/400 inch-pound on the torsion wire. The angularity of the balance axis remaining uncorrected may be computed as follows:

Let

F = force hung on arm.

β = angle of balance axis to vertical.

Then, taking moments about the vertical axis

$$F \sin \beta \times 18'' = 0.4 \times \sin \beta \times 18'' = 1/400$$

$$\text{or } \beta = 0^\circ\!.025.$$

Determination of Wind Direction in the Horizontal Plane

As a first approximation, the wind was assumed parallel to the axis of the tunnel. A vertical flat plate was mounted on the balance arm and carefully set parallel to a line drawn on the floor of the tunnel in the direction of its axis. The plate was inclined 8 degrees to right and left of this position and the transverse force measured on the balance. The observations were repeated for 6 degrees and for a second plate to eliminate errors due to irregularities in the plates. The transverse force on one side was greater than on the other, indicating an error in the assumed wind direction. A new line was drawn on the tunnel floor making an angle of 0.3 degrees with the original line. The observations for transverse force were repeated. It was then found that the average of transverse force readings taken for

equal angles to right and left of mid position differed 0.5 per cent from the mean of all readings. The extreme error of one observation, including the error of 0.5 per cent due to lack of sensitivity of the balance and the personal error of the observer, may then be as great as 1 per cent in case the two errors are cumulative.

It is not considered practicable to obtain a closer setting with the methods of alignment employed.

Setting Arms of Balance

Knowing the true direction of the wind, it is necessary to set the horizontal arms of the balance parallel and perpendicular to this direction. To do this the floating part of the balance was rotated by an adjustment provided by the design until the force recorded on the "drift" arm (the resistance) was equal for equal angles of the plate to right and left of the wind direction. The final setting indicates a remaining error of .2 per cent.

After making the slight adjustment required here the error in the transverse force measurement was not found to be increased.

Measurement of Air Velocity

The velocity of flow in the wind tunnel is measured by a pressure tube anemometer commonly called a double Pitot tube.

Our laboratory standard is a double Pitot tube presented by the director of the National Physical Laboratory, England. This tube was compared with the National Physical Laboratory standard which had been calibrated on a whirling arm by F. H. Bramwell.[1] Its constant had been determined to be unity to a precision of 0.1 per cent. Our tube was compared with it by a method allowing a precision of 0.25 per cent. A discrepancy of about 0.25 per cent was found. Its readings may then be taken as correct to this degree of precision. In all cases a uniform rectilinear current is implied.

The Pitot tube, in common with all anemometers, has the disadvantage of obstructing the channel, and where models are to be tested the channel should be kept entirely clear. The expedient of using a side hole in the channel is due to M. Eiffel.[2]

In a channel of uniform section, air is forced to flow practically parallel to the axis of the channel. Hence stream lines are all parallel and across any section, taken normal to the channel axis, there should be no component of velocity at any point. This statement is of course true only for a steady, uniform flow free from turbulence. The

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[1] Technical Report of the Advisory Committee for Aeronautics, London, 1912-13.

[2] La Resistance de l'Air et Aviation, Paris, 1912.

static pressure should be constant across a section, for if pressure differences existed there would be a transverse flow of air created. Tests in our wind tunnel showed constant pressure across a section to a good approximation. Incidentally the constancy of this static pressure across a section is a measure of the uniformity of flow.[1]

A small hole in the side of the tunnel can then be used to measure the static pressure, but the dynamic pressure measured by the impact end of the Pitot tube is

$$p + \frac{\rho v^2}{2g} = p_0, \text{ by Bernoulli's equation,}$$

where

p = pressure at any point in a stream line.
v = velocity at any point in a stream line.
ρ = density at any point in a stream line.
p_0 = pressure where v is zero.

In our wind tunnel a fan sucks air through the tunnel which is therefore all under suction. The air is discharged by the fan through a strainer into the building at one end, whence it returns at low velocity to the other end to pass again into the tunnel. At a point in the room the pressure transmitted by an impact tube would be

$$p_r + \frac{\rho v'_r{}^2}{2g} = p_0.$$

But the room is 30 times as large as the section of the tunnel, and when a wind of 30 miles is blowing in the tunnel there is only a gentle draft in the room of about 1 mile per hour. Thus the ratio $\frac{v'_r{}^2}{v^2} = \frac{1}{900}$ and the pressure in the room can be taken as

$$p_r = p_0 = p + \frac{\rho v^2}{2g}$$

neglecting $v'_r{}^2$.

[1] The static holes of the National Physical Laboratory Pitot tube were connected to an alcohol gage, and the velocity being kept constant, the tube was moved along the vertical center line of the tunnel. The following readings were taken:

Head in ½ mm. alcohol	Distance from wall
440.2	3″
438.0	6″
439.5	12″
439.8	18″
439.5	24″
441.2	30″
441.0	36″
441.0	42″
441.2	45″

If then we connect a hole in the side of the tunnel with one end of a liquid manometer, and leave the other end open to the room, the gage reading is proportional to the difference in pressure or to

$$p_r - p = p_0 - p = \frac{\rho v^2}{2g} \cdot$$

The reading of the manometer thus is a measure of the velocity.

Due to loss of head from friction in the mouth of the tunnel and in the honeycomb, the relation

$$p_r = p + \frac{\rho v^2}{2g}$$

is not strictly true. An unknown loss in friction would be represented by adding a term to indicate the friction head pressure. Then

$$p_r = p + \frac{\rho v^2}{2g} + p_f.$$

The use of the side plate method ignores the effect of p_f. A comparative test showed an error of 3 per cent when velocity was calculated from side plate readings. It is, therefore, necessary to calibrate the side plate and its manometer against the standard Pitot tube and its manometer.

The side plate used (fig. 1) consists of a thin brass disk about 3 inches in diameter set flush in the wall of the tunnel. The disk is flat and highly polished. Near its center, five holes 0.02 inch in diameter are drilled. These holes are connected with a brass tube soldered to the back of the plate and projecting through the side of the channel. Rubber tubing is used to transmit the static pressure from the small holes to one end of a manometer. As explained above, the other end of the manometer is open to the air in the room.

The pressures transmitted by the side plate have been found to respond very quickly to changes in velocity, and the method is even more sensitive than the Pitot tube. Naturally its precision is no better than that of the Pitot used for its calibration.

The pressure difference transmitted by the side plate is read on an inclined alcohol manometer on the Krell principle. Both the side plate and this alcohol manometer require calibration against a standard. For convenience, the side plate and its manometer were calibrated together against the standard Pitot tube and a Chattock manometer.

The standard National Physical Laboratory Pitot tube was mounted in the center of the tunnel in the place where models are tested. This tube was connected to the Chattock gage. The side plate in the wall opposite the tube was then connected to the alcohol

gage. The wind speed was then adjusted to 2, 4, 6, 8, etc., up to 40 miles per hour and both gages read. Some 100 readings were taken. From the Chattock gage readings the true speed was taken from its calculated curve (for standard air). The readings of the alcohol gage were then plotted on true speeds. The curve so made was then a calibration of the side plate and alcohol gage in combination. The Pitot tube and Chattock gage may now be removed, and in future

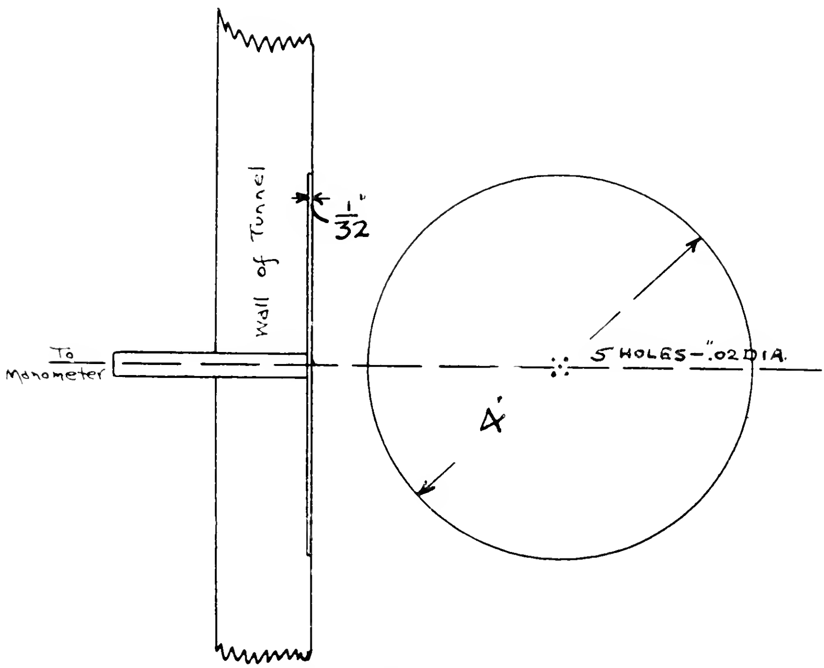


Fig. 1.—Side pressure plate.

model testing the alcohol gage readings may be used to measure the velocity at the center of the tunnel.

It is shown below that the velocity over the section varies about 1 per cent over a 2 foot 6 inch square at the center of the tunnel.

Chattock Micromanometer

The Chattock gage mentioned above has been adopted as our laboratory standard, but is used only for the calibration of other gages which may be preferred on account of ease of reading. The following notes on this gage are introduced here in the hope that someone may have use for a delicate pressure gage. Working draw-

ings will gladly be supplied to anyone contemplating the construction of such a gage.

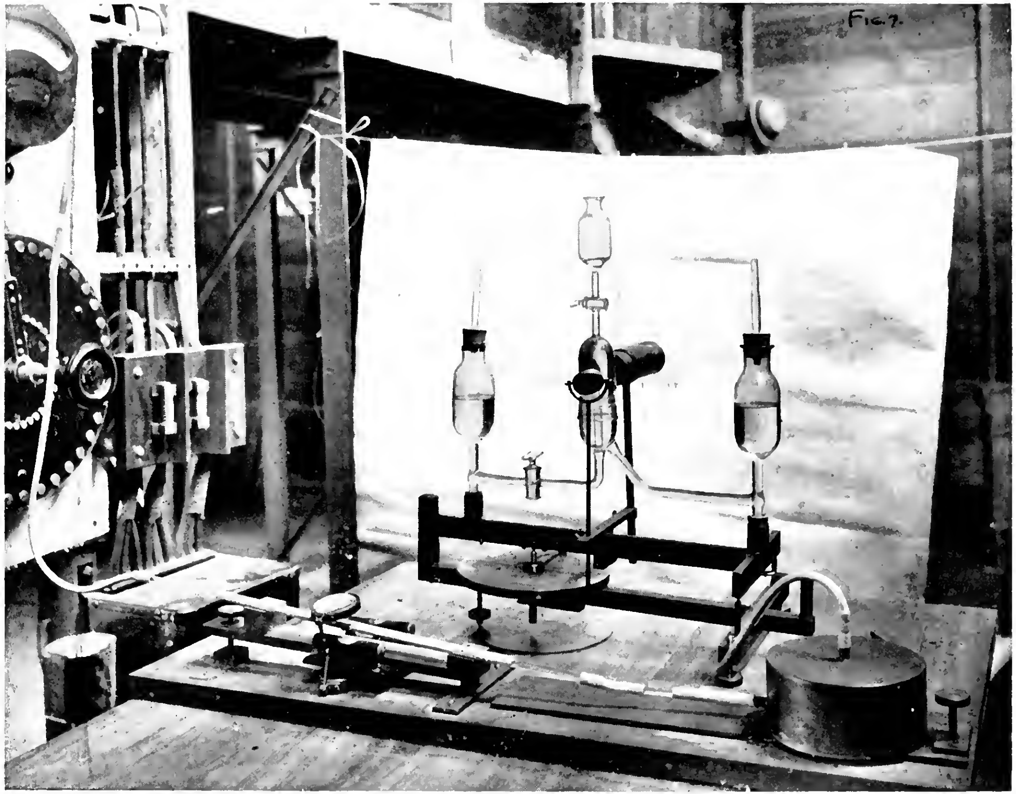
The Chattock micromanometer was devised by Professor A. P. Chattock and Mr. J. D. Fry for the precise measurement of very small pressures. The gage is described by Dr. T. E. Stanton in the *Proceedings of the Institution of Civil Engineers,* December, 1903. Dr. Stanton used this gage in his experiments on the air resistance of small plates.

The principle of the gage is that of the inclined liquid U-tube, but instead of giving the tube an initial pitch and observing the change of level of the liquid, the Chattock gage is fitted with an elevating screw and micrometer by which the gage is tilted to balance the pressure difference in its two ends. By reading on the micrometer the amount of tilt given, the head in inches of liquid is computed. By this means there is no motion of the liquid in the glass, and errors due to capillarity and viscosity are eliminated. Furthermore, the condition of the surface of the glass has no effect.

The gage (pl. 5 and fig. 2) consists of a glass U-tube mounted on a tilting frame *T*. The pressures to be measured are connected to the bulbs *A* and *C*, which are in communication with each other through a horizontal tube bearing a third bulb *B* at any intermediate point. The bulbs *A*, *C* and the lower part of *B* are filled with water. The upper part of *B* is filled with castor oil. The water in *B* and *C* is in free communication and hence the oil in *B* is at the pressure of *C*. The water in *A* is led through a thin walled tube through the bottom of *B* extending into the castor oil. An excess of pressure in *A* over the pressure in *C* will cause water to flow from *A* into *B*. A water bubble will then grow at *D* and expand into the oil. The gage can be tilted so that this bubble remains of uniform diameter. The pressures in *A* and *C* are then balanced. To provide this tilting the manometer proper is mounted on a tilting frame *T*, which pivots on the knife edges at *G* and is elevated by the screw *F*. The whole is carried on a bed frame *Z* fitted with three leveling screws *I*, a retaining spring *H*, and a scale *S*, on which may be read the full turns of the screw *F*.

A microscope, *M*, fitted with cross-hairs is mounted on the frame *T* and directed at the bubble *B*. A small mirror on the opposite side illuminates the surface of the bubble.

The screw *F* is fitted with a large drum divided into 100 parts. The screw has 20 threads to the inch. The gage is sensitive to one-half of a division on the drum, and hence to a movement of the screw of 1/4000 inch.



CHATTOCK GAGE AND ALCOHOL GAGE. USED FOR VELOCITY MEASUREMENTS

Before a measurement is taken, the bulbs *A* and *C* are opened to the air of the room and the frame tilted by moving the micrometer until the top of the bubble *B* is brought tangent to the horizontal cross-wire of the microscope. This is the zero reading. The bulbs *A* and *C* are then connected to the two parts of a Pitot tube and the frame tilted until the bubble is again on the cross-wire. The amount of tilt is then read on the micrometer.

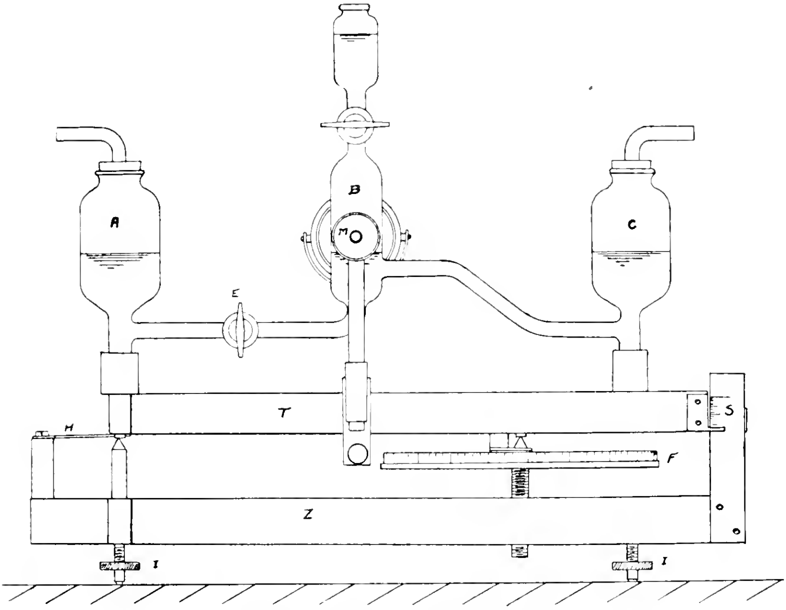


FIG. 2.—Chattock micromanometer.

Naturally too wide an excursion of the bubble will result in its rupture. The loss of a bubble transfers a drop of water from *A* to *B*, and hence a new zero reading must be found by balancing up again. To avoid sudden change of pressure and breaking of the bubble, a stop cock at *E* is fitted. This cock can be closed to make the instrument portable, and in taking a reading an approximate balance is made with *E* partly opened. The cock is then opened full.

The gage is filled with a solution of salt and water of s. g. 1.06. The addition of a little salt keeps the castor oil from growing cloudy.

Two gages were constructed, one by a skilled glass blower and the other by a student, with a view to determining the effect of workmanship and dimensions. The frame and stand were made

identical in the two gages, but the glass work was purposely altered. The tip of the tube at *B* was ground to a knife edge in one gage and in the other ground off square. One tube was .20 inch in diameter and the other .15 inch in diameter.

The two gages were connected to the same static pressure and gave readings identical to 0.25 per cent. It was found that the gage in no way is affected by minor variations in workmanship.

In the gage with the ground knife edge tip it was found that the bubble did not break so readily as in the gage with the square tip. It was suggested by Professor Gill that the tenacity could further be increased by coating the outside of the tube below the bubble with paraffin. This was tried and was found to be of great assistance. A height of bubble from three to four times the diameter of the tube at its base could be allowed without rupture. The reason for this is to be found in the fact that castor oil sticks very tight to glass but will not stick to paraffin. By the use of this wax the bubble could not creep over the edge of the tube and so slide down it causing a break. However, any large excursion of the bubble is to be avoided as tending to cause a slight change in the zero reading. In all tests the zero should be taken at intervals. The effect of the paraffin on the tip could not be detected in the readings of the gage.

The consistency of the gage readings with these various alterations in the base of the bubble as well as in the size of the bulbs and connecting tubes gives great confidence in this type of gage. It was not possible to calibrate this gage experimentally because there was no other gage available to measure it against which was equally sensitive. However, we have Professor Chattock, Dr. Stanton, and the National Physical Laboratory as authority for the calibration of the gage by calculation from the dimensions of its parts, and the density of the liquid. It may be noted that the density of the oil used has no effect on the principle of the gage and is not considered.

For the calculation of tilt, it is then necessary to measure the distance between the centers of the bulbs *A* and *C* and the distance from the knife edge *G* to the screw *F*. An error of 0.1 inch in either of these measurements is an error of 1 per cent in head or 0.5 per cent in velocity. There is no difficulty in getting these distances to the nearest hundredth of an inch. The screw thread was cut so precisely that it was impossible to detect any error in the pitch of the thread. The hole in *Z* was tapped with a standard Brown and Sharp tap. The calculation of the change in level of the surfaces of the liquid in

A and C is precise to 0.1 per cent. The density of the solution was taken on a Westfall balance to the same degree of precision.

Since the gage is sensitive to less than 0.1 per cent for heads of more than 0.3 inch, the measurement of velocity depends on the precision of the Pitot tube. The latter is good to probably 0.25 per cent in velocity. However, the air current always has some fluctuation at high speeds so that in the end the velocity measurement is limited in precision by the closeness with which such fluctuations can be averaged. In a very steady current, such as our wind tunnel, it was found that the error in estimating velocity was less than 0.5 per cent. The average of a number of observations is of course better than this.

Change in density of the salt solution is $\frac{1}{3}$ per cent for a change of 60 degrees F. in temperature. A temperature correction is ordinarily unnecessary.

An alcohol gage is a sensitive and consistent instrument, but requires calibration to eliminate errors due to viscosity and capillarity. The question of its suitability for precise work will be discussed later in another paper. It has the great advantage over the Chattock gage in that it requires no delicate manipulation to get a balance, no cross-wire and microscope, and with it it is possible to estimate the mean of fluctuations. The alcohol gage has been successfully used to measure air speeds as low as two miles per hour. It is shown with the Chattock gage in plate 5.

II. NOTES ON THE DIMENSIONAL THEORY OF WIND TUNNEL EXPERIMENTS

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Introduction

The forces which will act between a solid body and a fluid in contact with it in consequence of a relative motion of the two, cannot, except in a few of the simplest cases, be predicted by computation from the size and shape of the body, the relative velocity, and the physical properties of the fluid: the information can be obtained only from experiment. Such experiments may be expensive or impracticable, and it often appears desirable to get the required information, in advance of the final decision on the design of a structure which is to be subject to aerodynamic or hydrodynamic forces, by making preliminary experiments on a small model of the proposed structure.

In order that the results of such observations shall be interpretable as definite statements about the behavior of the full-sized original of which the model is a copy, certain requirements must be satisfied, and when they are satisfied the original and the geometrically similar model are said to be dynamically similar. The conditions for dynamical similarity are bound up with the general question of the possible forms of equations which describe relations subsisting among the physical quantities involved in physical phenomena.

Nature of the Problem to Be Discussed

Let us suppose that a solid body is moving, with the constant velocity S, through a fluid which is itself sensibly at rest at points far distant from the body; and let us consider the forces exerted on the body by the surrounding fluid. Since these forces are evidently due to the relative motion, they would remain unchanged if the body were held at rest and the fluid made to flow past it with the velocity $(-S)$. The boundaries of the fluid are supposed to be so distant from the solid body that no sensible disturbance reaches them, and their nature can then have no influence on the forces with which we are concerned and need not be further referred to. If the fluid is a liquid with a free surface, the foregoing condition requires that the moving body be so deeply immersed as not to cause any surface disturbances.

Let R be any force exerted by the fluid on the body; for example, the component in any specified direction of the force on some particular part of the solid surface; or, to make it more definite, let R be the total head resistance in the direction of motion. Then R will depend on and be completely determined by the relative speed, the size, shape, and attitude of the body, and the mechanical properties of the fluid; and there must be a definite relation connecting these various physical quantities, which can be described by an equation. We wish to consider the nature of this equation in so far as it is fixed by the natures of the separate quantities involved in it.

The Physical Quantities Which Influence Fluid Resistance

Let D be some linear dimension of the body, such as its greatest length. The shape of the body and its attitude, *i. e.*, its orientation with regard to the direction of motion can be specified by stating the ratios of a number of lengths to the particular length D. If these ratios are denoted by $r', r'', r''', \ldots$, etc., the size, shape, and attitude of the body are completely specified by the values of $D, r', r'', \ldots$, etc.

The properties of the fluid which determine its mechanical behavior are its density ρ, its viscosity μ, and its compressibility. Instead of the viscosity, it is generally more convenient to use the kinematic viscosity $\nu = \frac{\mu}{\rho}$ which will do equally well when ρ is given. And similarly, the speed C of sound waves in the fluid is fixed by the density and compressibility so that, conversely, C together with ρ fixes the compressibility. The properties of the fluid which concern us may therefore be specified by stating the values of the density ρ, the kinematic viscosity ν, and the acoustic speed C in the fluid.

We have now enumerated the quantities on which the force R may be supposed to depend, and if nothing has been overlooked there must be a complete relation connecting R with the other quantities. We may state the fact that such a relation subsists by writing the equation

$$f\,(R, S, D, r', r'', \ldots, \rho, \nu, C) = 0, \tag{1}$$

and our first task is to obtain from general principles any information we can about the form of this unknown function f, which will enable us to restrict the amount of experimentation required to finish the work of finding the form of the equation.

Application of the Principle of Dimensional Homogeneity

By the well known "principle of dimensional homogeneity," all the terms of a complete physical equation must have the same dimensions, and this fact enables us to simplify equation (1). Let Π represent a dimensionless product of the form

$$\Pi = R^{\alpha} S^{\beta} D^{\gamma} \rho^{\delta} \nu^{\epsilon} C^{\zeta}, \tag{2}$$

the numerical exponents α, β, γ, etc., being such as to satisfy the dimensional equation

$$[R^{\alpha} S^{\beta} D^{\gamma} \rho^{\delta} \nu^{\epsilon} C^{\zeta}] = [1] \tag{3}$$

when the known dimensions of R, s, D, ρ, ν, and C are inserted. Then it may readily be shown:[1] 1st, that since three fundamental units are needed as the basis of an absolute system for measuring the six kinds of quantity, R, S, D, ρ, ν, and C, the number of possible independent expressions of the form (2) is $6-3$ or 3; and 2d, that if these expressions are denoted by Π_1, Π_2, Π_3, any correct equation involving the quantities which appear in equation (1) and no others, must neces-

[1] Physical Review (2), 4, p. 345, October, 1914.

sarily, in order to have all its terms of the same dimensions, be reducible to the form

$$F(\Pi_1, \Pi_2, \Pi_3, r', r'', \ldots) = 0. \tag{4}$$

In addition to the dimensionless ratios r', r'', etc., there now appear in the equation only three instead of the original six variables, so that the labor of determining by experiment the form of the unknown function is much less than if we had to deal with all six variables.

The More Specific Form of the Equation of Fluid Resistance

The dimensions of the quantities on the familiar mass, length, time, or $[m, l, t]$ system are:

$$[R] = [mlt^{-2}], \qquad [\rho] = [ml^{-3}],$$
$$[S] = [lt^{-1}], \qquad [\nu] = [l^2t^{-1}],$$
$$[D] = [l], \qquad [C] = [lt^{-1}];$$

and we see by inspection that the expressions

$$\Pi_1 = \frac{R}{\rho D^2 S^2}, \quad \Pi_2 = \frac{DS}{\nu}, \quad \Pi_3 = \frac{S}{C}$$

are dimensionless products of the required form (2), and that they are independent. Accordingly, we know that equation (1) must be reducible to the form

$$F\left(\frac{R}{\rho D^2 S^2}, \frac{DS}{\nu}, \frac{S}{C}, r', r'', \ldots\right) = 0. \tag{5}$$

This equation is fundamental to the experimental study of the hydrodynamic or aerodynamic forces acting on totally immersed bodies.

Solving for Π_1 we now have

$$\frac{R}{\rho D^2 S^2} = \phi\left(\frac{DS}{\nu}, \frac{S}{C}, r', r'', \ldots\right), \tag{6}$$

in which the form of the unknown function ϕ remains to be found, if it needs to be found at all, by experiment or by other than dimensional reasoning.

Simplification When Compressibility May be Disregarded

A simplification is possible when the motion is not rapid enough to cause any sensible compression in the fluid. In this event it is immaterial what the compressibility is, so that $\frac{S}{C}$ may be omitted from consideration and equation (6) reduces to

$$\frac{R}{\rho D^2 S^2} = \phi\left(\frac{DS}{\nu}, r', r'', \ldots\right). \tag{7}$$

The approximation attainable when compressibility is thus left out of account depends on the value of $\frac{S}{C}$. If $\frac{S}{C}$ is a small fraction, as it nearly always is with liquids, equation (7) is a satisfactory substitute for (6). The speed of sound in air under ordinary conditions is of the order of 1100 feet per second, or 750 miles per hour. For rifled projectiles $\frac{S}{C}$ may be as high as 2.5 or even 3, so that equation (7) would be entirely misleading if used in studying projectile resistances. But at the speeds which occur in aeronautics, with the exception of propeller tip speeds, the ratio $\frac{S}{C}$ is a sufficiently small fraction that the air acts nearly like an incompressible fluid, *i. e.*, like a liquid of the same density and viscosity; and equation (7) may be used as a sufficiently approximate substitute for the more general equation (6).

Equation (7) supplies the basis for the experimental investigation of the aerodynamic problems which occur in connection with aeronautics and aviation by means of reduced scale methods.

Restriction to Geometrically Similar Bodies

Let us now confine our attention to a series of bodies of various sizes but all of the same shape, and presented to the wind in the same attitude. The bodies are geometrically similar, and any one may be regarded as a reduced or enlarged model of any other. The ratios r', r'', ... are now constants, so that equation (7) assumes the simpler form

$$\frac{R}{\rho D^2 S^2}=\psi\left(\frac{DS}{\nu}\right). \tag{8}$$

in which the form of the unknown function ψ of the single argument $\frac{DS}{\nu}$ remains to be determined by experimenting on bodies of the given series. The nature of this function will depend on the shape and attitude of the bodies but not on their size, if our disregard of compressibility, leading from (6) to (7), was a justifiable approximation.

The obvious procedure, in investigating ψ by analyzing the results of experiments, is to plot observed values of $\frac{R}{\rho D^2 S^2}$ against values of $\frac{DS}{\nu}$ and draw a curve through the points thus obtained. If we are using air of constant density and viscosity the experiments may con-

sist most simply in measuring, by the aerodynamic balance, the force R exerted on a given body at various values of the wind speed S. Variations of $\frac{DS}{\nu}$ may equally well be produced by varying D while S is constant, *i. e.*, by experimenting at a fixed speed but with a series of models of different sizes; or, D, S, ρ, and ν may all be varied simultaneously. But while such experiments furnish a desirable check on the results obtained when S alone is varied, they are not necessary, if compressiblity is negligible; for it is immaterial whether $\frac{DS}{\nu}$ is changed by changing D, S, or ν.

If the plotted points obtained in any of these ways do not all lie on a single curve, within their experimental errors, equation (8) is not accurate enough. And if the models have been exactly geometrically similar, we must conclude that compressibility has played some part in the phenomenon. This means that in the more general equation

$$\frac{R}{\rho D^2 S^2} = \phi\left(\frac{DS}{\nu}, \frac{S}{C}\right) \tag{9}$$

obtained by applying (6) to geometrically similar bodies, the effects of varying $\frac{S}{C}$ are not of entirely negligible importance.

Head Resistance Proportional to S^2; Viscosity Negligible

At ordinary speeds and for bodies that are not too small, experiment shows that in air of standard density, R is very nearly proportional to S^2. It follows that, to the degree of approximation to which equation (8) is valid, $\psi\left(\frac{DS}{\nu}\right)$ is merely a constant and is independent of the values of D, S, and ν. If we write

$$\psi\left(\frac{DS}{\nu}\right) = K,$$

equation (8) reduces to

$$R = K\rho D^2 S^2. \tag{10}$$

As is seen by referring to equation (7), K depends on the values of r', r'', ... , etc.; it is a shape factor for the given series of geometrically similar bodies in the given attitude.

It is to be noted that viscosity does not appear at all in equation (10), so that when the resistance is found to be proportional to the square of the speed, if compressibility is negligible the value of the viscosity is of no importance. This is not equivalent to saying that viscosity plays no part at all in the phenomena; for if viscosity did

not exist there would be no eddies of finite size, no dissipation, and at a constant speed no resistance. It means, rather, that the drag on the body by the fluid is due to the continual drain of energy needed to set up anew the turbulent eddying motion about the body; and that when these eddies have once been created it makes no difference how fast they are dissipated by viscosity after the body has left them behind.

The Critical Speed

In the foregoing case of resistance proportional to S^2, the plot of $\frac{R}{\rho D^2 S^2}$ as ordinate against $\frac{DS}{\nu}$ as abscissa gives, of course, a horizontal straight line for bodies of a given series. But if the experiments are carried down to smaller and smaller values of $\frac{DS}{\nu}$, a critical value may be reached where the relation ceases to hold and the character of the fluid motion changes very rapidly, though apparently not discontinuously, so that the function ψ ceases to be a constant for low values of $\frac{DS}{\nu}$. For a given body in a given medium this critical value of $\frac{DS}{\nu}$ corresponds to a critical speed S_c which may be computed *a priori* from the values of D and ν, if the critical value $\left(\frac{DS}{\nu}\right)_c$ has once been determined for bodies of the given shape by varying any one of the variables D, S, and ν, or all together. Eiffel's observations on spheres[1] confirm the foregoing statements when we take into consideration not merely a single speed for each diameter but the whole critical range within which the rapid change in the form of ψ occurs.

Mr. Hunsaker's observations on sharp cornered disks of different diameters, but the same thickness, are very interesting as showing the possible importance of such sharp edges or corners. The disks were not geometrically similar; but the corners at the edges were not only nearly similar but, inch for inch, very nearly identical. Accordingly, that part of the total resistance which may be regarded as due to the sharp corners at the edge reached its critical value always at about the same speed, irrespective of the diameter of the disk which was bounded by the edge. The rapid change in the total resistance near this speed seems to indicate that the "corner resistance" formed a considerable fraction of the whole.

[1] C. R. 155, p. 1597, December 30, 1912.

The occurrence of a critical speed for a given body in a given attitude is paralleled by the practically much more important phenomenon of the occurrence of a critical attitude at a given speed. Just as the nature of the fluid motion and the law of resistance of a given body change rapidly at a certain critical range of speed, so there are similar rapid changes in the motion and the forces at the critical angle of attack for a given aerofoil at a given speed.

Remarks on the Resistance of Flat Plates Normal to the Wind

From equation (8) it would appear that when R is proportional to S^2 it must also be proportional to D^2, and yet this does not seem to be true for flat plates normal to the wind. On the contrary, while $\frac{R}{\rho D^2 S^2}$ is nearly independent of S, the results of various observers indicate that it increases somewhat as the diameter of the plate increases from a few inches to a few feet. Leaving aside the improbable supposition that this effect is only apparent and due to observational errors, the most obvious explanation is that compressibility may not be entirely negligible. If that is the explanation, equation (9) and not equation (8) is the one to be used, and it is quite conceivable that $\phi\left(\frac{DS}{\nu}, \frac{S}{C}\right)$ might have such a form as to be independent of S without being entirely independent of D. Computations of the amount of compression to be expected at the speeds in question[1] seem to show that the discrepancies are too large to be accounted for in this way. But it may be remarked that in some of the details of the turbulence much higher speeds may occur than the speed of the wind as a whole. Hence compression might occur locally, in some parts of the field about the body, to such an extent as to modify the flow and so affect the resistance, even though computations based on the average speed of the wind might indicate that the effects of compression could not possibly be appreciable under the given experimental conditions.

Mr. Hunsaker's observations on circular disks suggest, however, that there may be another interpretation of the effect in question which does not oblige us to have recourse to the unlikely supposition that compressibility is of importance. If, as appears from these experiments, there is a critical range of speed determined by the form of

[1] See Bairstow and Booth: Rep. British Adv. Committee for Aeronautics, 1910-11, p. 21.

the edge and not dependent on the size of plate, it seems possible that some of the apparent discrepancies between $\frac{R}{S^2}$ = constant and $\frac{R}{D^2}$ = variable may be due to the experimental results of various observers having been influenced by such critical phenomena, which were not, however, sufficiently marked to attract attention.

To decide whether an explanation of this sort is applicable would require an experimental study of the forms of edge which have been used; for until the critical speeds for these edges—if they have any—have been investigated, it is impossible to say whether the speeds at which various experimenters have worked may have overlapped with these critical ranges. Nothing more definite can be said, at present, than that it is well to pay close attention to geometrical similarity: but it seems that a further experimental study of the resistance of flat plates, undertaken with the foregoing possibilities in mind, might lead to interesting results.

Dynamical Similarity

Let us suppose that we are confronted with a problem of design which requires our knowing, in advance, the head resistance, at a prescribed speed, of some body such as an air-ship which is too large for direct experiment. The question is, how to get the desired information from experiments on a small model which can be made at a permissible cost.

Returning to equation (9) or

$$R = \rho D^2 S^2 \phi \left(\frac{DS}{\nu}, \frac{S}{C} \right) \tag{11}$$

we notice that whatever be the form of ϕ, if its two arguments have the same values during two different experiments on geometrically similar bodies, the value of ϕ itself will be the same in both experiments. This observation leads to the notion of corresponding speeds and dynamical similarity.

Let us suppose that we require the resistance R of a body of size D at the speed S in a medium with the properties ρ, ν, C; and that we have a model of the size D_m, which can be run in a medium with the properties ρ_m, ν_m, C_m. Then if we run the model at a speed S_m, such that

$$\frac{D_m S_m}{\nu_m} = \frac{DS}{\nu} \text{ and } \frac{S_m}{C_m} = \frac{S}{C} \tag{12}$$

and observe the resistance R_m, we know by equation (11) that

$$\frac{R}{R_m} = \frac{\rho D^2 S^2}{\rho_m D^2_m S^2_m}. \tag{13}$$

For when equations (12) are satisfied, $\phi\left(\frac{D_m S_m}{\nu_m}, \frac{S_m}{C_m}\right) = \phi\left(\frac{DS}{\nu}, \frac{S}{C}\right)$, so that ϕ cancels out when we divide equation (11) for the full-sized original by the corresponding equation for the geometrically similar model. Speeds which satisfy equations (12) are "corresponding speeds," and when two geometrically similar bodies are run at corresponding speeds they are "dynamically similar."

If the speeds are low enough that compressibility may be disregarded, the value of $\frac{S}{C}$ is unimportant and the condition for corresponding speeds, which ensures dynamical similarity, is merely the first of equations (12). If we use only a single medium so that $\rho_m = \rho$ and $\nu_m = \nu$, the condition for corresponding speeds reduces to

$$\frac{S_m}{S} = \frac{D}{D_m}$$

and geometrically similar bodies will be dynamically similar if their speeds are inversely as their linear dimensions. Any great reduction in scale might therefore involve our running the model so fast as to make the effects of compressibility no longer negligible. But if the original is to be run in air while the model can be run in water, this difficulty may be avoided. For under ordinary conditions the kinematic viscosity of water is from 1/10 to 1/20 that of air, and for a model of given size the speed required for dynamical similarity with the original is reduced in the same ratio as the kinematic viscosity.

In practice the foregoing method of experimentation is usually unnecessary. For under ordinary working conditions the resistances of aeroplanes and their separate structural elements are so nearly proportional to the square of the speed, and the effects of compressibility are so small, that for practical purposes ϕ in equation (11) or in equation (9) may be treated as a constant and equation (10) used for computation, within any ordinary ranges of D and S. *Any* speeds may then be regarded as corresponding speeds, and geometrical similarity suffices by itself for dynamical similarity. If the constant K of equation (10) has been determined by experiments on any body of the given shape at any convenient speed, the same value may be used in equation (10) for computing the value of R for a different speed or a different size or both.

Complete Dynamical Similarity

The experience with flat plates, showing that even though R is proportional to S^2 it may not be to D^2, warns us to be cautious in

assuming that equation (10) may be relied on for great accuracy when the size D changes over a very large range; and it seems possible that it may sometimes be desirable to make experiments guided by equation (11) which holds for any series of geometrically similar bodies, whatever the speeds may be.

The conditions for dynamical similarity given by equations (12) can evidently not be satisfied if we work with only a single medium; for if $\nu_m = \nu$ and $C_m = C$, we have $S_m = S$ and $D_m = D$, so that no scale reduction is possible while preserving dynamical similarity. This difficulty may, in principle, be surmounted by running the model in water if the original is to run in air. Suppose, for instance, that the original is an air-ship which is to run 40 miles an hour in air, and let the model be run in water at such a temperature that its kinematic viscosity is 1/15 that of the air. We then have $\nu = 15\nu_m$ and the first of equations (12) gives us

$$15 D_m S_m = DS. \qquad (14)$$

The second condition requires that the speed of the model shall be the same fraction of the speed of sound in water as 40 miles per hour is of the speed of sound in air. Since sound travels about four times as fast in water as in air, the model must move at the very high rate of 160 miles per hour, or about 235 feet per second. With this condition that $S_m = 4S$, and the previous condition stated by equation (14), we have

$$D_m = \frac{1}{60} D.$$

A model to 1/60 scale run in water will then be dynamically similar to the original in air, if it is run four times as fast. Having thus satisfied equations (12) we may use equation (13); and if we set $\rho_m = 800\rho$ we have

$$\frac{R}{R_m} = \frac{1}{800} \times 60^2 \times \left(\frac{1}{4}\right)^2 = \frac{9}{32}.$$

The resistance of the original in air will therefore be about one-quarter of the resistance of the dynamically similar 1/60 scale model in water. How soon it will seem worth while to attempt experiments of this sort cannot be predicted, but the notion of dynamical similarity shows how the problem may be attacked.

The Pitot Tube

Hitherto we have let R be the total head resistance of a solid body, but if D is the diameter of the impact opening of a Pitot tube, $\frac{R}{D^2}$ may

evidently be regarded as a quantity which is proportional to the impact or velocity pressure p. Hence equation (6), as applied to the Pitot tube at rest in a current of fluid, may be written

$$p=\rho S^2\phi\left(\frac{DS}{\nu},\frac{S}{C},r',r'',\ldots\right). \tag{15}$$

and it is interesting to compare this with the known behavior of Pitot tubes and with the Pitot equation as ordinarily given.

In the first place, we know by experience that if the impact opening is the mouth of a long tube pointed up stream, the precise form of the tube and the shape and diameter of its mouth have no appreciable influence on the impact pressure recorded. This means not only that the shape variables r', r'', ..., etc., are of no importance and may be omitted from among the arguments of ϕ, but also that D is likewise of no importance, so that the argument $\frac{DS}{\nu}$ in which it appears may be omitted. Equation (15) thus reduces to the form

$$p=\rho S^2\psi\left(\frac{S}{C}\right). \tag{16}$$

When the fluid is nearly incompressible, like water, the compression caused by the impact pressure p will be so slight that it cannot affect the general behavior of the fluid. Hence compressibility may be left out of account and ψ treated as a constant, so that we have

$$S=\text{const}\times\sqrt{\frac{p}{\rho}}. \tag{17}$$

If p is measured as a head h of the liquid, we have $p=g\rho h$, and equation (17) reduces to

$$S=\text{const}\times\sqrt{gh}.$$

The value of the constant, which cannot be found by dimensional reasoning, is, in practice, $\sqrt{2}$ for a properly constructed tube.

If the fluid is a gas, equation (17) is still applicable when the speed is low. But when the speed is so high that the pressure p causes appreciable compression, $\frac{S}{C}$ cannot be neglected and we must return to equation (16). A form of $\psi\left(\frac{S}{C}\right)$ for high gas speeds may readily be found from thermodynamics, but so many approximations and unproven assumptions have to be made in the course of the argument that the results are not at all convincing.

III. THE PITOT TUBE AND THE INCLINED MANOMETER

By J. C. HUNSAKER

For aeronautical purposes the absolute measurement of velocity is of less importance than the measurement of impactual pressure. For this reason, an anemometer from which the velocity may be deduced from a pressure measurement is preferable to any of the vane anemometers which measure velocity directly, and require a somewhat laborious calculation of the density of the air before the effect of the wind can be evaluated.

The most common as well as the most convenient form of pressure anemometer is the double Pitot tube. Reference may be made to the papers of Taylor[1] and Zahm,[2] in which it is shown that the equation to a stream line in any perfect gas may be simplified in the case of air by considering the air incompressible for velocities below 100 feet per second. The simplified expression connecting pressure and velocity in moving air is then Bernoulli's equation as used in hydraulics:

$$\frac{\rho v_1^2}{2g}+p_1=\frac{\rho v_2^2}{2g}+p_2,$$

where v_1 and p_1 are velocity and pressure at any point, and v_2 and p_2 are corresponding values for some other point in the same stream line.

Let us choose the point where v_2 is zero, then

$$\frac{\rho v_1^2}{2g}+p_1=p_2.$$

In air this is the barometric pressure. Let us change the notation so that $\frac{\rho v^2}{2g}+p=p_0=$ barometric pressure, a constant.

p is now the pressure in the unchecked stream, the "static" pressure, and p_0 is the pressure in the impact tube where the current is brought to rest. This is called "dynamic" pressure.

The Pitot tube is a device for transmitting the pressure difference, $p_0-p=\frac{\rho v^2}{2g}$, from which the velocity may readily be calculated. The quantity $\frac{\rho v_2^2}{2g}$ is commonly called "velocity" pressure.

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[1] Experiments with Ventilating Fans and Pipes, by D. W. Taylor, Naval Constructor, U. S. Navy, Trans. Soc. Naval Architects and Marine Engineers, 1905.

[2] Measurement of Air Velocity and Pressure, A. F. Zahm, Ph. D., Physical Review, December, 1903.

If one end of an open tube be pointed into a stream of air and the other end be attached to a manometer, the total dynamic pressure will be recorded. On the other hand, if a tube with closed end be pointed into the wind and further fitted with a conical or parabolic tip, the stream line is only slightly deflected and distorted. If then small holes or slots be cut in this tube at a distance well back from the tip, the wind should blow past these openings and the interior of the tube should be subjected to the static pressure of the stream. This pressure can be measured by connecting the tube to a manometer.

It is generally accepted from the results of tests that any open-ended tube of any size, if pointed fairly into the wind, will correctly transmit the dynamic pressure.

It is equally common knowledge that the correct transmission of the static pressure is not so simple. Widely different values are obtained with different forms of tube and static orifice, and many tubes, such as the Dines and the Recknagel, must be calibrated against some standard. It is obvious that the nose of the tube should be of easy form, that the tube should not be large in diameter, and that it should be carefully polished in order that the air stream may pass undisturbed. The best form of entrance will introduce some disturbance, so that such static openings as are used should be placed well back from the nose on the cylindrical portion. The form and size of the openings will be discussed later.

The Pitot tube may consist of two separate tubes or a double tube made up of a pair of concentric tubes, the dynamic tube being enclosed within the static tube. Since the dynamic tube transmits the pressure $p+\frac{\rho v^2}{2g}=p_0$, and the static tube transmits p, it is sufficient to connect the two tubes to the two ends of a U-tube filled with liquid. The reading of the instrument is then proportional to the difference between the pressures transmitted and hence to $\frac{\rho v^2}{2g}$. Knowing the density, the velocity may be computed. The density of air depends on the pressure, temperature, and humidity. Avoidance of the necessity for calculating the density for ordinary aerodynamical tests would be of great assistance.

Elimination of Density of Air

It is generally accepted that the forces produced by a fluid in motion with reference to any solid object depend on the size, shape, and attitude of that object, the velocity, the density of the fluid, and

its viscosity, and upon nothing else for ordinary transportation speeds.

The most general expression[1] for this statement which satisfies the theory of dimensions is

$$R = \rho L^2 V^2 f\left(\frac{VL\rho}{\mu}\right).$$

in which

L denotes the length of any linear dimensions of the solid,
V, the relative velocity of solid and fluid,
ρ, the density of the fluid,
μ, the coefficient of viscosity of the fluid,

and f is a function of the single variable $\frac{VL\rho}{\mu}$.

It will be noted that the compressibility of the fluid has been neglected.

The value of $f\left(\frac{VL\rho}{\mu}\right)$ is very nearly constant for bodies of a given shape in a given orientation when the motion of the fluid is sufficiently turbulent. Experimentally it is found that $R \propto V^2$, nearly, and hence not only is $f\left(\frac{VL\rho}{\mu}\right)$ nearly constant, but the influence of viscosity is small. The changes in $f\left(\frac{VL\rho}{\mu}\right)$ with change of scale, density, and viscosity are hence in the nature of a correction.

For objects moving through the air at very low speed, especially objects of easy form, turbulence is not marked and viscosity is of importance. Consequently for such tests the assumption of f constant is not justified.

However, for aeroplane wings, parts, etc., moved through the air at high speeds the resistance to motion is largely due to turbulence, R varies nearly as ρV^2 and $f\left(\frac{VL\rho}{\mu}\right)$ is constant nearly. Therefore, we may assume that for the ordinary work of an experimental wind tunnel, forces to be measured will vary as the density of the air. Likewise, the manometer reading obtained from a Pitot tube will vary as the density of the air.

It has been decided to adopt a standard density for air to be used throughout. Velocity computed from a manometer reading is then referred to this standard air, and forces measured on the balance are

[1] Helmholtz, Wissenschaftliche Abhandlungen, Vol. I, p. 158; O. Reynolds, Phil. Trans. Roy. Soc., 1883, p. 935; Lord Rayleigh, Phil. Mag., 1899, p. 321.

referred to the same standard. Standard air is taken to be dry air under the following conditions:

Barometric pressure, 29.921 inches mercury.
Temperature, 62 degrees Fahrenheit.
Density, 0.07608 pound per cubic foot.

Comparison of Pitot Tubes

Opportunity was taken to compare the National Physical Laboratory standard Pitot tube, calibrated by Bramwell,[1] with several forms of tube in use by engineers in the United States.

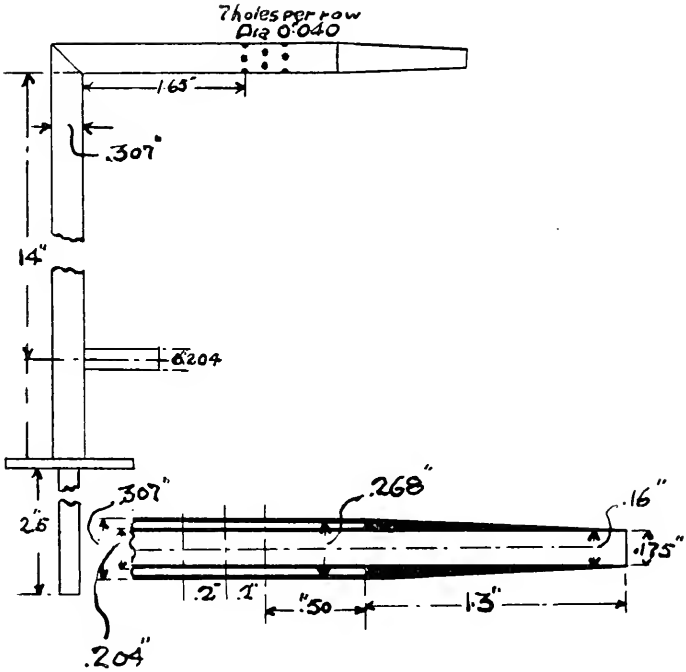


Fig. 3.—N. P. L. tube.

The tube under investigation was mounted in the center of the tunnel and connected by rubber tubing with a Chattock micromanometer. Care was taken to eliminate leaks in the leads, and to point the tube parallel to the axis of the tunnel. A steady wind was then blown through the tunnel and its velocity read from the alcohol gage connected with the side suction plate which had already been calibrated as described in a previous paper. The velocity from the side

[1] Technical Report of the Advisory Committee for Aeronautics, London, 1912-13.

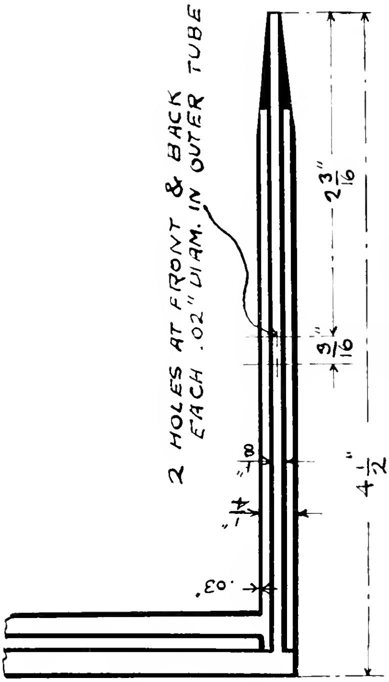


FIG. 4.—A. B. C. tube.

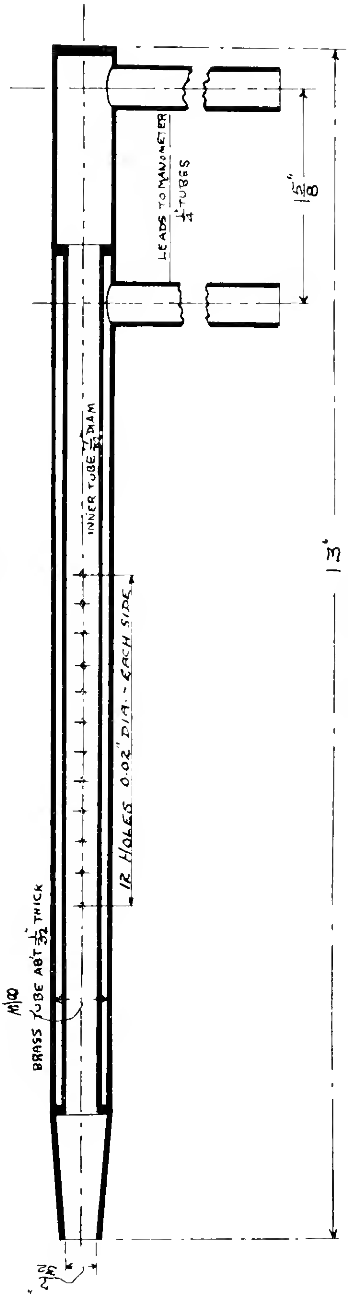


FIG. 5.—Tube A.

plate was then compared with the velocity indicated by the Pitot tube under test. Comparisons were made at a number of speeds for each tube.

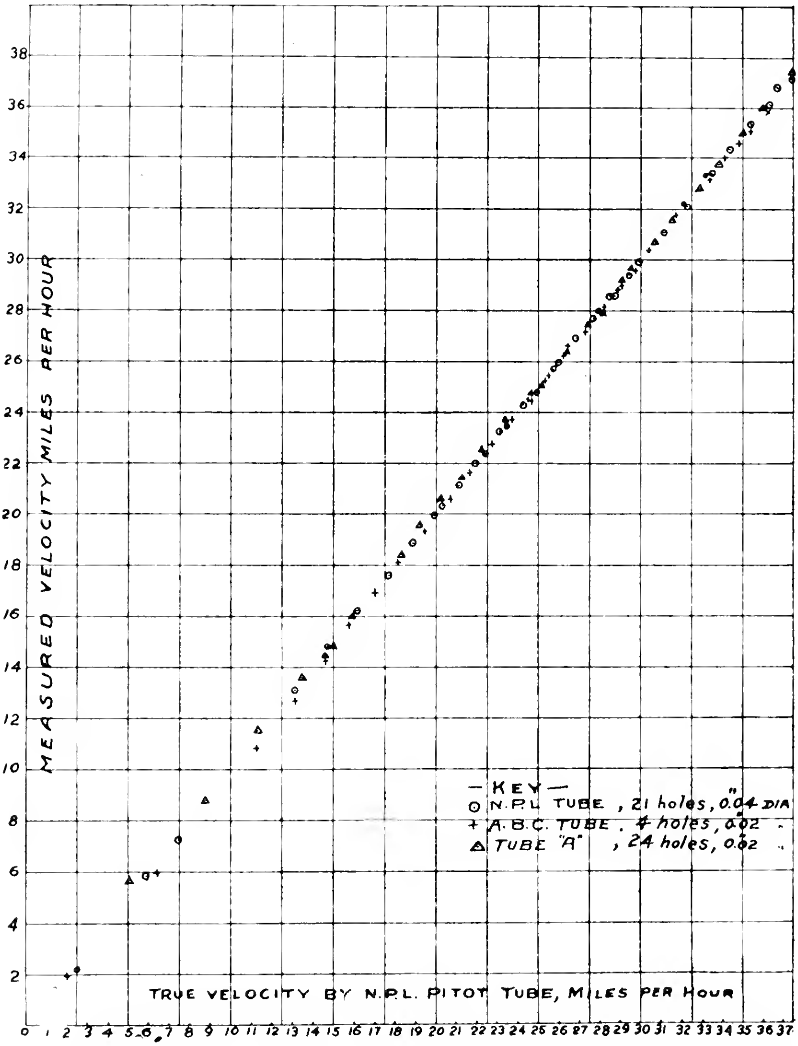


Fig. 6.—Comparison of three forms of Pitot tube. Each point represents one observation, and serves to bring out the sensitivity of the Pitot tube and manometer measurements employed.

It was demonstrated that the velocity is correctly measured by any tube having an easy entrance and a long cylindrical portion parallel to the wind in which a number of small holes are drilled to transmit

the static pressure. The arrangement of the holes appeared to have no effect. Long slots in the tube introduced large errors if the tube were not pointed fair into the wind. The size of the tube appeared to be immaterial.

The results of comparison of three tubes with different arrangements of holes from 4 to 24 in number are shown on figure 6. The tubes are shown on figures 3, 4, and 5. The agreement is very close—within ½ per cent at velocities above 10 miles per hour.

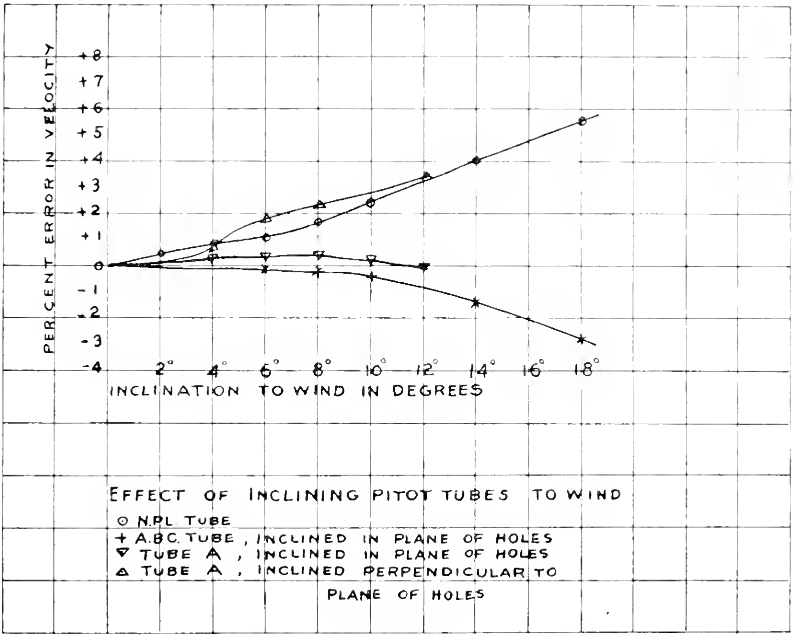


FIG. 7.—Effect of inclining Pitot tubes to wind.

Tests were made to determine the effect of inclining the tubes to the wind. The tubes with holes show an error of but 1 per cent for an inclination of 4 degrees. The results are shown on figure 7.

The following conclusions may be drawn from these tests:

(1) Static openings should be small holes about 0.03 inch in diameter to minimize effects of bad alignment or turbulence.

(2) An error of 2 degrees in aligning the tube causes no important change in velocity reading.

(3) If a tube is correct at two speeds it remains correct at all others within the range of our experiments.

(4) Holes should be symmetrically distributed on the cylindrical portion of the tube.

(5) Any new type of tube should be calibrated against a standard.

The Inclined Manometer

Granted that a Pitot tube is at hand which will correctly transmit the static pressure, measurements of velocity are no better than the manometer used. The ordinary U-tube filled with water, gasoline, alcohol, or other light liquid shows a head of less than 1 inch for ordinary velocities. To obtain the velocity head within 1 per cent, it would be necessary to read the displacement of the meniscus to within 0.01 inch. This is hardly practicable, and various devices are used to magnify the reading. It is at once apparent that if the U-tube be canted at an angle of 1 in 20, a 1-inch head of liquid corresponds to 20 inches on the scale. With an inclined tube, the diameter of the tube must be kept small in order to obtain a good meniscus for reading. On the other hand, in any gage in which the imperfections in the glass produce changes in capillarity, the liquid sticks at some places. A large tube tends to reduce this source of error. The American Blower Company recommend an inclined U-tube filled with gasoline for use with the Pitot tube. This type involves the simultaneous reading of the meniscus level in each leg of the tube, a somewhat difficult feat in an unsteady current.

The German "Krell" manometer is filled with alcohol colored to give a visible meniscus. One leg of the U-tube is an inclined glass tube, and the other is a reservoir bottle whose section is some 400 times the section of the tube. Hence, as liquid rises in the glass tube the depression in the reservoir is unimportant. Only one meniscus level then need be read.

An inclined tube manometer on the Krell principle was constructed, and an investigation made of its errors by comparing it with a Chattock manometer known to be nearly correct. This alcohol manometer is shown in figure 8. It is seen to include a reservoir *R* mounted on a hinged plate with leveling screw. By means of the latter, the liquid in the tube is brought to the zero of the scale at the beginning of a test, thus making a zero correction unnecessary. The glass tube *T* is likewise mounted on a brass plate pivoted at the knife edge *K*, and adjusted in pitch by the screw *S*. To the brass plate are attached permanently two small machinists' spirit levels L_1 and L_2, set at 3 degrees and 6 degrees to the axis of the tube. The corresponding pitch is roughly 1 in 10 and 1 in 20. For a low velocity

measurement (below 30 miles per hour) the screw S is turned until the level L_1 shows horizontal. The tube is then inclined 3 degrees. The instrument is thus quite independent of the leveling of the table or bench on which it may be used. Connection between the reservoir and glass tube is made by a short piece of rubber tube. Displacement of the liquid in T is read on a scale of 600 half millimeters attached to the frame. This manometer was made by a skilful instrument maker, and great care was taken to set the spirit levels at the correct angles. The best grade of German glass tubing was used, and each tube was carefully cleaned with strong sulphuric acid and potassium bichromate.

If there are no appreciable errors in the leveling, the correct head of liquid (alcohol, 95 per cent, stained red with fuchsine dye) is

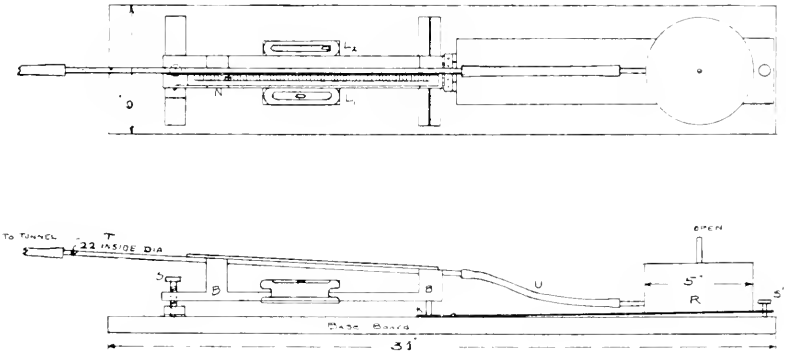


FIG. 8.—Alcohol manometer.

given from the geometrical construction. Thus: Head of liquid = displacement in $T \times$ sine of inclination. A small correction can be made for the depression of the liquid in R as the level in T rises. This also can be computed from the dimensions.

The density of the alcohol was taken on a chemist's "Westfall Balance" to a precision of 0.1 per cent. The effect of surface tension is to cause the level in T to be slightly higher than the level in R when the two ends of the manometer are under the same pressure. This is not an error in the instrument, since the zero setting takes account of it.

Tests were made by connecting both the reservoir end of the alcohol gage and one leg of the Chattock gage to the same static pressure made by a water column. In this way errors due to fluctuations of pressure were eliminated. The Chattock gage readings were taken as a standard for reference. The same Chattock gage

was used in all tests. The alcohol gage was fitted with a straight glass tube 0.15 inch in diameter. The tube was clean and dry. The velocity calculated from the alcohol gage was found to be 12 per cent low. The tube was then wet by blowing the liquid to the top of the scale and then allowing it to settle back to zero. Readings taken subsequently were only 4 per cent low. The experiment was repeated using a glass tube 0.17 inch in diameter, clean and wet. The velocity recorded was 4 per cent low. A different tube, but of same diameter, was then put in the alcohol gage. Its average readings were found to be 10 per cent low. Examination of the glass tube showed two minute cracks in the glass hardly to be seen with the naked eye. A glass tube 0.2 inch in diameter was then tested and read 2 per cent low. A tube 0.22 inch in diameter read 1.5 per cent low. A tube 0.25 inch in diameter could not be used on the 3-degree pitch, as the alcohol would not form a meniscus.

In all, some 1,000 check observations were made, and the following conclusions drawn:

(1) The inclined type of liquid gage as commonly employed in ventilation work is not an instrument of precision.

(2) For consistent results, the glass tubing used must be free from all slight flaws on the inner surface which might cause changes in capillarity throughout the bore.

(3) The tube must be uniform in diameter.

(4) The tube must be as large as it is possible to use and still get a good meniscus.

(5) For alcohol at 3 degrees inclination an internal diameter of 0.22 inch is suitable.

(6) The maximum precision with such a gage used to measure air speeds from 4 to 40 miles per hour is about 1.5 per cent on velocity.

(7) The alcohol gage properly constructed is consistent and very sensitive.

(8) The alcohol gage may be used as an instrument of precision when calibrated against a standard.

In its final form with 0.22-inch tube, this alcohol gage was found to measure speeds within 1.5 per cent. Such precision is ample for engineering work, and this type of gage is recommended for a cheap portable instrument. For a laboratory standard, however, an error of 1.5 per cent cannot be accepted. Since the gage responded to changes of velocity of less than $\frac{1}{4}$ per cent, its sensitivity is such that it may be calibrated against a better manometer, and when calibrated, may be as precise as the standard.

IV. ADJUSTMENT OF VELOCITY GRADIENT ACROSS A SECTION OF THE WIND TUNNEL

By H. E. ROSSELL, Asst. Naval Constructor, U. S. Navy, and
D. W. DOUGLAS, S. B.

In any wind tunnel experiments, it is important that the velocity of the air striking different parts of the model shall be the same. Consequently, after developing precise methods for measuring velocity, the cross-section of the tunnel was explored to detect variations in velocity from point to point.

The procedure was as follows: The side plate described above (Report I, page 10) was connected to the Chattock gage. One observer by regulating the motor field rheostat kept the velocity as nearly constant as possible, indicated by keeping his gage reading constant. A speed of about 28 miles per hour was selected as standard. The Pitot tube (National Physical Laboratory tube) was mounted on a standard and moved parallel to itself along vertical lines 6 inches apart. Great care was taken to point the tube in the axis of the wind. The Pitot tube was connected to the alcohol gage. A velocity reading was taken every 6 inches by a second observer. The same two observers made the entire test.

The first preliminary tests showed the velocity over the section for a constant static pressure, as shown on the Chattock gage to be far from uniform. The velocity near the sides was higher than near the center. Such a result could be caused by the honeycomb at the suction mouth of the tunnel. The air entered the mouth in converging lines of flow as was shown by the direction taken by fine silk threads. The honeycomb was at the very end of the tunnel, and probably straightened out this flow too soon to allow a sufficient volume of air to reach the center.

To assist the air to flow more to the center, the honeycomb was shoved 1 foot into the tunnel. The effect was satisfactory, but not enough.

The honeycomb was then shoved 1 foot farther into the tunnel. The exploration of velocity over the section showed that a fair result had been obtained, and it was concluded that no advantage would be gained from further movement of the honeycomb.

Since models not greater than 18 inches in span are to be used, the useful part of the tunnel is included within a square 2 feet on a side. The parts of curves, drawn through the experimental points, which passed through the 2-foot square, show an average velocity of 27.8

miles per hour. The per cent difference between the velocity at each point and the average velocity of 27.8 miles per hour was then calculated. The results are shown marked on the section of the tunnel in figure 9. It is seen that the variation in velocity over the useful part of the tunnel is from + 1.1 to — 1.2 per cent. It is believed that this

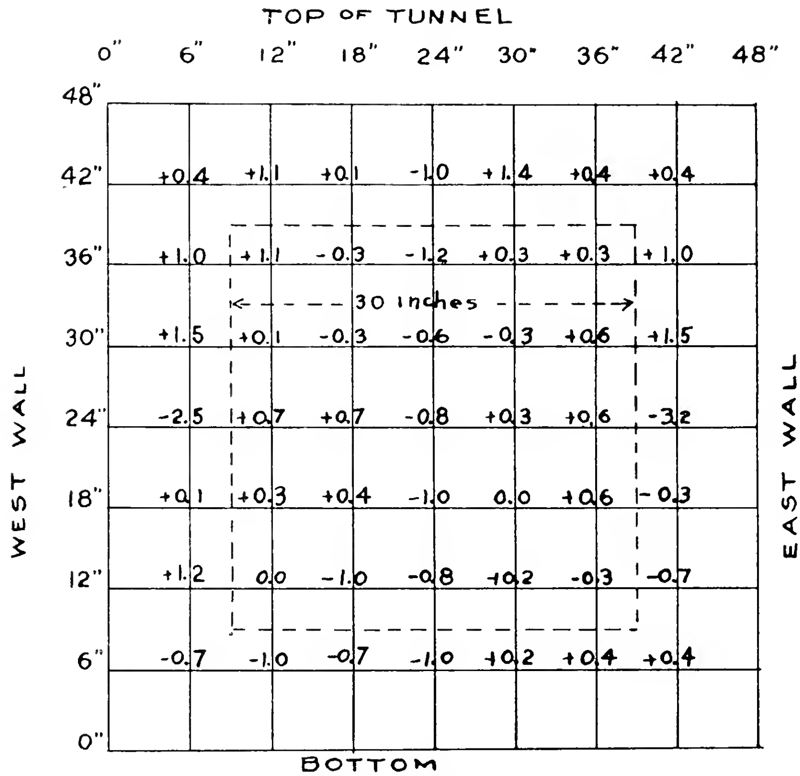


FIG. 9.—Variation of velocity across section of wind tunnel. Points represent per cent above or below mean velocity in dotted square which is 27.8 miles per hour.

variation is not too great for our purposes, and that the uniformity of flow compares well with that to be found in other wind tunnels.

With regard to the effect of the variation in velocity across the section, it may be stated that such a small variation in velocity is only of importance in tests on the moment tending to turn a model aeroplane away from or into the wind. An excess of velocity on one wing would give a tendency of the model to show a "lee helm." However, if the model be reversed and the experiment repeated, the

average of the measurements taken in both tests will have eliminated any error due to lack of symmetry in the flow.

Rotation or twist of the air in the tunnel has not been detected, and if present must be small. The honeycomb at entrance and baffles at exit are designed to prevent the air spinning with the propeller.

V. CHARACTERISTIC CURVES FOR WING SECTION, R. A. F. 6

By H. E. ROSSELL, Asst. Naval Constructor, U. S. Navy, C. L. BRAND, Asst. Naval Constructor, U. S. Navy, and D. W. DOUGLAS, S. B.

In order to furnish a final check upon the calibration of instruments, the alignment of tunnel and balance and general methods of testing, it was considered desirable to repeat the determination of the aerodynamical constants published by the British Advisory Committee for Aeronautics, Report 1912-13, for the wing profile, designated as R. A. F. 6.

Two models 18 inches span by 3 inches chord were cut in brass, and carefully filed and scraped to form. The surface was highly polished to remove tool marks.

Each model was mounted vertically in the wind tunnel and its "lift" and "drift" forces measured on the balance for angles of the chord to the wind from -4 degrees to $+18$ degrees.

The moment of the resultant force about the vertical axis of the balance was measured on the torsion wire. It was then possible to determine the direction of the resultant from the ratio $\frac{\text{lift}}{\text{drift}}$, the magnitude by $\sqrt{\text{Lift}^2+\text{Drift}^2}$, and its line of action from: $\frac{\text{observed moment}}{\text{magnitude}}=$ perpendicular distance from axis.

The center of pressure is usually defined arbitrarily as the intersection of the resultant force with the plane of the chord. This point was calculated for each incidence.

On figure 10 are plotted the values of lift and drift coefficients, defined by:

$$K_y=\frac{\text{Lift}}{AV^2}, \quad K_x=\frac{\text{Drift}}{AV^2},$$

where A is the wing area in square feet, and V the velocity of the wind in miles per hour. Lift and drift are in pounds force, hence K_y

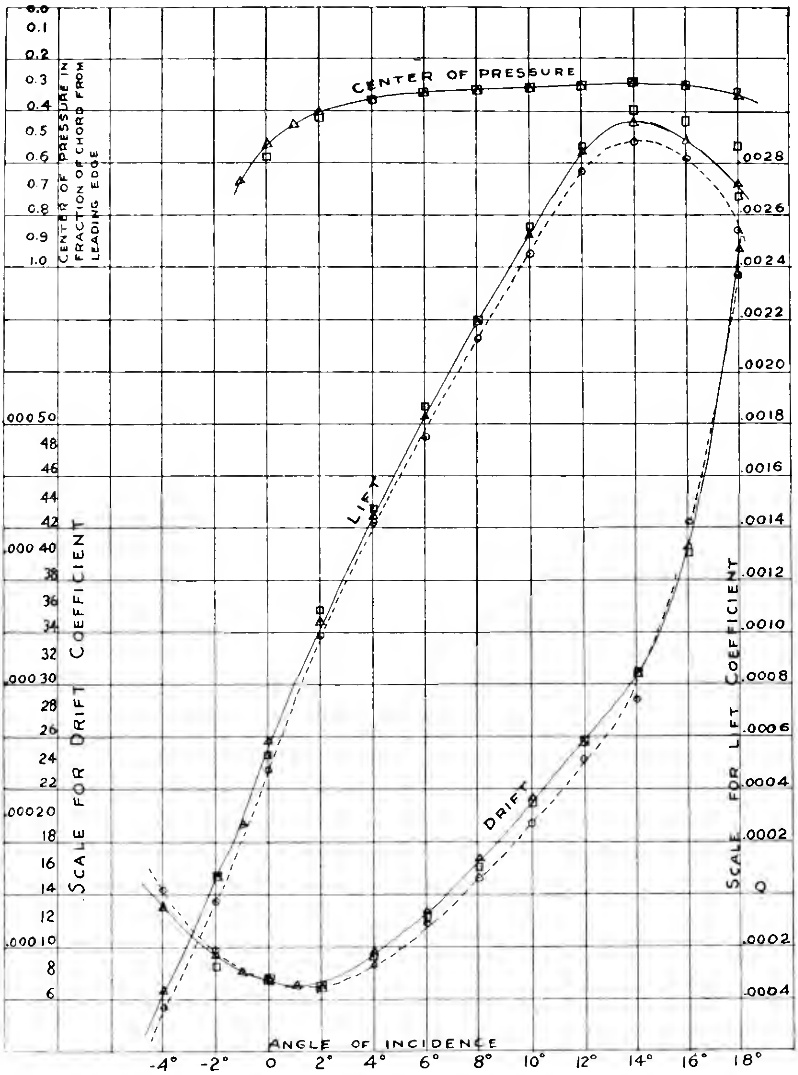


FIG. 10.—Wind characteristics.

Aerofoil R. A. F. 6.
Velocity 29.85 miles per hour.
Density of air 0.07608 pound per cubic foot.
⊡ Nat. Phys. Lab. observations.
—— △ Mass. Inst. Tech. observations, model A.
- - - ⊙ Mass. Inst. Tech. observations, model B.

is the force in pounds on 1 square foot due to a wind of 1 mile per hour, of air of standard density (*i. e.*, .07608 pound per cubic foot).

The "center of pressure" is also shown on figure 10 in terms of distance from leading edge in fraction of chord.

On the same sheet are shown the experimental points published by the National Physical Laboratory for this wing, using the same size model and the same speed, *i. e.*, 29.85 miles per hour.

It is seen that there is only slight discrepancy between the lift observations up to an incidence of 14 degrees, the useful range in aviation. The English points lie from 1 to 3 per cent higher than the corresponding points for model B, and coincide with those of model A.

Similarly for the drift observations there is very good concordance up to 14 degrees.

The curve of center of pressure coefficient is in practical coincidence with the English observations.

It appears that undetected differences in workmanship and finish between two models may cause a change in coefficients of not more than 3 per cent. Actual observations are precise within one-half of 1 per cent. Consequently, our results may be considered sufficiently precise for purposes of aeroplane design.

VI. STABILITY OF STEERING OF A DIRIGIBLE

By J. C. HUNSAKER

When floating in the air with no way on, a dirigible takes an attitude such that the center of gravity lies on the vertical passing through the center of buoyancy of the envelope or gas bag. The ship is then in stable equilibrium. When under way, the thrust of the propellers is balanced by the resistance of the air. In order that the attitude shall not be changed, the moment of the propeller thrust and the moment of the air resistance taken about any point must balance one another. It is convenient to take axes of coordinates, vertical, transverse, and horizontal, located at the center of buoyancy. If it be assumed that the ship is in equilibrium on her course, then the component forces along and moments about the three axes through the center of buoyancy are each zero.

This is equivalent to the statement that the motion of the ship is one of pure translation in the direction of the fore and aft axis of the envelope, which axis is horizontal.

Any angular deviation of this axis will call into play moments about the axes chosen above as well as forces along those axes. The forces will cause the trajectory of the center of buoyancy to be deflected, and the moments will swing the ship's axis. The motion is stable if the forces and moments tend to restore the original attitude and state of motion.

If we consider only the moments produced by angular deviation of the ship's axis, we may say that the ship is stable if the moments tend to restore the original attitude, and unstable if they tend to magnify any initial deviation.

With a model held fixed in a current of air by a spindle passing through the center of buoyancy, the stability of any attitude is measured by the moments about the spindle.

An elongated ellipsoid, as can be shown by hydrodynamic theory,[1] has three positions of equilibrium in a wind, corresponding to the directions of the three axes. Only one position, however, is stable, and the body tends to place itself broadside to the wind. For torpedo-shaped bodies, the stable position is intermediate between the broadside-on position and the desired bows-on position, and for any such body there is a stable "drift-angle" at which the body tends to hold its major axis to the wind. This angle may be between 50 and 90 degrees to the wind.

A feathered arrow with weighted head is stable for a translation along the direction of its shaft. In general, it is not practicable to fit sufficient fin surface at the stern of a dirigible envelope to give it such weather-cock stability, but it is possible to reduce the "drift-angle" to 20 degrees.

A dirigible must be steered both in a vertical and in a horizontal plane and its stability of route requires both horizontal and vertical fins and rudders.

A wooden model of a dirigible hull was fitted with rudders and fins in accordance with usual practice and tested in the wind tunnel at 35 miles per hour. The fin and rudder area was then adjusted until a satisfactory combination was obtained.

The principal interest in the research lies in the fact that it is generally possible to base the designed fin and rudder area upon such experimental wind tunnel tests instead of rule of thumb.

The model was mounted in the wind on a vertical spindle passing through the center of buoyancy as calculated from the plans. The

[1] Lamb, Hydrodynamics, p. 121.

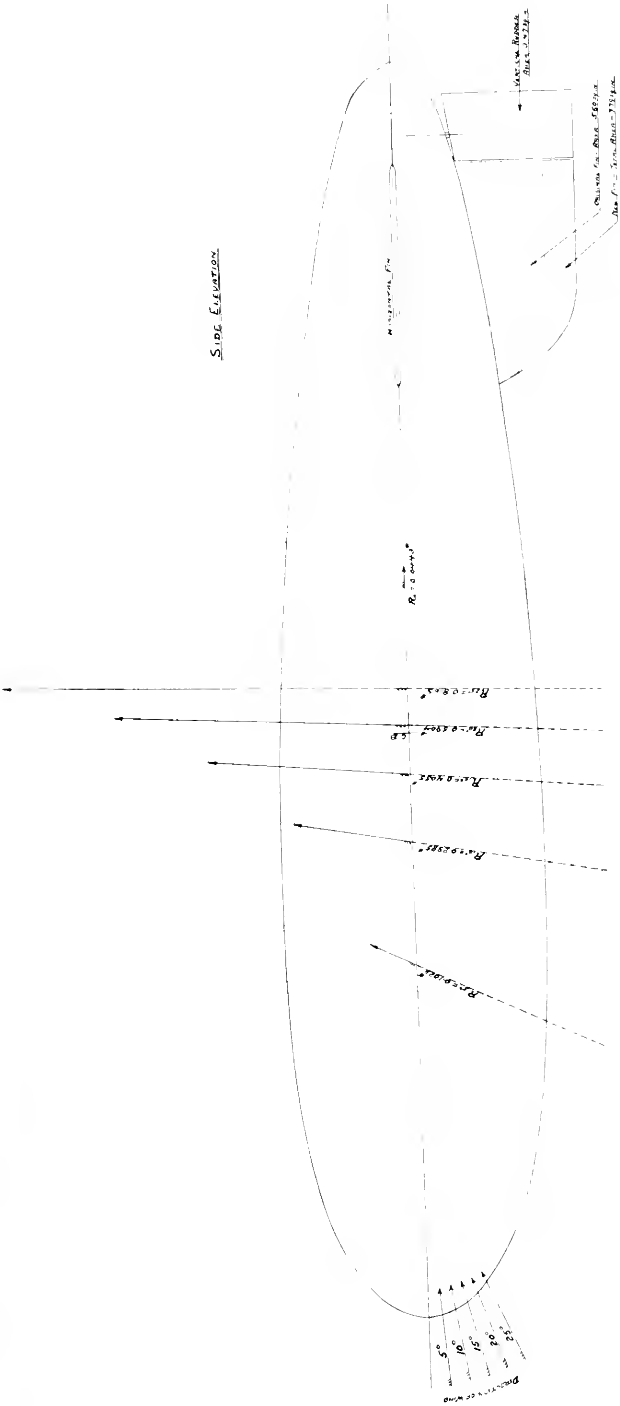


FIG. 11.—Resultant forces on model dirigible envelope for angles of pitch. Forces in pounds for 35 M. P. H., standard air (dry air), 62° F., 29.921" barometer.

moment about the vertical axis through the spindle, M, was measured on a torsion wire with the model inclined 0, 5, 10, 15, and 25 degrees to right and left of the tunnel axis. Similarly the longitudinal and lateral components of resultant wind force in the horizontal plane were observed, *i. e.*, R_x and R_y.

The total resultant force is then

$$R = R_x^2 + R_y^2.$$

The direction of this force is at the angle

$$\theta = \tan^{-1} \frac{R_y}{R_x}$$

measured from the axis of the tunnel. The resultant force R has an arm A, the perpendicular distance from the center of buoyancy to the line of action of R. Thus

$$A = \frac{M}{R}.$$

The force R is then determined in magnitude, direction, and line of application, and is laid out graphically on the model drawing for each angle of inclination.

Experiments were made with the model swung on its vertical axis through the center of buoyancy to obtain the yawing moments, and with the model on its transverse axis to obtain the pitching moments.

The resultant forces for angles of pitch are shown graphically on the side elevation of the model, figure 11. It appears that with the horizontal fins fitted, the resultant forces pass forward of the center of buoyancy. The model is, therefore, unstable when moving in the direction of its longitudinal axis, and if left to itself will take a "drift angle" of about 20 degrees up or down. In practice this pitching moment is counterbalanced by the powerful righting couple due to the weight of the car suspended beneath the envelope.

For example: If the center of gravity of the whole ship be a distance d below the center of buoyancy, the righting couple for a pitch of θ degrees is

$$M_s = Wa \sin \theta$$

where W is the total weight.

On the other hand, the upsetting moment on the envelope due to wind forces is a function of the inclination, as shown, and the velocity squared, or

$$M_e = KV^2 f(\theta)$$

where K is a constant.

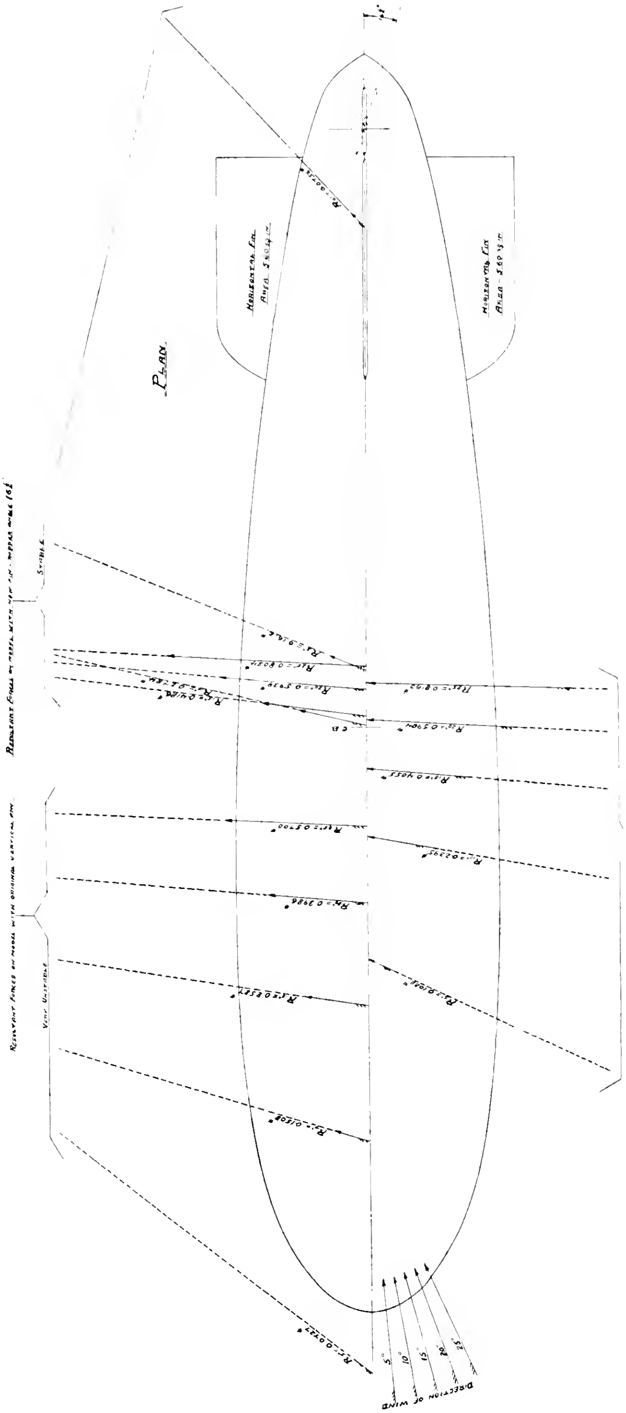


Fig. 12.—Resultant forces on model dirigible envelope for angles of yaw. Forces in pounds for 35 M. P. H., standard air (dry air), 62° F., 29.921" barometer.

For longitudinal stability, the following inequality must be satisfied:

$$M_s > M_e$$
$$Wa \sin \theta > KV^2 f(\theta).$$

For given incidence, $f(\theta)$ and $\sin \theta$ are constant, and hence the righting moment due to weights is constant. The upsetting moment of the wind increases as the square of the velocity. At some critical velocity the upsetting moment may preponderate and the ship become unmanageable.

This is the critical velocity first pointed out by Colonel Renard, which led to serious difficulties in the early dirigibles.

For a given design, it is possible from wind tunnel tests to determine this critical velocity, and by suitable additions to the horizontal fin area or lowering of the car to insure that in operation the critical velocity can never be reached.

The tests as described above were repeated for angles of yaw. In the first series, a single vertical fin of 5.6 square inches area was fitted as shown in figure 11. The resultant forces are drawn on figure 12. The model is very unstable, and tends to swing to the right or left of its course until the axis makes an angle of about 40 degrees to the wind. The fin area is obviously insufficient. It will be noticed that the resultant force for 5 degrees yaw passes outside the model. This is no doubt due to the arbitrary mechanical definition of resultant force. The resultant force as drawn merely represents that force which, acting along the line shown, will have the same moment about the center of buoyancy as the complicated distribution of pressure about the model. This experimentally observed moment might as well be represented by a couple.[1]

To improve the stability of steering, a larger vertical fin and a vertical rudder were next fitted and the test repeated. The rudder was fixed in the plane of the fin. The resultant forces are shown on the lower part of figure 12. It appears that the drift angle has been reduced from 40 to 20 degrees, but that the ship is still unstable. It is not practicable to fit more fin surface, and the remaining instability must be met with the rudder.

The rudder was then set at $16\frac{1}{2}$ degrees to the keel line and the test repeated. The resultant forces shown on figure 12 are seen to lie

[1] This position of the resultant force at small angles was predicted by Sir George Greenhill, and later verified experimentally by L. Bairstow, Tech. Report of the Advisory Committee for Aeronautics, p. 35, London, 1910-11.

abaft the center of buoyancy. The ship is therefore stable. At 10 degrees angle of yaw, the resultant force passes nearly through the center of buoyancy, and if the ship should get in this position the pilot would have to use slightly more than $16\frac{1}{2}$ degrees rudder angle to bring her back.

The conclusion from these tests is that with the size rudder and fin fitted, 7.79 and 3.47 square inches, the ship can be held on her course by the use of not more than $16\frac{1}{2}$ degrees of rudder.

The importance of an adequate vertical rudder is apparent, since the weights give no restoring moment for yawing as they do for pitching.

It appears impossible in practice to give sufficient vertical fin area to hold the ship on her course without use of the helm.

VII. PITCHING AND YAWING MOMENTS ON MODEL OF CURTISS AEROPLANE CHASSIS AND FUSELAGE, COMPLETE WITH TAIL AND RUDDER, BUT WITHOUT WINGS, STRUTS, OR PROPELLER

By J. C. HUNSAKER and D. W. DOUGLAS

A wood model of the Curtiss tractor body, complete with all appendages except wings, was constructed to a scale 1 inch to the foot and 24.5 inches actual length. The model was held in its ordinary position in the wind tunnel by a vertical spindle attached to the balance. The angle of yaw was varied, and observations were made to determine the components of force directed down stream and across the stream, as well as the twisting moment about a vertical axis passing through the supporting spindle. The axis of the model was kept in a horizontal plane during this test.

The model was then removed from the spindle shown in figure 13, side elevation. A new spindle was set into the side of the model as shown in figure 13, plan. The model was again placed in the wind tunnel but now lay on its side. Rotation about the new spindle axis made it possible to set the model at various angles of pitch, the angle of yaw being held constant. Measurements were made as above of down stream and cross stream components of force, and moment about the axis of the spindle.

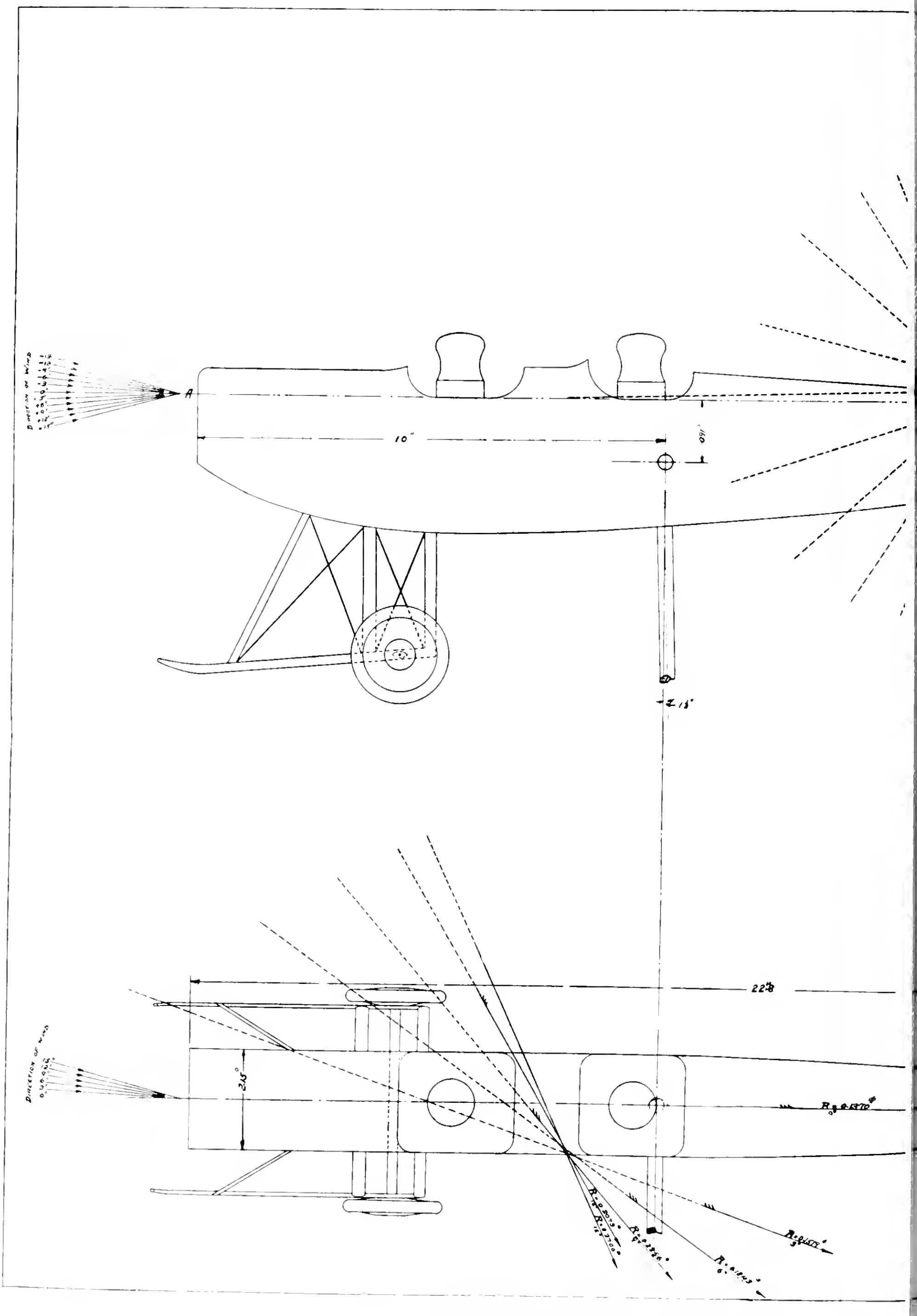
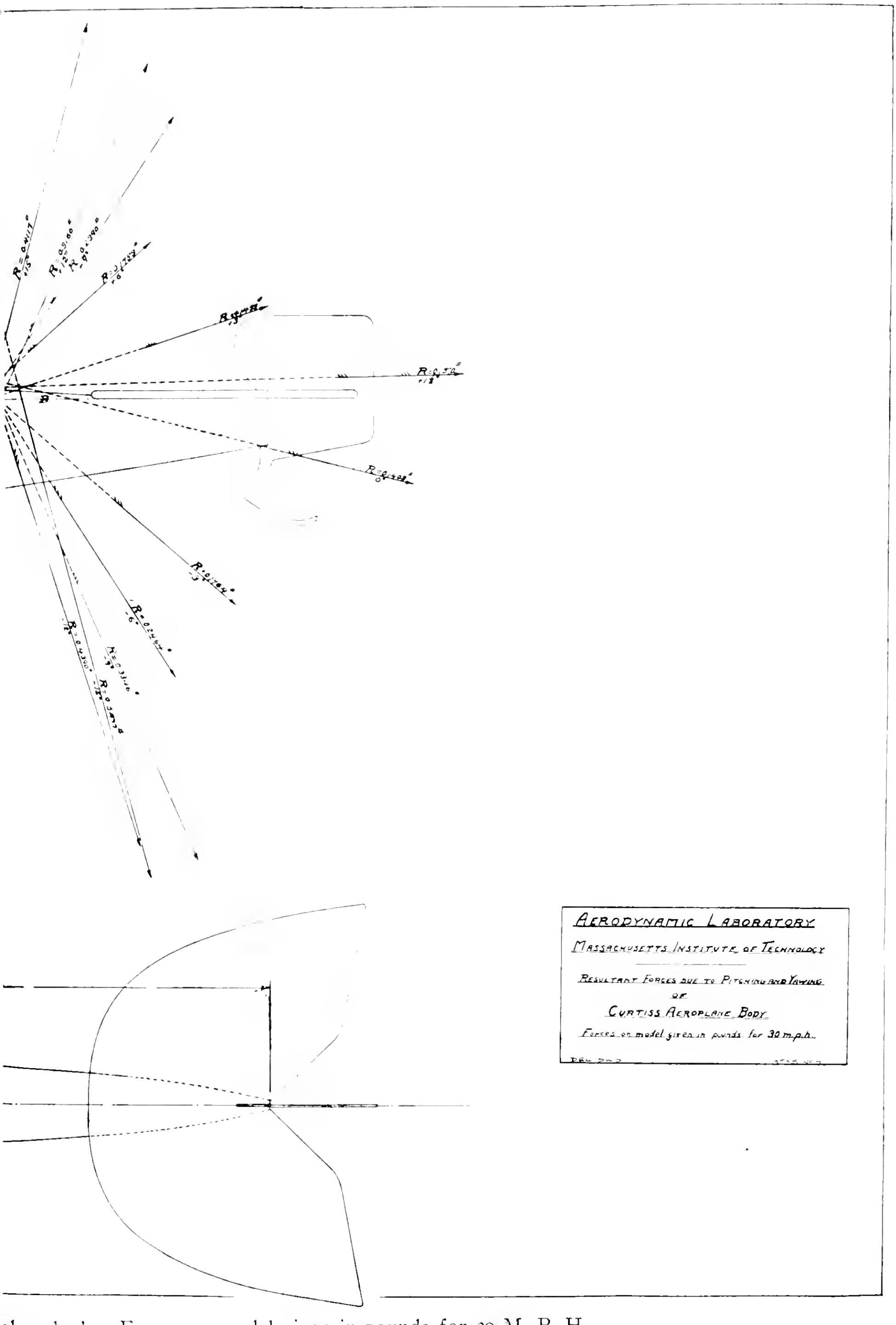


FIG. 13.—Resultant forces due to pitching and yawing of Curt



plane body. Forces on model given in pounds for 30 M. P. H.

The forces and moment measured in the first test are called

R_x, drift component,

R_y, cross wind component,

M_z, yawing moment;

and in the second case,

R_x, drift component,

R_z, lift component,

M_y, pitching moment.

Forces are measured directly in pounds and moments in pounds-inches on the model for a wind velocity of 30 miles per hour. Density of air is 0.07608 pound per cubic foot.

For any angle of pitch we may substitute for the lift, drift, and pitching moment a resultant force vector defined as that force which is the mechanical equivalent of these. It is to be noted that we here deal only with forces in the plane of symmetry of the aeroplane.

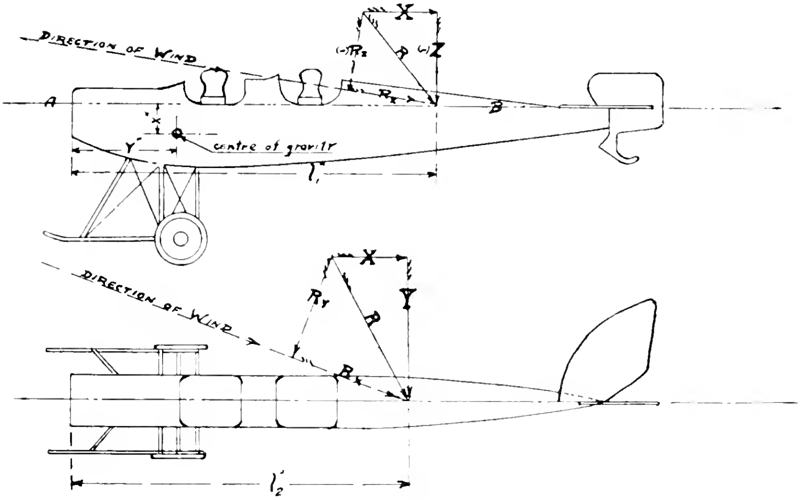
Similarly in the second test, where the angle of pitch was kept constant at 1.5 degrees, the drift, cross wind force, and yawing moment are represented by a vector in the horizontal plane passing through the intersection of the body axis and vertical spindle.

These two resultant force vectors may be called for convenience pitching resultant and yawing resultant. They are shown in position, magnitude, and direction on the two views of figure 13.

The artifice of representing a force and a moment by a vector makes it immaterial where the axes of support of the model were originally taken. In aeroplane design, the axis of the propeller usually is made to pass near the center of gravity, and it is hence necessary to locate the center of gravity at such a point that angular deviations from normal flight attitude will produce moments about the center of gravity tending to restore the original attitude.

The usual location of the center of gravity well forward in the body will insure that the pitching resultants pass to the rear of the center of gravity and that the pitching is stable. See figure 13, side elevation, where the pitching resultants are shown graphically.

For convenience, the resultant force on the model is given on figure 14 in terms of lift and drift components marked R_z and R_x. The resolution of the forces is shown on the upper figure. It appears from figure 15 that the drift R_x is practically constant from $+2$ degrees to -2 degrees pitch, but the lift R_z is zero only for $1\frac{1}{2}$ degrees pitch. It would be of some advantage to fly the machine at full speed with the body "tail heavy" $1\frac{1}{2}$ degrees.



ANGLE OF PITCH	Rx	Rz	X	Z	l_1''
0°	0·1365	-·0340	·1365	-·0340	18·56
+3°	·1383	+·0370	·1362	+·0442	16·80"
+6°	·1453	+·1050	·1335	+·1196	16·54"
+9°	·1615	+·1760	·1321	+·1991	16·82"
+12°	·1854	+·2570	·1279	+·2898	17·05"
+15°	·2227	+·3460	·1256	+·3918	16·80"
-3°	·1395	-·1080	·1336	-·1151	17·00"
-6°	·1543	-·1920	·1334	-·2071	17·00"
-9°	·1784	-·2830	·1319	-·3084	[illegible]
-12°	·2180	-·3810	·1340	-·4182	16·95"
-15°	·2727	-·4750	·1405	-·5293	17·51"

PITCHING MOMENT $''\#$ (about C.G.) = $X \times x'' - Z \times (l_1'' - Y'')$

ANGLE OF YAW	Rx	Ry	X	Y	l_2''
0°	0·1365	0	0·1365	0	—
3°	·1454	0.0440	·1429	0·0515	5·12"
6°	·1599	·0915	·1495	·1077	6·72"
9°	·1796	·1525	·1535	·1788	7·18"
12°	·2034	·2255	·1521	·2628	7·51"
15°	·2358	·2850	·1541	·3364	7·61"

YAWING MOMENT $''\#$ (about C.G.) = $Y \times (l_2'' - Y'')$

NOTE :- ALL FORCES ON MODEL ARE IN POUNDS, AT 30 M.P.H.

FIG. 14.

It is also possible to resolve the force R into components X along the axis of the machine and Z perpendicular thereto. Since a force may be resolved at any point in its line, we choose the intersection of the line of action with AB. This point is a center of pressure and is a distance "l_1" from the nose of the machine. See upper figure of figure 14. A force is completely defined by X, Z, and l_1, and these quantities are given in the upper table. The component X has no moment about the center of gravity if the center of gravity be on line AB, and if the center of gravity be a distance y from the nose, the pitching moment is $(l_1-y)Z$. Upward forces are plus. In general, for a center of gravity with coordinates x, y as shown on figure 14, the pitching moment is:

$$X_x - Z(l_1 - y).$$

By use of figure 14 and this formula, a curve of pitching moments can be readily obtained for any assumed position of the center of gravity.

In a similar manner the model was held at a pitch angle of $1\frac{1}{2}$ degrees to the horizontal wind, and placed at angles of yaw from 15 degrees right to 15 degrees left. Observations right and left have been averaged. The resultant forces are shown in direction, magnitude, and application in the lower figure of figure 13. The cross wind and drift components R_y and R_x are tabulated on figure 14.

As above, the components along the axis of the body X, and at right angles Y, are also tabulated. The intersection of the line of the resultant force with the vertical plane of symmetry is taken as a center of pressure and is a distance l_2 from the nose of the body. The yawing moment about the center of gravity placed a distance y from the nose is hence

$$Y(l_2 - y).$$

It is apparent from the last test that the aeroplane is directionally unstable, unless the center of gravity be well forward of the forward passenger's seat. The addition of a propeller and the fin effect of biplane struts will augment this tendency to instability of steering, and require a still farther forward position of the center of gravity.

Forces are given in pounds on the model for a wind speed of 30 miles. If the model is to a scale of 1 inch to the foot, the forces on the full-size body will be 144 times as great and at 60 miles four times greater. Thus the resistance of this body at 60 miles would be about

$$0.137 \times 144 \times 4 = 79 \text{ pounds},$$

on the assumption that the resistance varies as the square of the

speed. The true resistance would be less than this on account of the exponent of V being somewhat less than two. On the other hand, in

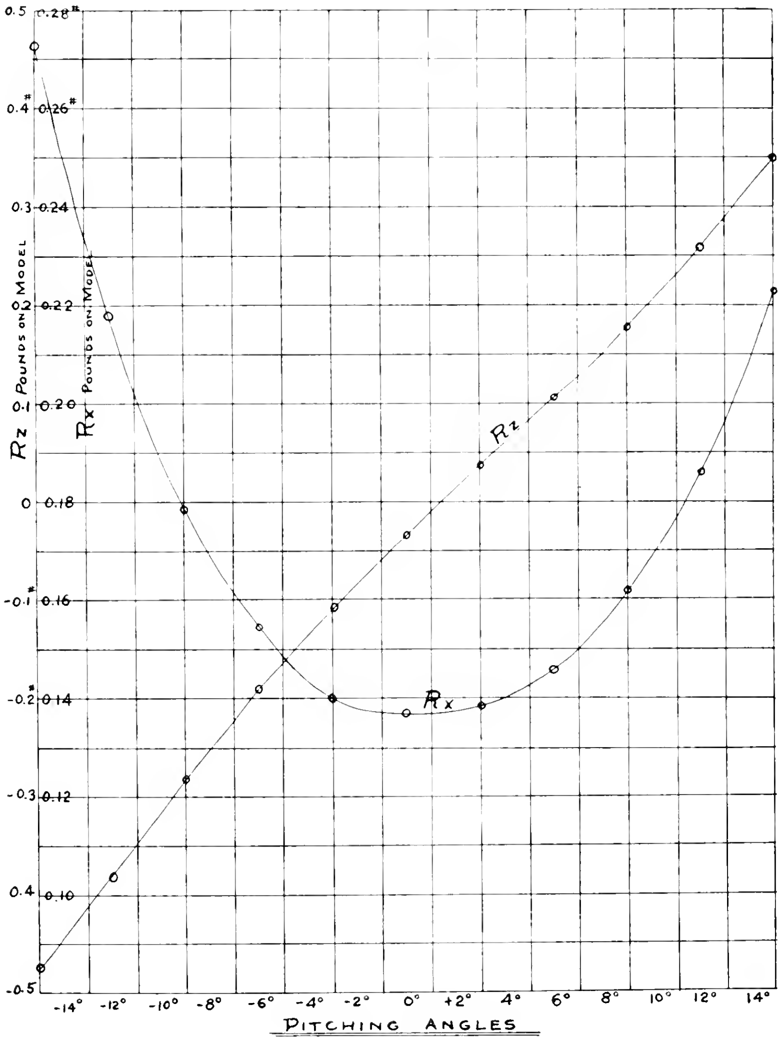


FIG. 15.

a tractor type aeroplane, the body resistance may be considerably augmented by the slip stream from the propeller. The assumption of the "square law" is usual in design.

To show the precision of the experimental work, curves of R_x, R_y, and R_z, plotted on pitch and yaw are given on figures 15 and 16. The

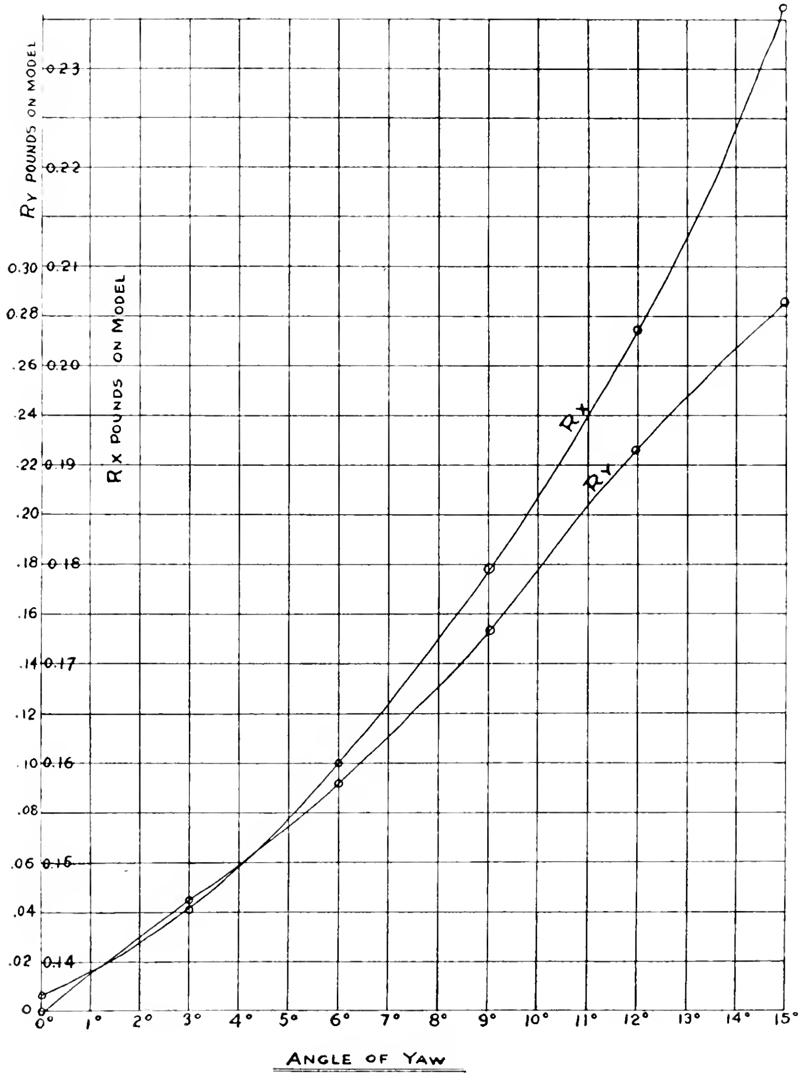


FIG. 16.

individual observations are there shown by the small circles which appear to permit a fair curve to be passed through them.

VIII. SWEPT BACK WINGS[1]

By H. E. ROSSELL and C. L. BRAND, Asst. Naval Constructors, U. S. Navy

Part I.—Lift, Drift, and Center of Pressure

To determine the effect on the aerodynamical properties of a wing of sweeping back the wing tips, as in the so-called German "Pfeil" aeroplanes, a series of tests was made on the model described as R. A. F. 6. The right and left halves of the wing were swept back 10, 20, and 30 degrees to the normal position as shown in figure 17.

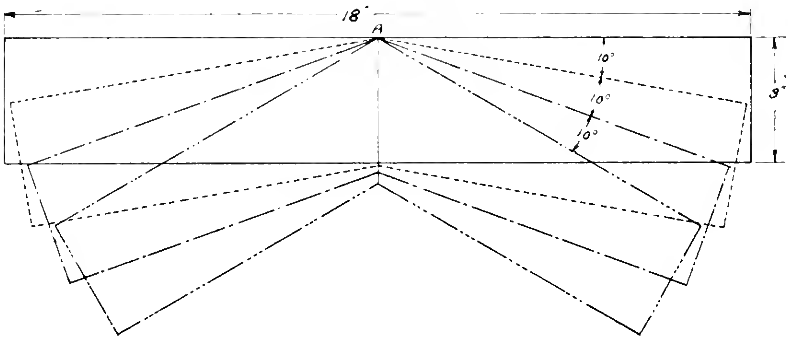


Fig. 17.

———— I.
- - - - - II.
— - — - — III.
—— - - - —— IV.

The first part of the investigation deals with the variation in lift and resistance coefficients, and the movement of the center of pressure for the different models as the angle of incidence changes. The wind velocity of 29.85 miles per hour of standard air[2] remains in the plane of symmetry of the wing.

The curves of the above characteristics for the four wings are given in figures 18, 19, 20.

The ratios of lift to resistance coefficients are given in figure 21.

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[1] Abstract from a research submitted for the degree of Master of Science in the Department of Naval Architecture, at the Massachusetts Institute of Technology.

[2] Standard air is of density 0.07608 pound per cubic foot.

The following key will serve to identify the wings:

Wing	Swept Back
I	0 degrees
II	10 degrees
III	20 degrees
IV	30 degrees

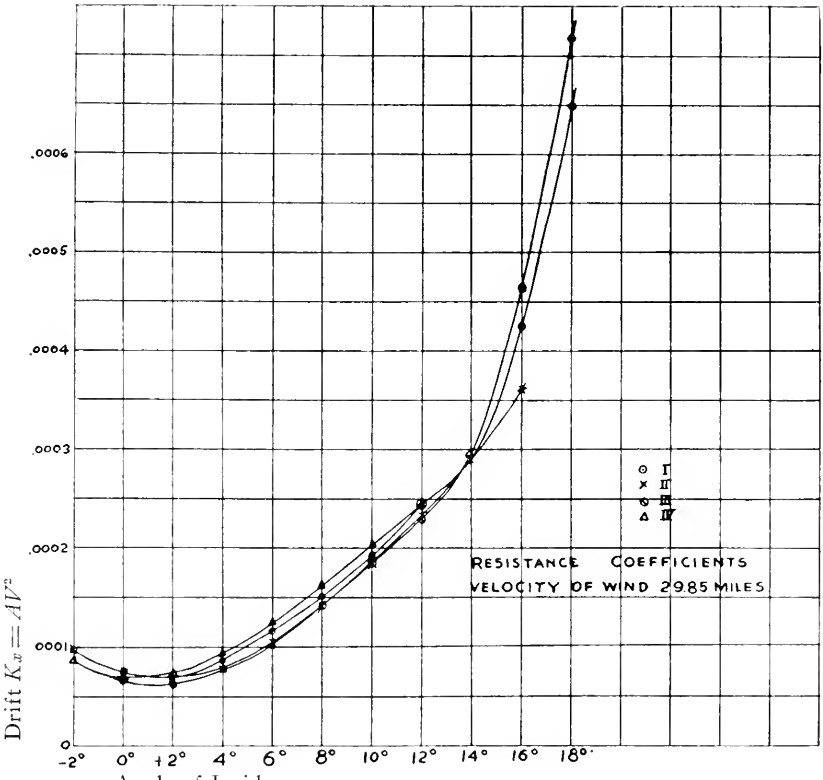
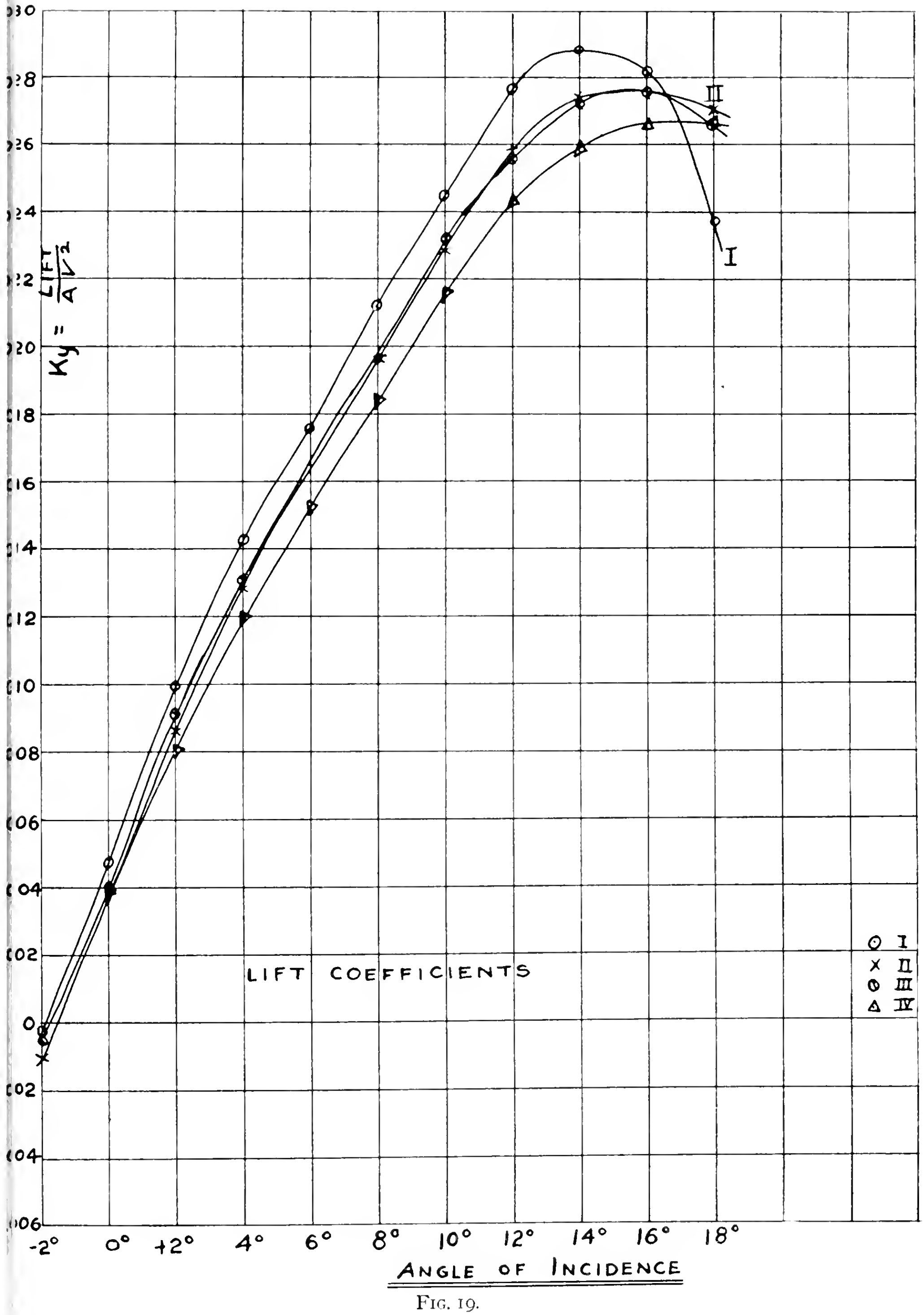


FIG. 18.

Coefficients are given as pounds force per square foot per mile per hour velocity.

It appears that a sweep back of 10 degrees and 20 degrees is of no disadvantage from considerations of lift and resistance. There is, however, no gain. For a sweep back of 30 degrees, the lift coefficient is diminished and the ratio of lift to resistance is seriously reduced. What advantages there may be as regards lateral sta-



LIFT COEFFICIENTS
$Ky = \frac{LIFT}{AV^2}$
ANGLE OF INCIDENCE
-2° 0° +2° 4° 6° 8° 10° 12° 14° 16° 18°
I
II
III
IV

FIG. 19.

bility for wings swept back 30 degrees are therefore paid for by a considerable loss in effectiveness. The ratio $\frac{K_y}{K_x}$ of 16.5 for the normal wing and for No. II is reduced to 12.9 for No. IV.

The center of pressure motion is shown by the curves of figure 20, in which it appears that the motion is similar for all the wings tested.

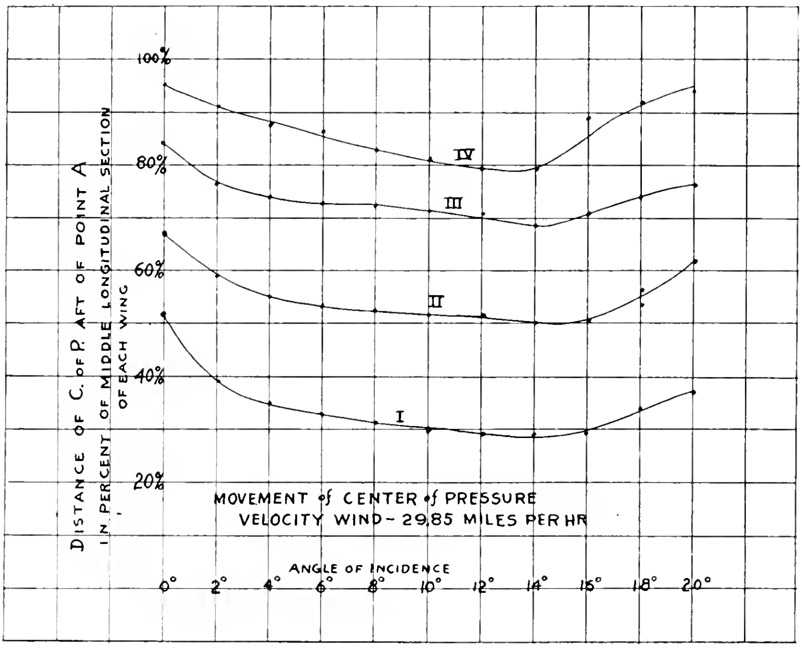


FIG. 20.

The center of pressure motion gives rise to the same degree of longitudinal instability. The center of pressure is referred to the forward point of the middle longitudinal section of each wing, and it appears that for a given angle of incidence the center of pressure is thrown to the rear by about two-tenths of the chord for a sweep back of 10 degrees, and four-tenths of the chord for a sweep back of 20 degrees. The center of gravity of an aeroplane will, therefore, have to be placed farther to the rear if swept back wings are used.

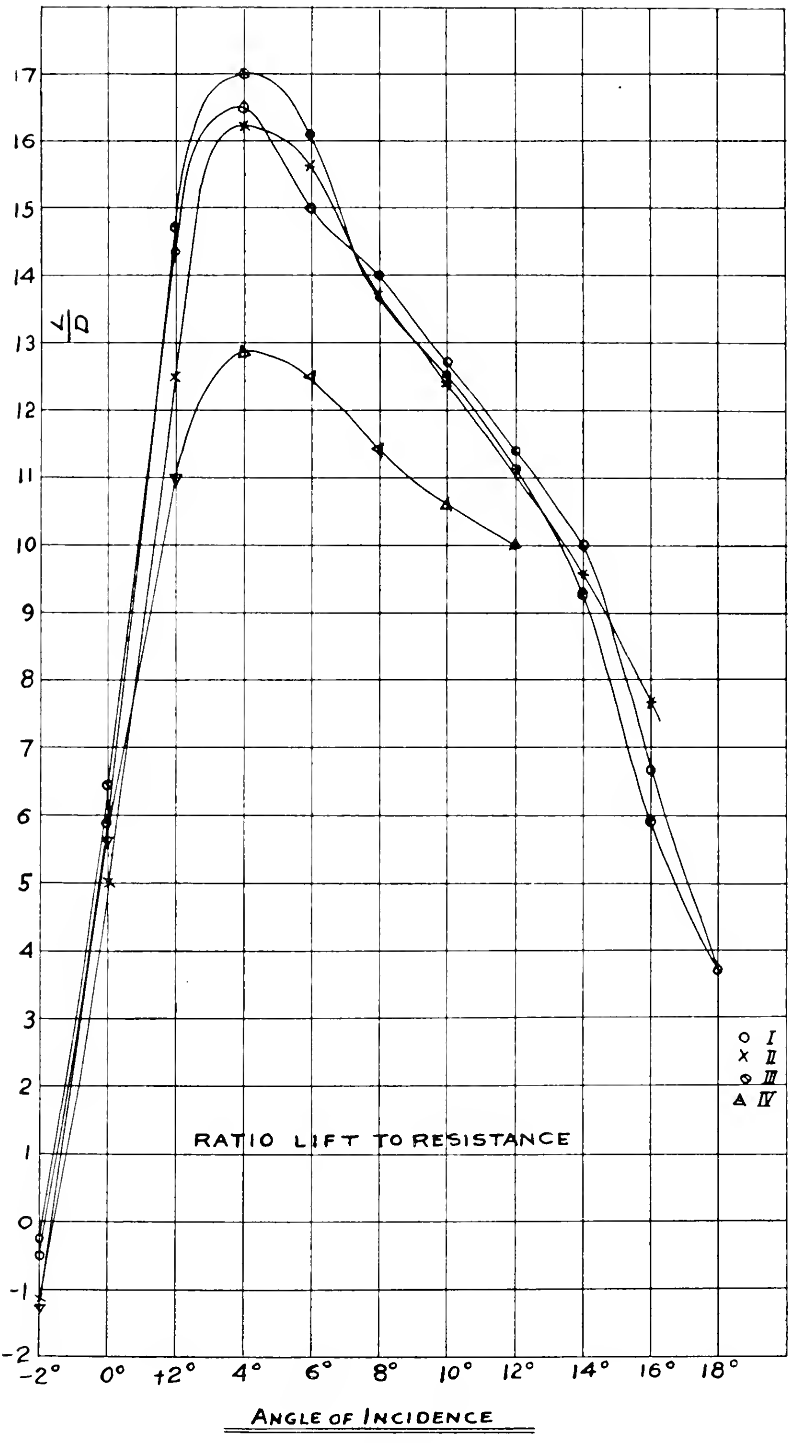


Fig. 21.

Part II.—Rolling, Pitching, and Yawing Moments

The object of this test is to determine the effect on stability of sweeping the wing back to various angles, as described in Part I of this report.

The wing uncut was first mounted horizontally on the balance, and forces and moments as enumerated below were measured for angles

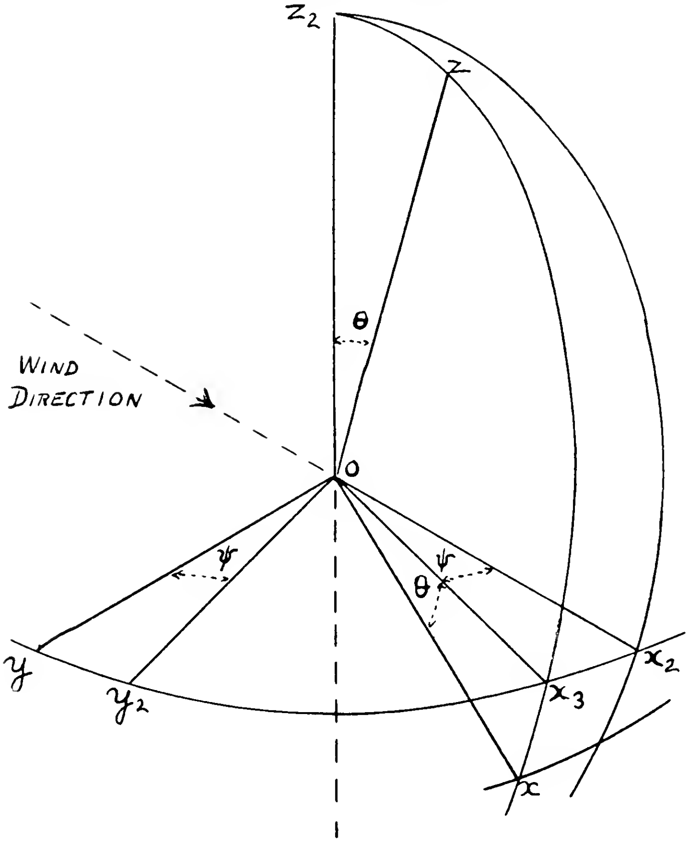


Fig. 22.

of yaw of 0, 5, 10, 15, 20, 25, and 30 degrees in both directions for each of the following angles of incidence: 0, 3, 6, and 9 degrees. The velocity of wind was kept constant at 29.85 miles per hour. The same measurements were made with wing models swept back to 10, 20, and 30 degrees. The method of manipulating the balance to make these measurements is given in detail in Report 68, Technical Report of the Advisory Committee on Aeronautics, 1912-13 (London).

It is convenient to calculate the forces and moments about moving axes fixed relative to the wing. The axes are lettered *OX*, *OY*, and *OZ*. In figures 22 and 23, point *O* we have located at the middle of the leading edge of each wing. *OX* is the fore and aft, *OY* the transverse, and *OZ* the normal axis of the wing. When the wing has zero incidence and yaw, the position of the axes is shown in figure 22 by

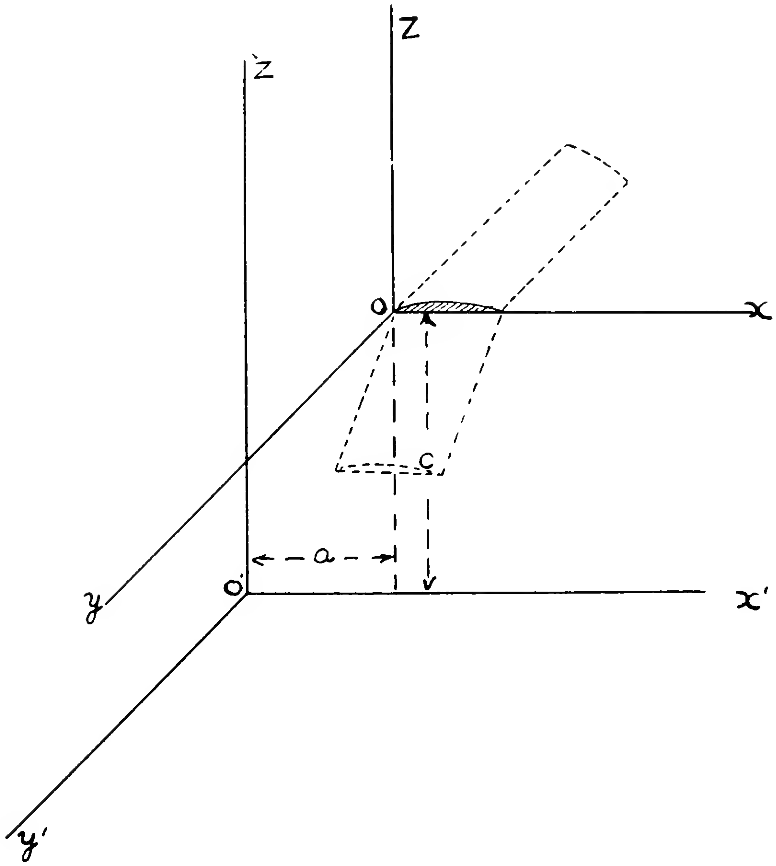


FIG. 23.

OX_2, OY_2, and OZ_2. Now suppose the wing to be turned about OZ_2 through a positive angle of yaw ψ. Then the axes will be in the position, OX_3, OY, and OZ_2. Now if the wing is given an angle of incidence of θ about OY, the axes will swing into the positions OX, OY, OZ.

In making the calculations it is easier first to refer the forces and moments to a set of axes parallel to OX, OY, and OZ, which we have

designated as $O'X'$, $O'Y'$, and $O'Z'$. The origin of these axes is coincident with the origin of axes about which rolling and pitching moments were measured.

Forces along axes OX, OY, OZ are called AX, AY, AZ, A being the area of model in square feet; and moments about these axes are called L, M, and N. Forces along and moments about axes $O'X'$, $O'Y'$, and $O'Z'$ are represented in the same way, *i. e.*, AX', AY', etc.

The notation used for the measured forces and moments is as follows:

(1) Force along $O'Z_2'=V_F$. Measured on vertical force lever.
(2) Moment about $O'Z_2'=M_Z$. Measured on torsion wire.
(3) Moment about $O'Y'=V_P$. Measured on vertical force lever.
(4) Moment about $O'X_3'=V_R$. Measured on vertical force lever.
(5) Moment about an axis parallel to $O'Y_2'$ and distant l below it $=M_D$. Measured on drift beam.
(6) Moment about an axis parallel to $O'X_2'$ and distant l below it $=M_C$. Measured on cross wind beam.

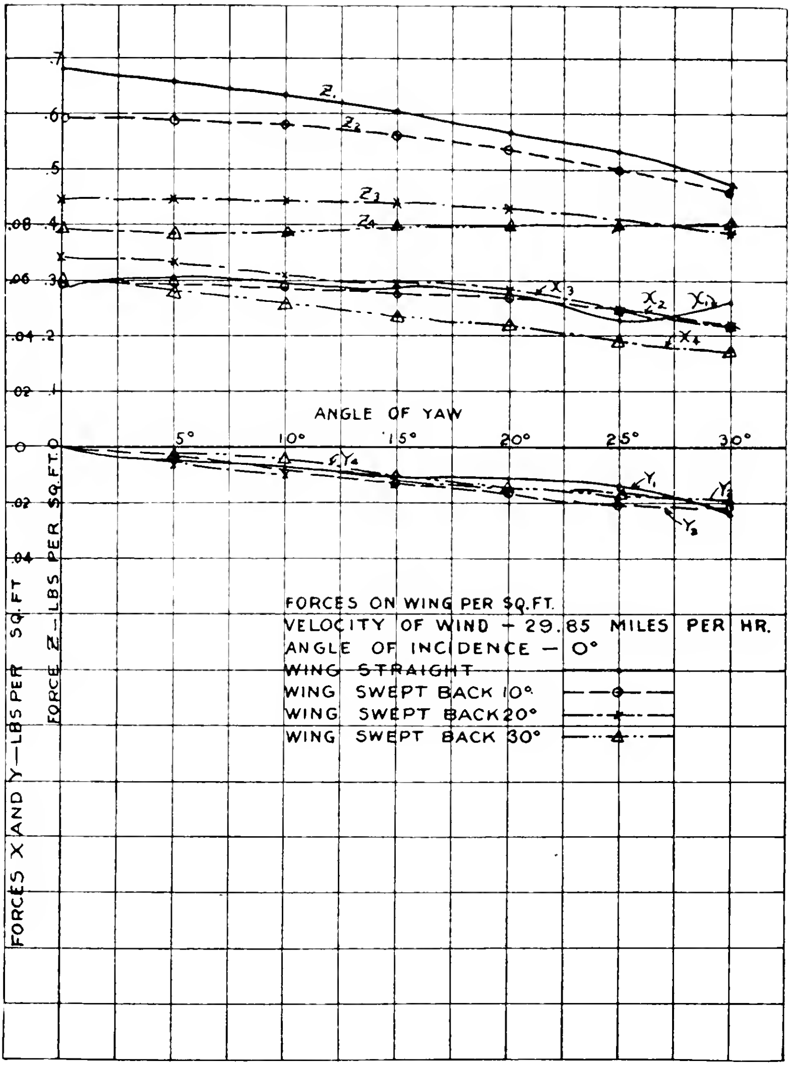
The forces and moments desired can then be calculated by the following equations:[1]

$$L'=V_R \cos\theta - M_2 \sin\theta.$$
$$M'=V_P.$$
$$N'=V_R \sin\theta + M_Z \cos\theta.$$
$$AX=-V_F \sin\theta + (M_D \cos\phi - M_C \sin\phi - V_P)\frac{\cos\theta}{l}.$$
$$AY=\frac{V_R - M_C \cos\phi - M_D \sin\phi}{l}.$$
$$AZ=V_F \cos\theta + (M_D \cos\phi - M_C \sin\phi - V_P)\frac{\sin\theta}{l}.$$
$$L=L'+cAY.$$
$$M=M'-cX+aAZ.$$
$$N=N'-aAY.$$

Where a and c are the X and Z coordinates of the origin O taken from axes $O'X'$, $O'Y'$, and $O'Z'$.

Figures 24 to 31 show the forces X, Y, Z, and moments L, M, N plotted on angles of yaw as abscissæ for constant angles of incidence of 0, 3, 6, and 9 degrees. At any incidence the chord of the wing at its

[1] Technical Report of the Advisory Committee for Aeronautics, London. 1912-13. Report No. 75.



FORCES ON WING PER SQ.FT.
VELOCITY OF WIND — 29.85 MILES PER HR.
ANGLE OF INCIDENCE — 0°
WING STRAIGHT
WING SWEPT BACK 10°
WING SWEPT BACK 20°
WING SWEPT BACK 30°
ANGLE OF YAW
FORCES X AND Y—LBS PER SQ.FT
FORCE Z—LBS PER SQ.FT.

Fig. 24.

center coincides with the OX axis. In figure 32, X, Z, and M are plotted with angles of incidence as abscissæ and with 0 degrees yaw.

The notation used is as follows:

Subscript 1—wing straight.
Subscript 2—wing swept back 10 degrees.
Subscript 3—wing swept back 20 degrees.
Subscript 4—wing swept back 30 degrees.

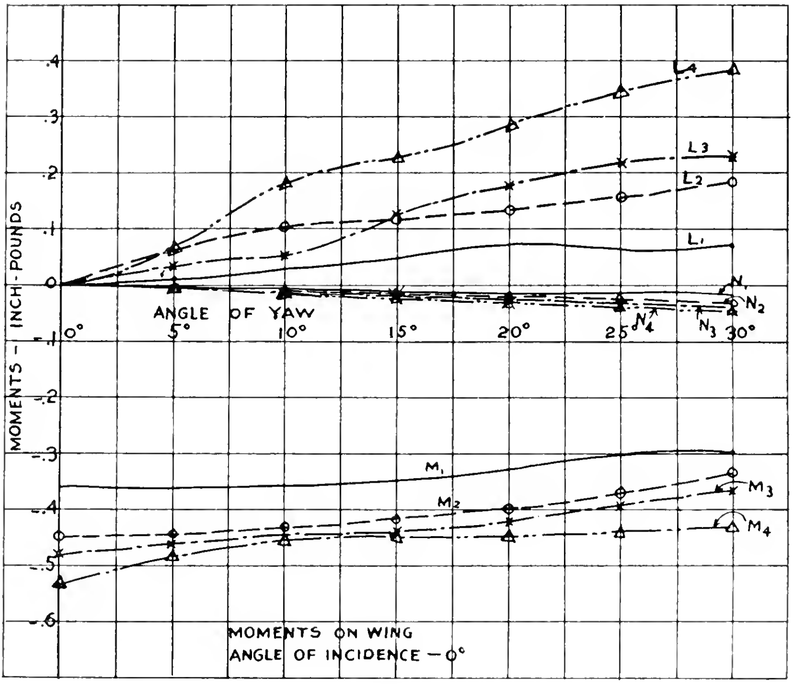


FIG. 25.

For the application of the theory of the dynamical stability of aeroplanes, reference should be made to the excellent papers by Mr. L. Bairstow.[1]

It is shown that for longitudinal stability the rate of change of the quantities X, Z, and M with angle of incidence determines the so-called resistance derivatives. It appears from the observations made on swept back wings that there is no appreciable change in the slope of curves of X, Z, and M plotted on angle of incidence for the

[1] Technical Report of the Advisory Committee for Aeronautics, London, 1912-13. Report No. 79.

different wings. Figure 32 shows some difference in form for curves M plotted on angle of incidence. It must be borne in mind, however, that the origin of these curves is at a greater distance from the center of pressure the farther the wing is swept back. A better idea of the variation in pitching moment with change in incidence can be obtained by plotting the curves with the estimated center of gravity

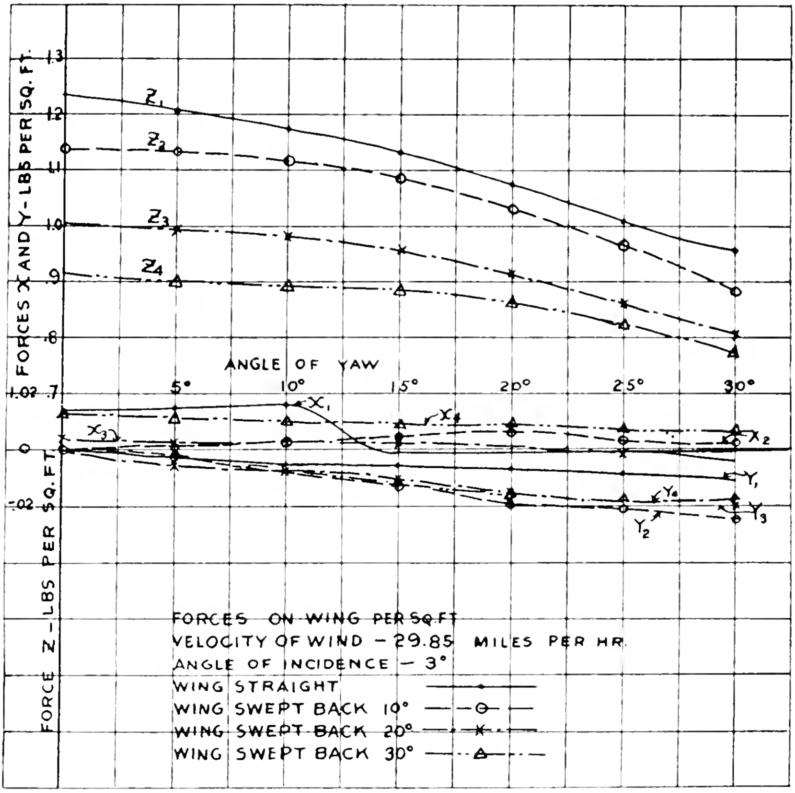


FIG. 26.

of the machine as the origin. Figure 20, Part I of this report, shows the movements of the centers of pressure to be similar for all wings. The conclusion is that sweeping back wings has no effect on the corresponding derivatives of X, Z, and M, and consequently no effect on longitudinal stability. It is of course to be expected that swept back wings may slightly increase the radius of gyration for pitching and so affect the motion indirectly. Aerodynamically, however, no advantage is to be expected.

For lateral stability the derivatives affected by the wings are the rate of change of Y, L, and N with angle of yaw. From considerations of symmetry, Y, L, and N are not affected by angles of roll.

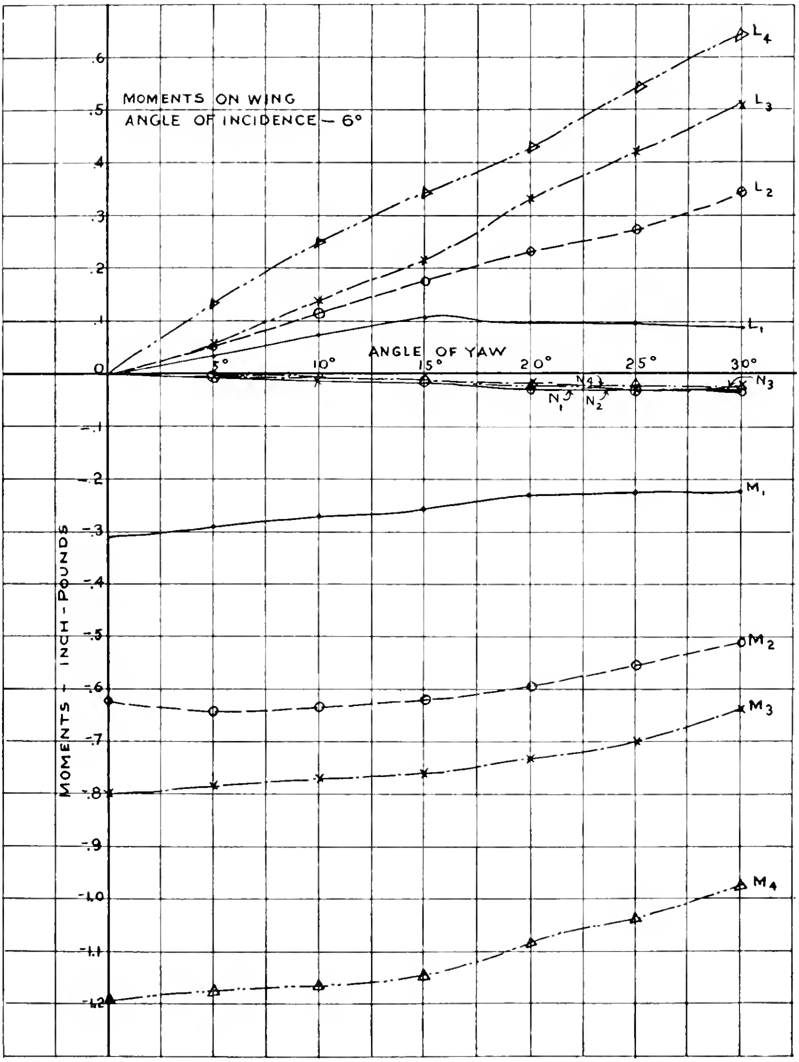


FIG. 27.

The curves of lateral force Y give evidence of difficulties in experimentation. The force Y on the model was in the neighborhood of 0.004 pound. It does not appear that there is any consistent

change in Y due to sweep back which can be brought out. In any case the force Y is so small that small changes would be of no interest.

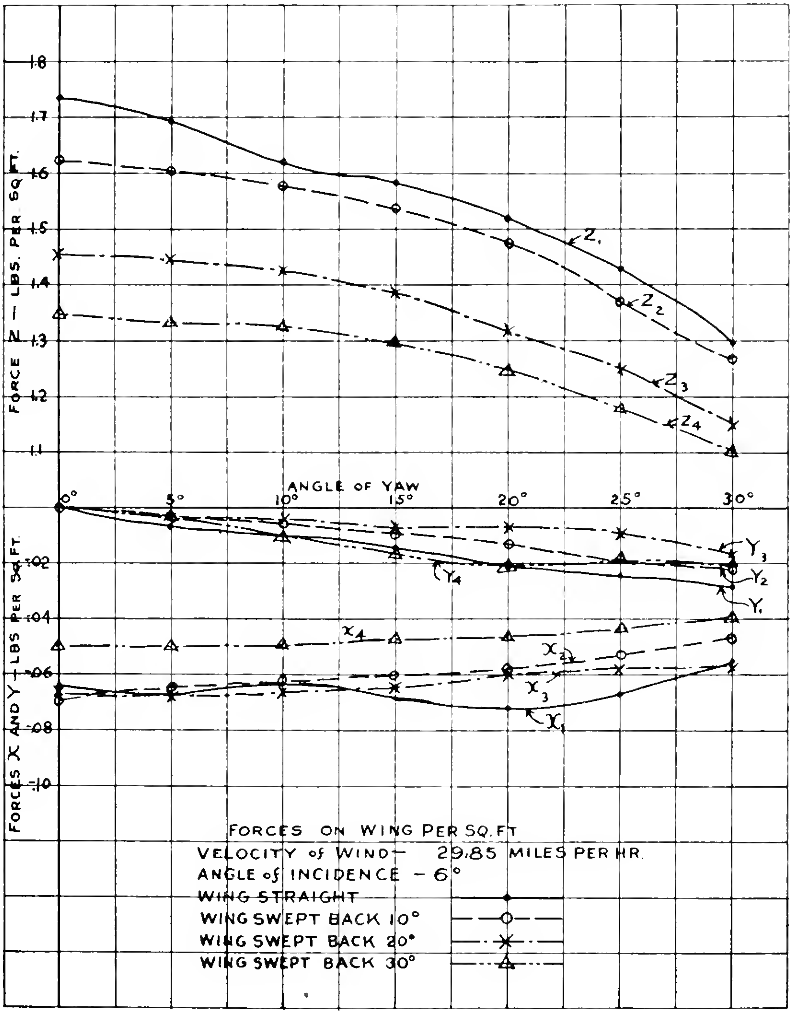


FIG. 28.

The curves of yawing moment N with angle of yaw show an extremely small moment, which was obviously impossible to measure with sufficient precision to detect changes between the various wings.

Indeed, as in the case of lateral force Y, the effect of changes in N produced by the wing is of no account.

On the other hand, L, the rolling moment produced as the wing is yawed out of its course, is large and of the greatest importance, as it

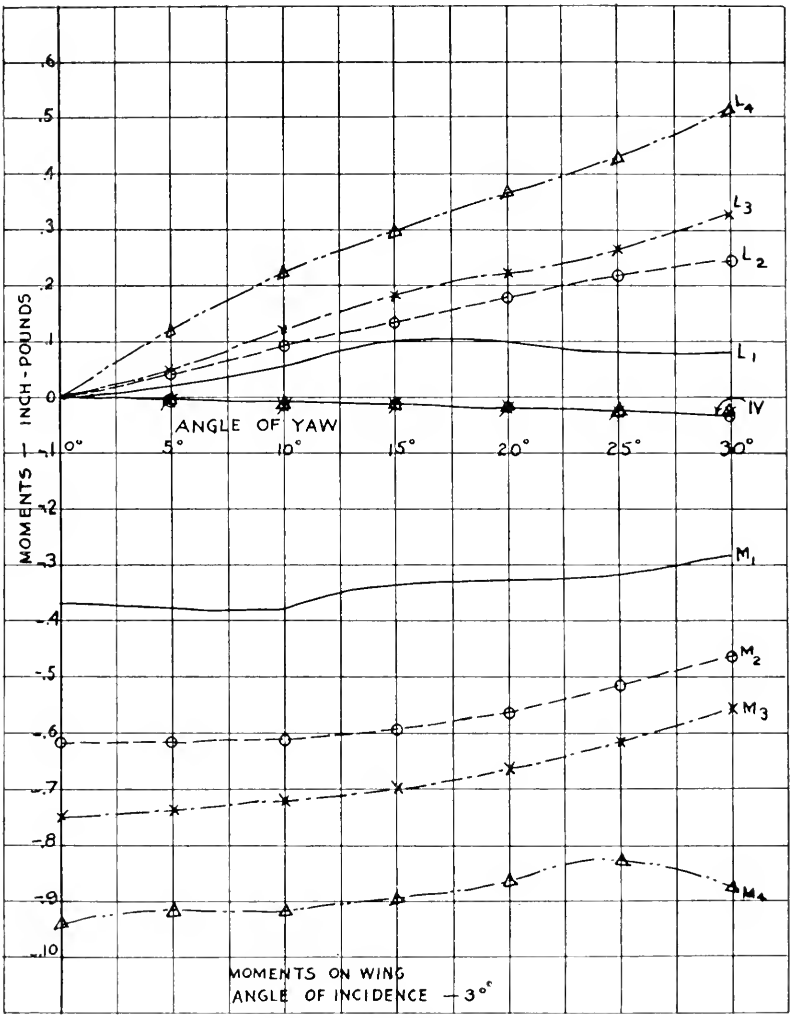


FIG. 29.

interconnects directional and lateral stability. For a normal wing flying at any reasonable incidence a deviation from the course causes a small rolling moment tending to bank the machine in a manner appropriate to making the turn. The couple may be called a " natural

banking" moment. This "natural banking" moment is increased nearly 100 per cent if the wing has a sweep back of but 10 degrees. The moment for a sweep back of 20 degrees is still greater.

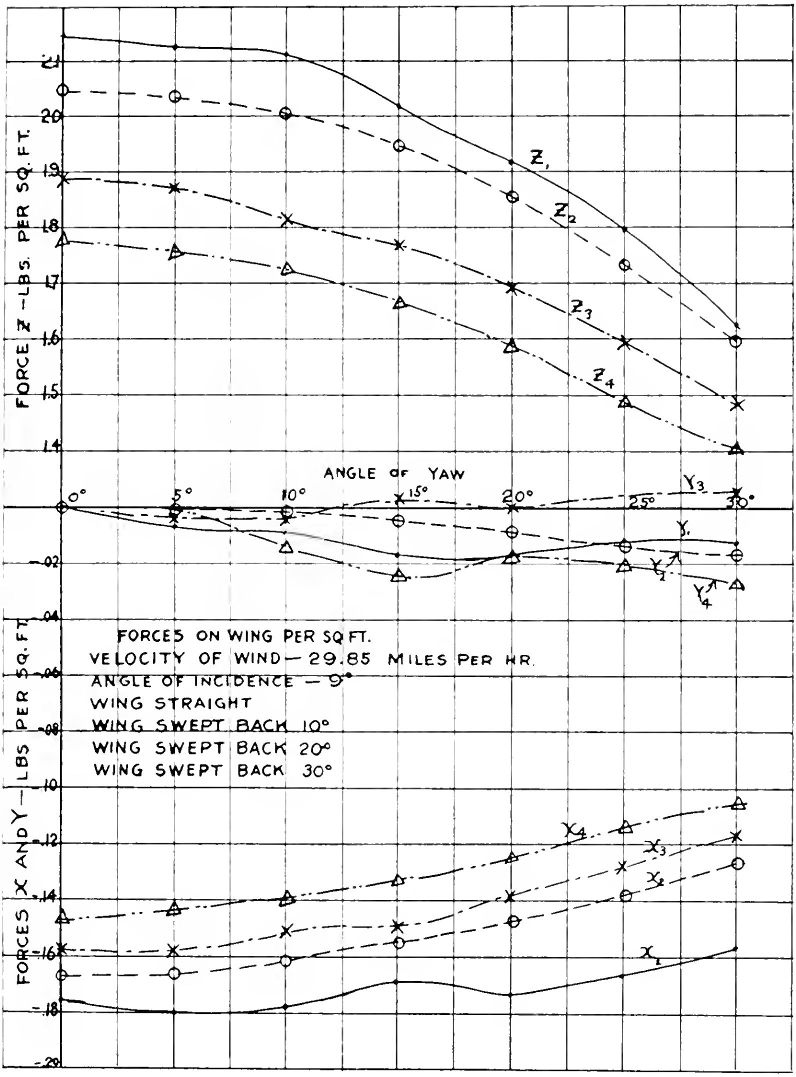


FIG. 30.

In view of the structural difficulties in making strongly swept back wings, and the loss of effectiveness as carrying surface for a sweep of more than 20 degrees, one would conclude that if any sweep back be used, about 10 or 15 degrees is sufficient.

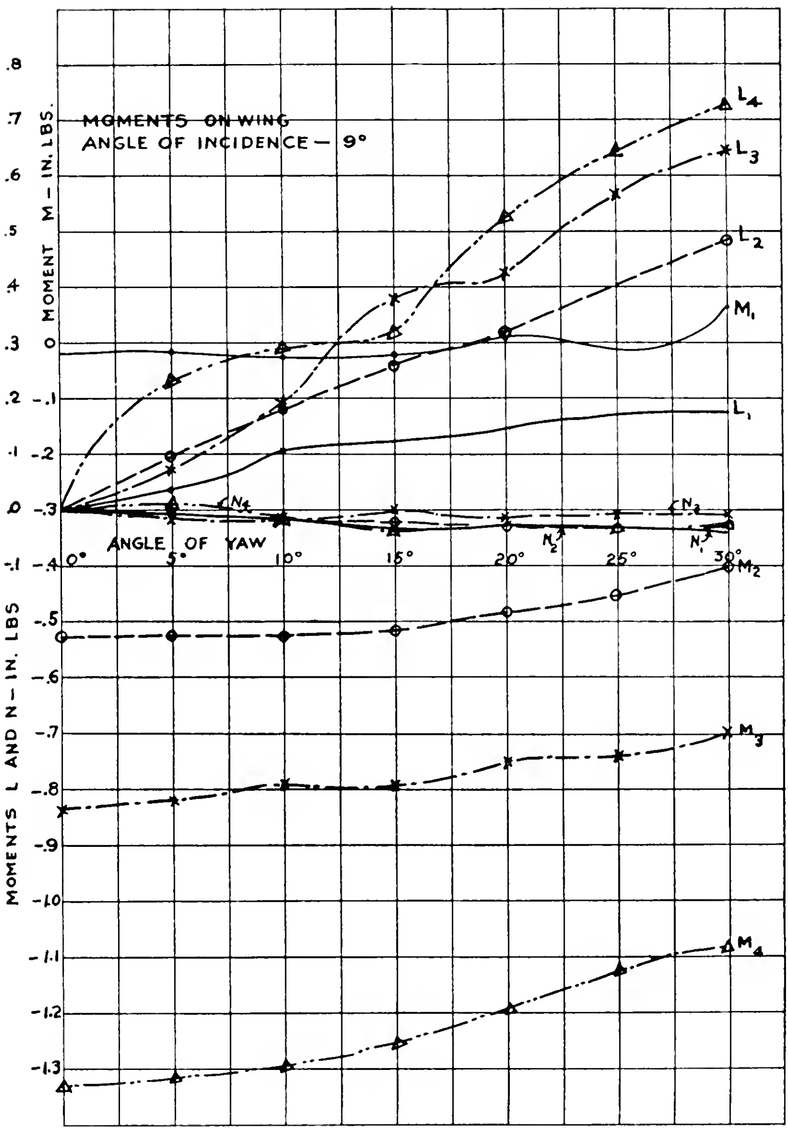

MOMENTS ON WING
ANGLE OF INCIDENCE — 9°
MOMENT M – IN. LBS.
MOMENTS L AND N – IN. LBS
ANGLE OF YAW
.8
.7
.6
.5
.4
.3
.2
.1
.0
-.1
0
-.1
-.2
-.3
-.4
-.5
-.6
-.7
-.8
-.9
-1.0
-1.1
-1.2
-1.3
0°
5°
10°
15°
20°
25°
30°
L_4
L_3
L_2
M_1
L_1
N_4
N_3
N_2
N_1
M_2
M_3
M_4

FIG. 31.

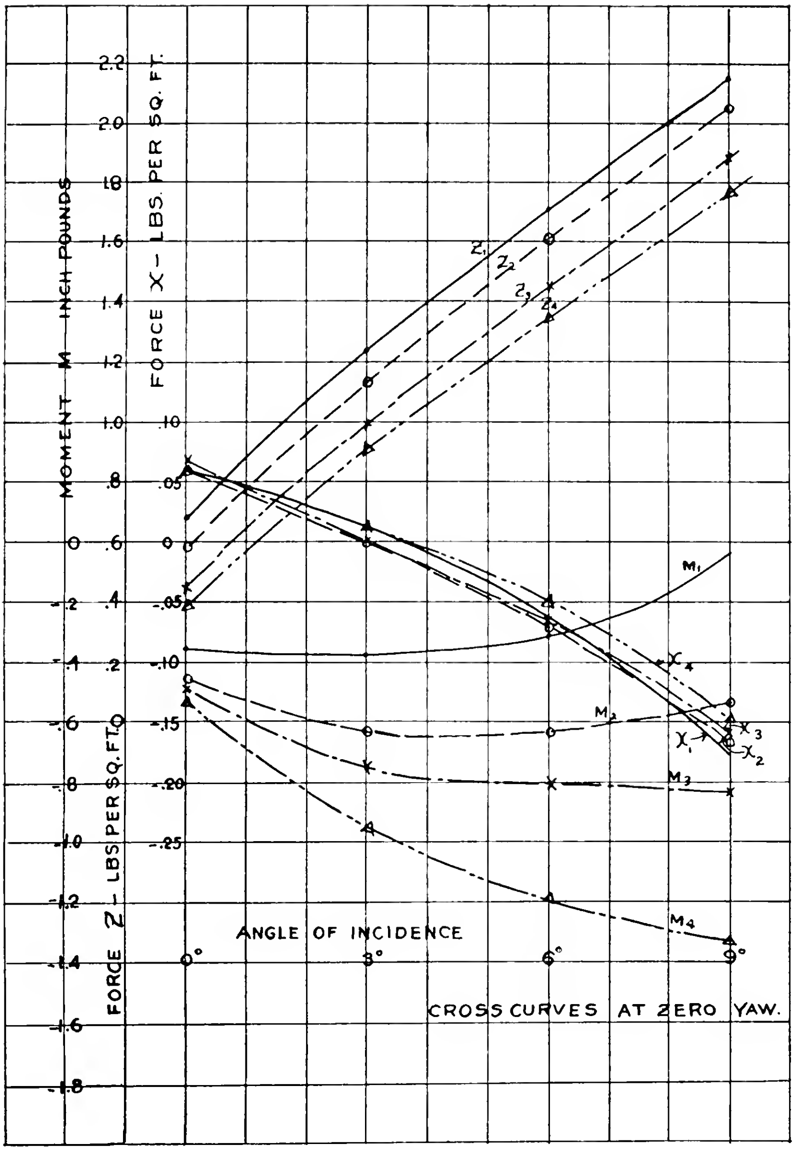

MOMENT M - INCH POUNDS
FORCE X - LBS. PER SQ. FT.
FORCE Z - LBS PER SQ. FT.
ANGLE OF INCIDENCE
CROSS CURVES AT ZERO YAW.
0°
3°
6°
9°
M1
M2
M3
M4
Z1
Z2
Z3
Z4
X1
X2
X3
X4

FIG. 32.

The banking moment is appreciable on the aeroplane, as may be shown by the following calculation:

Incidence 6 degrees, yaw 15 degrees, sweep 10 degrees, moment L_1 on model .18 pound-inch at 29.85 M. P. H., moment on biplane of 400 square foot area L_2 at 60 M. P. H.

$$\frac{0.18}{12} \times 2\left(\frac{200}{.364}\right)^{1.5} \times \left(\frac{60}{29.85}\right)^2 = 1{,}560 \text{ pound-feet.}$$

The greatest value of swept back wings is to be found on a side slip. If by any accidental cause an aeroplane with swept back wings is heeled over, it begins to side slip toward the low side. The apparent wind is no longer from dead ahead, but is a little to one side as compounded from the velocity of side slip and velocity of advance. The effect is the same as if the machine had yawed from its course, and with swept back wings the "natural banking" moment becomes a much greater righting moment than with straight wings. The machine then has a degree of lateral stability.

For lateral stability, swept back wings can be made to give a righting couple without introducing a lateral force or yawing couple. The absence of a yawing couple and lateral force is of advantage if the machine is to be kept on its course as it rolls.

Naturally, it is purely a matter of judgment whether too large a righting couple is disadvantageous. In a gusty wind, a machine with swept back wings will tend to roll as side gusts strike it. This may be uncomfortable for the pilot, but if his aileron control be powerful, he can always overcome the rolling moment due to the wings. Approaching a landing, this is especially necessary. In the air, it would be both unnecessary and fatiguing for him to fight the natural rolling of his machine.

A "natural banking" moment can be obtained by the use of a vertical fin above the center of gravity, or by giving the wings an upward dihedral angle. The equivalence of these methods has not yet been determined.

These tests bring out simply the fact that with a sweep back of 10 degrees an appreciable righting moment may be expected without change in any of the other aerodynamical properties of the straight wing.

No reference is made here to the "rotary derivatives" or changes in forces and moments produced by angular velocity. The damp-

ing effect of wings on such oscillations can hardly be appreciably affected by sweep back, and accordingly this question has not been investigated.

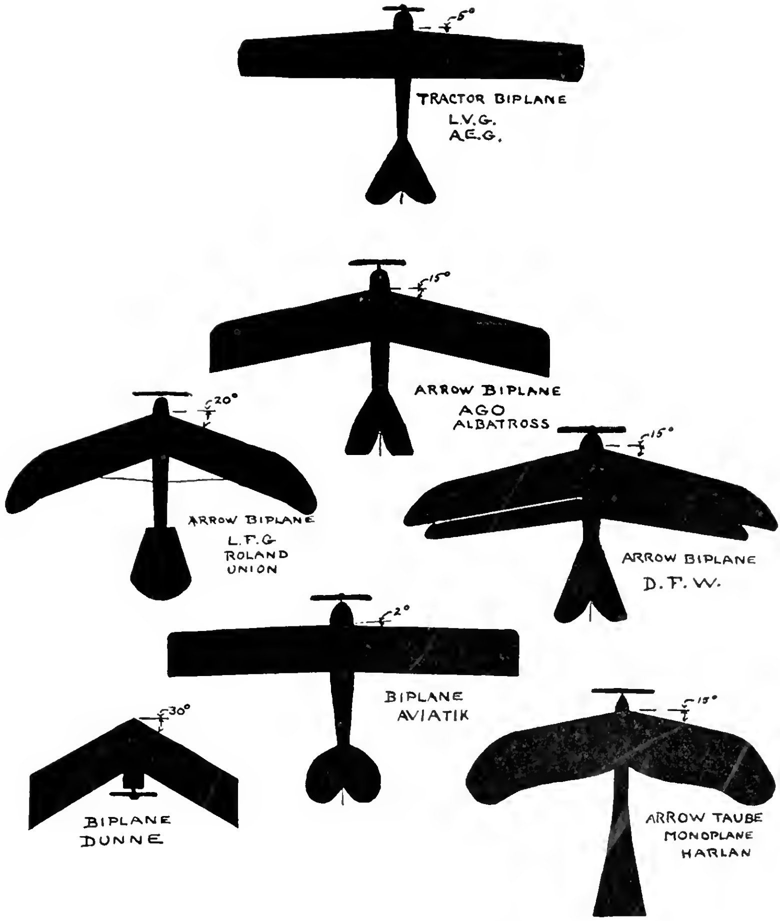


FIG. 33.—Recent aeroplanes.

Figure 33 shows some types of aeroplanes used in Germany at the present time, and also the English Dunne machine. Both the German machines and the Dunne are reported to be laterally stable in normal flight.

IX. EXPERIMENTS ON A DIHEDRAL ANGLE WING

By J. C. HUNSAKER and D. W. DOUGLAS

Following up experiments by Rossell and Brand showing the effect on the lateral stability of sweeping back the wings of an aeroplane, additional tests have now been made to determine whether the righting moment given by the above procedure cannot be better obtained by another method. To this end, a brass model 18 inches by 3 inches having curvature, known as R. A. F. 6,[1] was made identical in every way with the straight wing tested by Rossell and Brand, except that each half of the wing was inclined upwards and outwards $2\frac{1}{2}$ degrees from the horizontal. This gave a dihedral angle upward of 175 degrees. The use of this amount of angle has come into general practice in aeroplane design.

The wing was mounted horizontally on the balance and forces and moments were measured for angles of yaw of 0, 5, 10, 15, 20, 25, and 30 degrees to right and left and at angles of incidence of 2, 4, 6, and 9 degrees. The velocity of the wind was kept constant at 30 M. P. H. standard air. The method of manipulating the balance to make the necessary measurements is described fully in Report 68, Technical Report of the Advisory Committee for Aeronautics, 1912-13 (London). The calculations, from the measured forces and moments, to obtain the final results, were made as is outlined in the preceding report on Swept Back Wings, by Rossell and Brand (page 61).

The axes along and about which the calculated forces and moments act are as follows:

OX—longitudinal axis coincident with chord at middle section.

OY—transverse axis of wing, perpendicular to OX.

OZ—normal axis of wing, perpendicular to OX and OY.

The origin O is the intersection of the axis OX with the leading edge of the wing.

The forces are:

X—acting along OX, positive when in direction of wind.

Y—acting along OY, positive when tending to increase side slipping to left.

Z—acting along OZ, positive when acting upwards.

The moments are:

L—acting about OX, rolling moment, positive when wing tends to take proper bank for turn to right.

[1] Technical Report of the Advisory Committee for Aeronautics, London, 1912-13.

M—acting about OY, pitching moment, positive when wing tends to stall.

N—acting about OZ, yawing moment, positive when wing tends to turn to right away from wind.

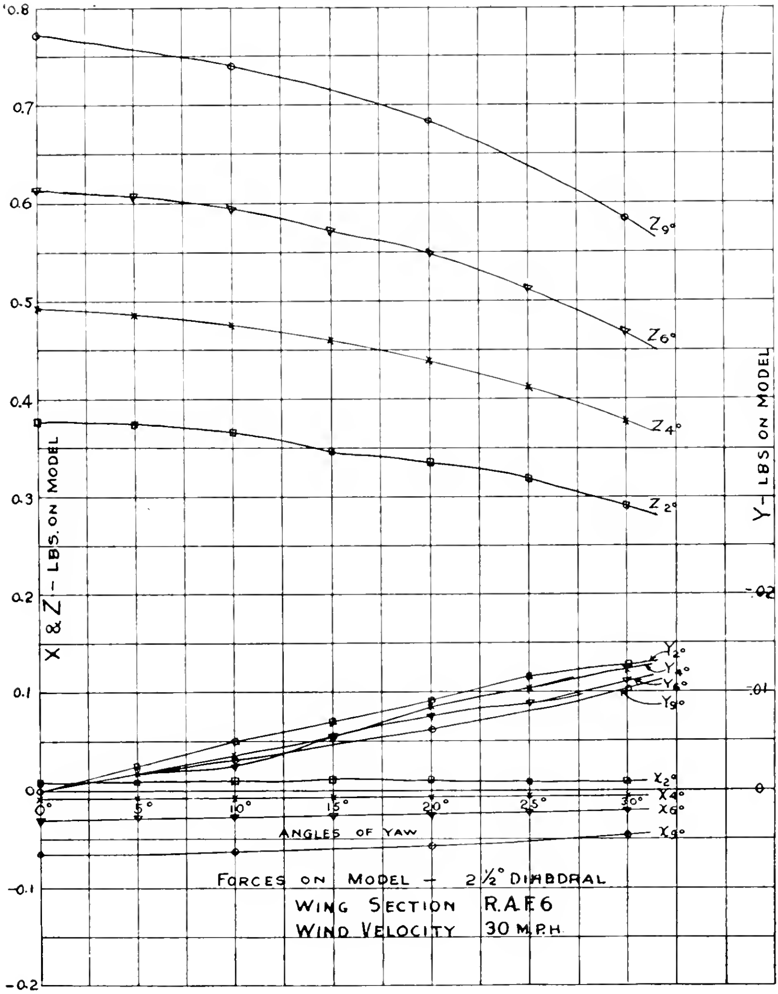


FIG. 34.
(Subscripts designate angle of incidence.)

In figure 34 we can see the changes of the forces X, Y, and Z with yaw and incidence. It will be noted that Y is plotted to a scale of ordinates ten times greater than X and Z. This force, although negative and hence acting to resist a side slip, is so small in magni-

tude that it is negligible. Z falls in magnitude with increase of yaw angle as is to be expected, and as was the case with the swept back wings.

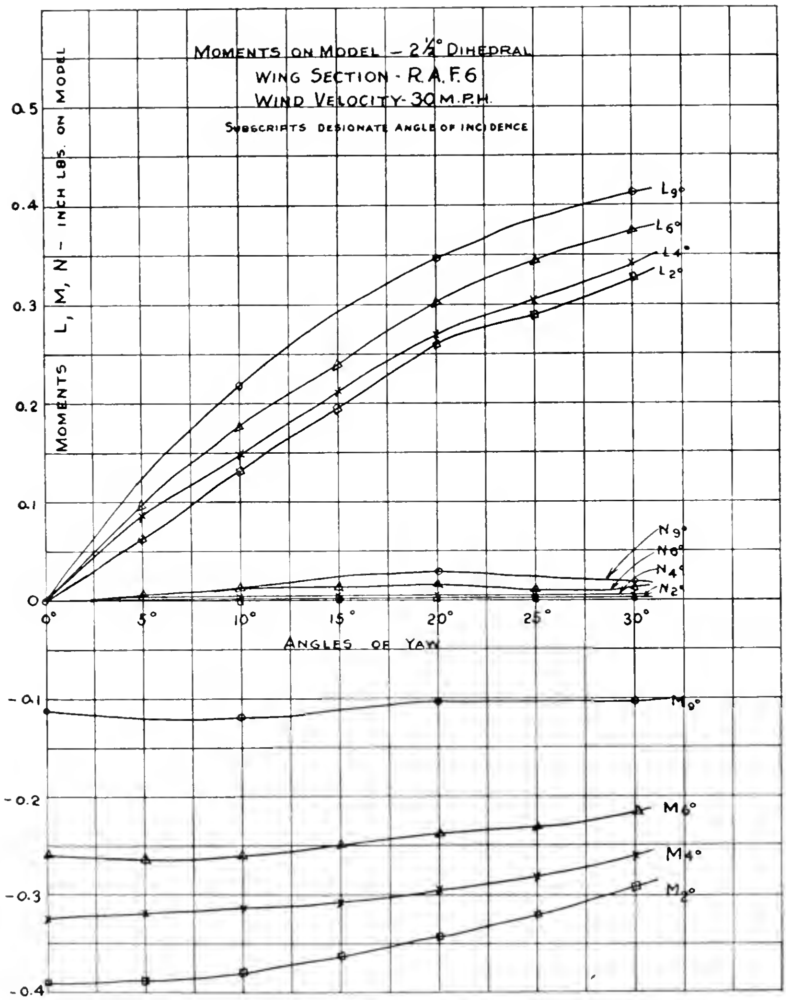


FIG. 35.

Figure 35 shows the moments on the wing. N, the yawing moment, is very small and does not change appreciably with the incidence of the wing.

M, the pitching moment, shows at the smaller angles of incidence a tendency for the machine to stall in side slipping. At the larger

angle of incidence of 9 degrees, however, this stalling tendency disappears. This is a good feature, as it shows that the wing should not get into a bad stalling position when side slipping. The dihedral angle seems in this respect superior to the swept back wing, which has a stalling tendency at all incidences.

The curves for rolling moments, *L*, show for all incidences a rolling moment which increases rapidly with angle of yaw and with the incidence. This moment is a natural banking moment, and hence is one which is favorable for lateral stability. Comparing the rolling moments curve at 6 degrees incidence, for a normal wing, with the dihedral curve, it is seen that the magnitude is from two to three times greater in the case of the dihedral. Comparing this same dihedral rolling moment curve with those for the swept back wings (fig. 29), it is seen that the dihedral gives a rolling moment of magnitude about equal to that obtained with 20 degrees swept back wings, except at a large angle of yaw.

As it is much more difficult structurally to build a 20-degree swept back wing than a dihedral, and as the latter is as effective, it seems that the dihedral is of more value for purposes of lateral stability.

It is of interest to note in this connection that Professor Langley's "aerodrome" of 1902, as well as his previous power-driven models, were given dihedral angle wings inclined upwards by about 6 degrees.

X. CRITICAL SPEEDS FOR FLAT DISKS IN A NORMAL WIND

By J. C. HUNSAKER

1. Prefatory Note on Normal Flow Past a Circular Disk

By E. B. WILSON, Professor of Mathematics, Massachusetts Institute of Technology

Theoretical hydrodynamics cannot yet give a very satisfactory quantitative account of Mr. Hunsaker's experiments to which this note is attached. The classical theory treats two cases which are qualitatively somewhat like nature: First, symmetric continuous flow; second, asymmetric discontinuous flow. In both cases the flow is irrotational and the fluid incompressible. In the first case, owing to the symmetry of the lines of force, there is theoretically no resultant pressure to urge the disk down stream. Such a condition may possibly approximate to nature in cases of slow flow under high pressures. In the second case, as the method of solution depends on

the theory of a complex variable, the problems treated are those of two dimensional flows, and here the whole fluid is divided by a surface of discontinuity (for velocity) into a moving and a stationary portion with the stream lines in the moving portion diverging instead of closing in behind the obstructing object. This state of affairs may be found in nature to a certain approximation in the case of jets. It is clear that the theory of the continuous solution is entirely inapplicable in the discussion of the pressure exerted on the disk by the fluid, and that for that problem the discontinuous solution would be necessary. And it is equally clear that for a discussion of the possibility of a critical velocity the discontinuous solution is disadvantageous, and the continuous solution must in the present state of our knowledge be used for what information it may afford.

We shall henceforth assume that the fluid is incompressible, that the motion is irrotational, symmetric fore and aft of the disk, and identical in all planes through the axis of the disk; that the eddies which are known to form are to be disregarded; and that the viscosity may be neglected. We know that the ordinary continuous symmetric solution must break down at least as soon as cavitation appears, and we may with reasonable safety assume that the velocity $u=U$ of the stream sufficient to induce incipient cavitation at the edges of the disk will be an upper limit for the critical velocity found by Mr. Hunsaker. (There is unfortunately no assurance that the upper limit may not seriously exceed that critical velocity.) Now if the disk has a perfectly sharp edge, cavitation will take place for any velocity of the general stream, no matter how small that velocity may be. To get any result of value we must therefore replace the disk by a body of rounded contour, such as an ellipsoid of revolution. The theoretical problem which we shall solve is therefore this: To find the velocity U at which cavitation begins in the case of an extremely flat ellipsoid of revolution whose axis coincides with the general course of the stream; and for this we shall determine a very simple approximate expression in terms of the axes of the ellipsoid. For an ellipsoid 6 inches by 1/16 of an inch we shall find $U=22$ foot-seconds, or thereabouts, and this result will be discussed in the light of the experiments.

As the motion is irrotational, by assumption, there is a velocity potential ϕ, and as the density is assumed constant, the velocity potential satisfies Laplace's equation,

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0.$$

We have to find a solution of this equation which is the same at all points of circles about the axis of rotation (owing to the assumed rotational symmetry), which reduces to ux at all points at great distances from the ellipsoid (the axis of x being taken along the axis of revolution and u being the general velocity of the stream), and which satisfies the condition that the normal derivative $\frac{d\phi}{dn}$ vanishes at each point of the ellipsoid (as the flow must be tangential to the ellipsoid).

We first introduce cylindrical coordinates with the axis of revolution as axis. Then

$$y=r\cos\theta,\quad z=r\sin\theta,$$

and

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}=0.$$

As we are interested only in solutions which do not depend on θ, Laplace's equation reduces to

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}=0. \tag{1}$$

We next replace x and r, the rectangular coordinates in a plane through the axis, by a system of coordinates (ζ, μ) derived from the system of ellipses and hyperbolas in the xr-plane confocal with the ellipse in which that plane cuts the ellipsoid.[1] If c be the distance from focus to center, we write

$$x=c\mu\zeta,\quad r=c\sqrt{1-\mu^2}\sqrt{\zeta^2+1}. \tag{2}$$

The elimination of μ and ζ respectively gives

$$\frac{x^2}{c^2\zeta^2}+\frac{r^2}{c^2(\zeta^2+1)}=1 \text{ and } \frac{r^2}{c^2(1-\mu^2)}-\frac{x^2}{c^2\mu^2}=1. \tag{3}$$

A small value of ζ gives a narrow ellipse; a large value, a large ellipse. The set of ellipses may therefore be represented by values of ζ from 0 to ∞. A small value of μ gives a sharp hyperbola nearly coincident with $x=0$, a value of μ equal to 1 gives the line $r=0$, the axis of revolution. By assigning different signs to μ and to one of the radicals in (2) we may represent all points (x, r) of the plane; but the symmetry of the figure is such that we may work only in the

[1] This system of coordinates is that used by Lamb, Hydrodynamics, 2d ed. (1895), p. 150, for treating the motion of an ellipsoid of revolution in a fluid at rest at infinity. We could use Lamb's analysis and make a correction to bring the ellipsoid to rest in a moving fluid. It seems as easy to solve our problem independently with all the simplifications it admits.

quadrant in which x and r are positive, and hence deal only with positive values of μ and the radicals.

In terms of the coordinates (ζ, μ) equation (1) becomes

$$\frac{\partial}{\partial\zeta}\left[(\zeta^2+1)\frac{\partial\phi}{\partial\zeta}\right]+\frac{\partial}{\partial\mu}\left[(1-\mu^2)\frac{\partial\phi}{\partial\mu}\right]=0, \qquad (4)$$

as a straightforward change of variable will show.[1] We follow the usual method of integration and try for particular solutions of the form[2]

$$\phi=Z(\zeta)M(\mu),$$

a product of a function of ζ by a function of μ. Then (4) becomes

$$\frac{1}{Z}\frac{d}{d\zeta}\left[(\zeta^2+1)\frac{dZ}{d\zeta}\right]+\frac{1}{M}\frac{d}{d\mu}\left[(1-\mu^2)\frac{dM}{d\mu}\right]=0.$$

Here the variables are separated and the equation could not hold identically unless the two parts were equal and opposite constants. If we set

$$\frac{d}{d\mu}\left[(1-\mu^2)\frac{dM}{d\mu}\right]+n(n+1)M=0, \qquad (5)$$

$$\frac{d}{d\zeta}\left[(\zeta^2+1)\frac{dZ}{d\zeta}\right]-n(n+1)Z=0, \qquad (6)$$

we see that the first is Legendre's equation and the second a slight modification of it. For $n=0, 1, 2, \ldots\ldots$, the polynomial solutions of (5) are respectively constant multiples of 1, μ, $3\mu^2-1$, ... ; and of (6), 1, ζ, $3\zeta^2+1$, ...

A consideration of (2), or of the figure made up of the confocal ellipses and hyperbolas, shows that for large values of ζ, *i. e.*, in the distant portions of the plane, $c\zeta=\rho$ and $\theta=\cos^{-1}\mu$ are approximately polar coordinates with the x-axis as polar axis. Then in these regions we have approximately

$$\frac{\partial\phi}{\rho\partial\theta}=-\sin\theta\frac{\partial\phi}{\rho\partial\mu}=-\text{(tangential velocity along } \rho=\text{const.)}.$$

[1] It is easier to transform (4) into (1), and still easier to express directly in terms of ζ and μ the condition of continuity; for if ds_e and ds_h are elements of arc along the ellipses and hyperbolas, the velocities are $-\frac{\partial\phi}{ds_e}$ and $-\frac{\partial\phi}{ds_h}$, and the flux is

$$\frac{\partial}{\partial s_h}\left(r\frac{\partial\phi}{\partial s_e}\right)+\frac{\partial}{\partial s_e}\left(r\frac{\partial\phi}{\partial s_h}\right)=0,$$

which is readily expressed in terms of ζ, μ.

[2] See, for example, Wilson's Advanced Calculus, Chapter XX.

By the hypothesis this velocity is $-u \sin\theta$, where u is the velocity of the stream. Hence, when ζ is large, we must have for all values of μ approximately

$$\frac{\partial\phi}{c\zeta\partial\mu} = \frac{Z}{c\zeta}\frac{dM}{d\mu} = -u, \text{ independent of } \mu. \tag{7}$$

It follows that M cannot be of higher order than 1, and the only possibilities for n are 0 and 1. The solution for ϕ must therefore be of the form

$$\phi = Z_0 + \mu Z_1, \tag{8}$$

where Z_0 and Z_1 are solutions of (6) for $n=0$ and 1 respectively. Moreover, for the radial velocity in distant regions we have

$$-\frac{\partial\phi}{c\partial\zeta} = -\frac{dZ_0}{cd\zeta} - \mu\frac{dZ_1}{cd\zeta} = u\cos\theta = u\mu. \tag{9}$$

We have next to express the condition that flow along the ellipsoid shall be tangential, *i. e.*, that the normal derivative $\frac{d\phi}{dn}$ shall vanish. The normals to the elliptical section of the ellipsoid are the hyperbolas along which μ is constant. The normal dn is

$$dn = \sqrt{dx^2 + dr^2} = c\left(\mu^2 + (1-\mu^2)\frac{\zeta^2}{\zeta^2+1}\right)^{\frac{1}{2}} d\zeta = c\sqrt{\frac{\mu^2+\zeta^2}{\zeta^2+1}}\, d\zeta.$$

The particular ellipse which is the profile of the disk is determined by some value ζ_0 of ζ. Then

$$\frac{d\phi}{dn} = c\sqrt{\frac{\zeta_0^2+1}{\mu^2+\zeta_0^2}}\left(\frac{dZ_0}{d\zeta} + \mu\frac{dZ_1}{d\zeta}\right)_{\zeta=\zeta_0} = 0.$$

As this equation holds for every μ, we have

$$\left(\frac{dZ_0}{d\zeta}\right)_{\zeta=\zeta_0} = 0, \quad \left(\frac{dZ_1}{d\zeta}\right)_{\zeta=\zeta_0} = 0. \tag{10}$$

The equations (7), (9), (10) should suffice to determine what values of Z_0 and Z_1 are needed in (8) to represent the flow.

The general solution of (6) for $n=0$ may be obtained at once by integration,

$$Z_0 = C_1 + C_2 \tan^{-1}\zeta. \tag{11}$$

The general solution for $n=1$ may be obtained from the above-mentioned particular solution ζ by the substitution $Z = Z'\zeta$, which gives

$$Z_1 = K_1(\zeta\tan^{-1}\zeta + 1) + K_2\zeta. \tag{12}$$

From (10), (11), (12) we find:

$$\frac{C_2}{\zeta_0^2+1} = 0, \quad K_1\left(\tan^{-1}\zeta_0 + \frac{\zeta_0}{\zeta_0^2+1}\right) + K_2 = 0.$$

Hence $C_2=0$, and the ratio $K_1:K_2$ is determined. From (7) and (9) we have the approximate equations

$$\frac{K_1(\zeta\tan^{-1}\zeta+1)+K_2\zeta}{c\zeta}=-u,\quad K_1\left(\tan^{-1}\zeta+\frac{\zeta}{\zeta^2+1}\right)+K_2=-cu.$$

Hence $K_1\frac{\pi}{2}+K_2=-cu$ and

$$K_1=\frac{-cu}{\cot^{-1}\zeta_0-\frac{\zeta_0}{\zeta_0^2+1}},\quad K_2=\frac{cu}{\cot^{-1}\zeta_0-\frac{\zeta_0}{\zeta_0^2+1}}\left(\tan^{-1}\zeta_0+\frac{\zeta_0}{\zeta_0^2+1}\right).$$

As the solution for Z_0 reduces to a constant by $C_2=0$, the value of ϕ may be taken as merely μZ_1 in (8) ; or

$$\phi=\mu\left[\frac{cu}{\cot^{-1}\zeta_0-\frac{\zeta_0}{\zeta_0^2+1}}\right]\left[-\zeta\tan^{-1}\zeta-1+\zeta\left(\tan^{-1}\zeta_0+\frac{\zeta_0}{\zeta_0^2+1}\right)\right].$$

We are dealing only with very small ellipsoids and hence ζ_0 is small. Hence ϕ reduces approximately to

$$\phi=\frac{c\mu u}{\frac{\pi}{2}}[-\zeta\tan^{-1}\zeta-1+2\zeta_0\zeta].$$

The velocity along the ellipse $\zeta=\zeta_0$ may be obtained like the normal velocity. The element of arc ds is

$$ds=\sqrt{dx^2+dr^2}=c\left[\zeta_0^2+\frac{\mu^2(\zeta_0^2+1)}{1-\mu^2}\right]^{\frac{1}{2}}d\mu=c\sqrt{\frac{\zeta_0^2+\mu^2}{1-\mu^2}}\,d\mu,$$

$$-\frac{d\phi}{ds}=\frac{2u}{\pi}\sqrt{\frac{1-\mu^2}{\zeta_0^2+\mu^2}}[1].$$

The value of this velocity is greatest when $\mu=0$, *i. e.*, at the ends of the ellipse, as was to be expected; and its value is then $\frac{2u}{\pi\zeta_0}$. The value of ζ_0 may be expressed in terms of the axes of the ellipse. From (3),

$$a=c\sqrt{\zeta_0^2+1},\quad b=c\zeta_0.$$

Indeed the value is $\zeta_0=\frac{b}{c}=\frac{b}{a}$ approximately. Hence the velocity is approximately

$$\text{Maximum velocity}=\frac{2u}{\pi}\frac{a}{b}.$$

By Bernoulli's principle for a stream line or Kelvin's theorem on irrotational motion we have

$$\frac{p}{\rho}+\frac{1}{2}v^2=\text{const.}$$

When cavitation begins, $p=0$, $v=2a\dfrac{U}{\pi b}$. The value of $\dfrac{p}{\rho}+\dfrac{1}{2}v^2$ at large distances from the disk may be taken as $\dfrac{p_0}{\rho}$, where p_0 is the atmospheric pressure in pounds per square foot. Hence we find

$$\frac{p_0}{\rho}=\frac{1}{2}\,\frac{4a^2U^2}{\pi^2b^2},\quad U=\frac{\pi b}{a}\sqrt{\frac{p_0}{2\rho}} \tag{13}$$

As numerical data we may take $p_0=(22\times 12)^2$, $\rho=.08$. Then

$$U=2100\,\frac{b}{a}\text{ foot-seconds.} \tag{14}$$

In the case of an ellipsoid 6 inches by 1/16 inch, the velocity U is about 22 foot-seconds. For smaller disks of the same minor axis the velocity U should be larger in inverse ratio to the size of the major axis.

The upper limit of 22 foot-seconds here found for the critical velocity is not surprisingly above the actual range of 10 to 20 foot-seconds found by Mr. Hunsaker. But a noteworthy result shown on his diagrams is that the critical velocity does not vary proportionately to the reciprocal of the diameter of the disk—it is practically constant—and had we taken to test the theory his smallest disk, we should have had an upper limit decidedly above the experimental value. This lack of accord between his experiments and the present theory can hardly be regarded as surprising. His disks were all equally sharp upon the edge, and all considerably sharper than an ellipsoid of the same length and breadth, particularly in the smaller disks. Once the edge is sharp enough to start cavitation at a given velocity of the general stream, the motion becomes such that we can no longer expect our theory to hold, and it is entirely possible that the thickness of the disk is unimportant above a certain value. The effect of a 6-inch disk 1/16 of an inch thick may not differ appreciably from that of one 1/32 of an inch thick. The effects of the changing density of the air and of viscosity might also, and probably would, be of some importance. On the whole we may consider the correlation of experiment and theory as fairly satisfactory; at least it is good enough to indicate strongly the existence of a critical velocity for these disks at reasonably small velocities of the stream.

2. Critical Speeds for Flat Disks in a Normal Wind

The value of tests on the resistance of small models in a wind tunnel depends upon the precision with which such results may be applied to larger models at different speeds. The resistance of thin disks normal to the wind is not of great practical interest in aeronautics, but certain theoretical conclusions of general importance may be drawn from experiments with disks.

The resistance of the air depends upon the density, viscosity, and compressibility of the fluid. Except in gunnery, the velocities in common use do not approach the velocity of sound in air, and hence compressibility is neglected.

Two well-defined types of flow are recognized. The first is the so-called stream line flow in which the fluid flows around the body in steady lines, closing in behind it so that no turbulent wake is formed. Such flow may be expected about a fish-shaped body at low speeds. The second form is called the discontinuous or turbulent flow, in which eddies are formed on the body and are carried down stream as a turbulent wake. Such flow may be expected about bodies of abrupt form moving at high speeds.

If no eddies are formed, the resistance to motion is a viscous drag depending on the viscosity of the fluid, a linear dimension, and the velocity.

With the formation of eddies, additional resistance is caused by the energy lost in imparting kinetic energy to the fluid in the eddies. It is observed that, in general, eddies are left behind and so represent a loss of energy. The eddy making resistance should depend on the kinetic energy imparted to the fluid and hence should vary as the density, the square of the velocity, and the extent of the disturbance. The latter should be proportional to the square of a linear dimension of the body, or the cross-section of the wake.

In any real case both viscous and eddy resistance are present. For bodies of easy shape, the turbulence is not great and we should expect viscosity to play an important part, and the resistance to vary less rapidly than V^2.

However, for a thin disk with its face normal to the wind we should expect eddy making to be violent and the resistance to vary with the density of the air, area of face of disk, and square of velocity. Under such conditions the viscous drag might be negligible.

At very low speeds, if it be possible for the fluid to turn the corner and close in behind the disk, eddy making may be so much reduced

that the viscous drag is important. An entirely different form of flow may then exist, and the resistance should no longer vary as the square of the velocity. The change from turbulence to steady flow should be marked by a critical velocity. The resistance of disks, aero-

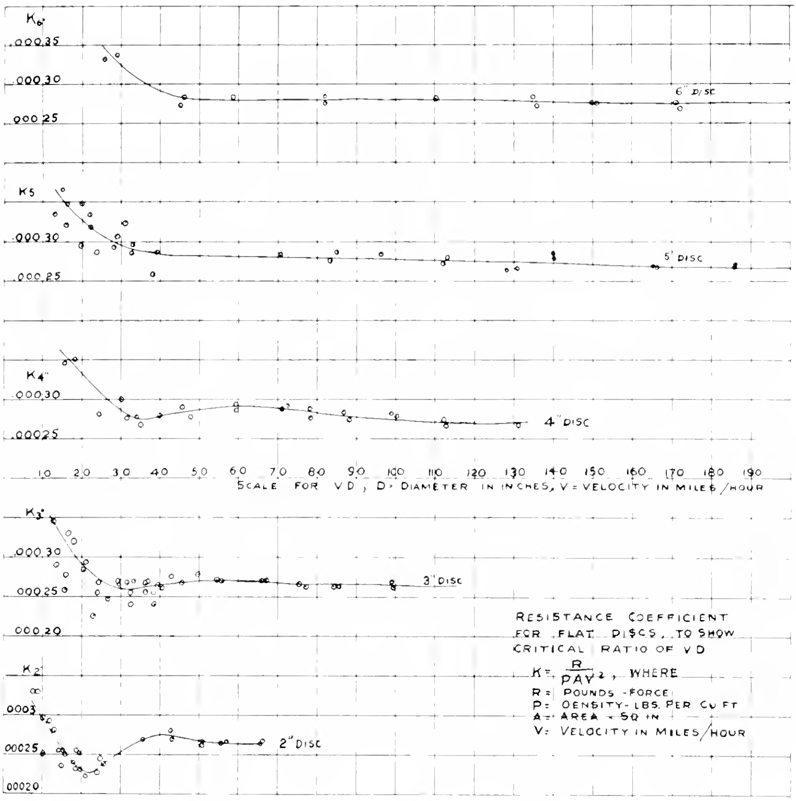


FIG. 36.

plane wings, and all sharp-edged objects is usually represented by a formula as follows:

$$R = K\rho AV^2,$$

where

- R is resistance in pounds,
- ρ is density in pounds per cubic foot,
- A is area in square inches,
- V is velocity in miles per hour,
- K a coefficient assumed constant.

To verify this law of resistance, it was decided to test in the wind tunnel a series of thin disks placed normal to the wind. Disks of sheet brass 1/16 inch thick with square edges and diameter 2, 3, 4, 5, and 6 inches were used. The resistance of each disk was measured for a series of speeds. The measurements were repeated with the face of each disk reversed and the results averaged.

If resistance be correctly represented by the formula above, the constant K computed from the observed resistances should remain constant. In figure 36 the values of K computed from each observation are plotted on the product VD as abscissæ, when D is the diameter in inches. It appears that K does not remain constant even for a given disk. The points represent actual observations, and it is seen

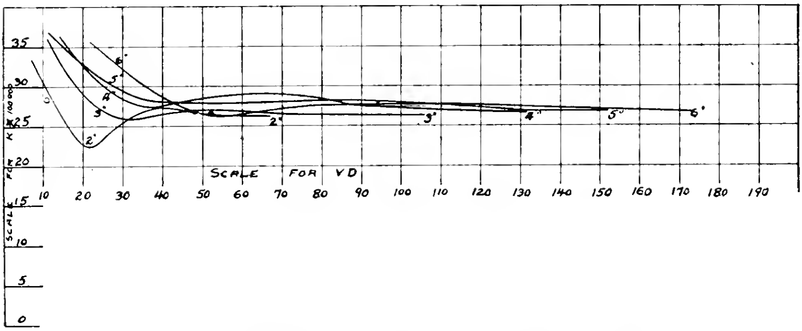


FIG. 37.—Resistance coefficient as a function of VD.

$$K = \frac{R}{\rho A V^2}$$

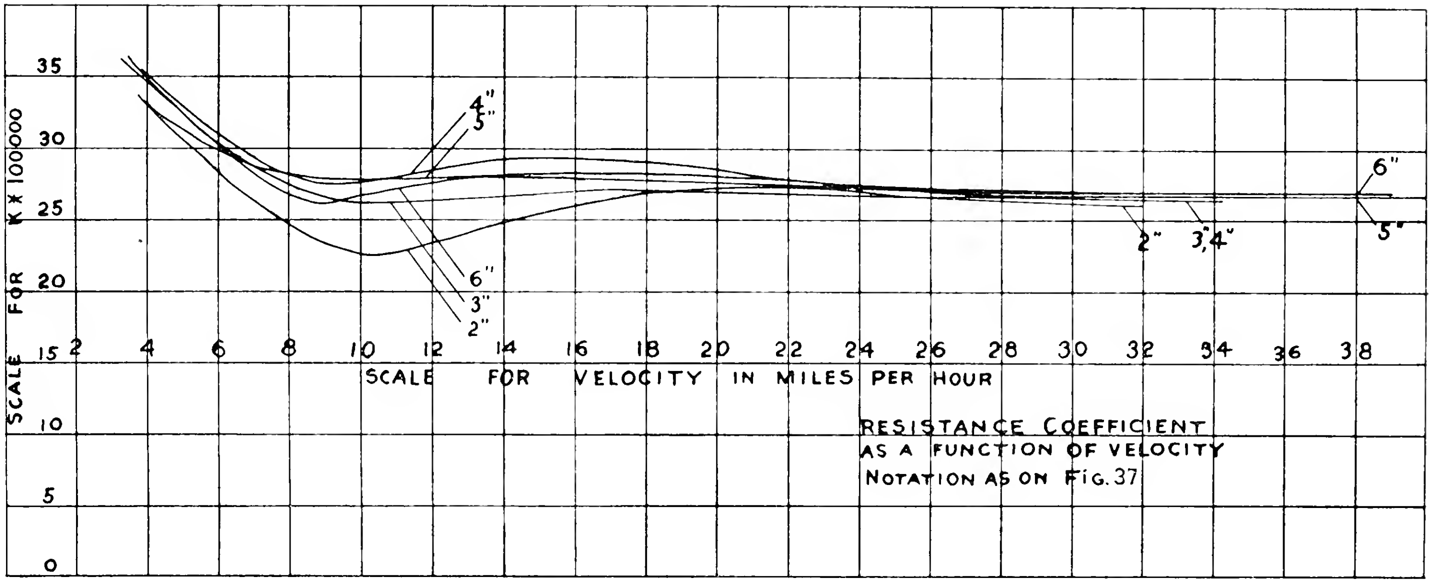
where

R = force in pounds.
P = density in pounds per cubic foot.
A = area in square inches.
V = velocity in miles per hour.
D = diameter in inches.

that below a value of VD of 40, K becomes very erratic. A great many check observations were taken in this region, but the flow seems to be unstable and K cannot be determined with precision. The mean curves are replotted on figure 37 on VD as abscissæ, and again on figure 38 on V as abscissæ. The critical velocity apparently discovered seems to be about 9 miles per hour for all the plates.

Theoretically, if the resistance due to viscosity be important the coefficient K should be a function of the product VD. Thus, Lord Rayleigh has suggested[1] that where both density and viscosity are

[1] Phil. Mag., p. 66, July, 1904.



SCALE FOR K × 100000
35
30
25
20
15
10
5
0
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38
SCALE FOR VELOCITY IN MILES PER HOUR
4"
5"
6"
3"
2"
6"
5"
2"
3",4"
RESISTANCE COEFFICIENT
AS A FUNCTION OF VELOCITY
NOTATION AS ON FIG. 37

Fig. 38.

important, the most general form of the resistance equation which satisfies dimensional requirements is the following:

$$R=\rho AV^2 f\left(\frac{VL}{\nu}\right),$$

when L is a linear dimension, such as diameter of disk; ν the kinematic viscosity of the air, taken constant; and f an unknown function of the single variable $\left(\frac{VL}{\nu}\right)$.

This expression may be written

$$R=\rho AV^2\phi(VD), \text{ when } \nu \text{ is sensibly constant.}$$

The coefficient K should then be a function of VD and the curves of figure 37 should coincide. This is not the case, and it may be concluded that the effect of viscous drag is not important, or that the disks are not geometrically similar since the thickness remains constant. The critical velocity appears by figure 38 to be about the same for each disk.

For velocities above 27 miles per hour, the value of K remains practically constant for all speeds and all disks. The usual formula can, therefore, be applied if model tests are run at speeds greater than 27 miles per hour. Our wind tunnel testing will in future be conducted at speeds between 27 and 40 miles per hour.

The values of K for larger disks appear to be larger than those for the smaller disks. This discrepancy may be eliminated when it is considered that the 6-inch disk obstructs 1.25 per cent of the tunnel and should cause us to underestimate the mean velocity past the disk by about 1.25 per cent, and consequently K as computed will be 2.5 per cent too large. The actual discrepancy between values of K for the 2-inch and 6-inch disks is about 3.5 per cent, leaving 1 per cent to be laid to experimental errors.

It seems safe to conclude that for speeds above 27 miles the coefficient K remains constant and the same for all disks.

The same conclusion was reached by Stanton[1] and by Riabouchinski,[2] but Eiffel[3] states that K is greater for larger surfaces. His tests in the wind tunnel certainly show an increase of K with area, but

[1] T. E. Stanton, Proceedings of the Institution of Civil Engineers, Vol. 156, Part II, London, 1904.

[2] Bulletin de l'Institut de Koutchino, Moscow, 1912.

[3] La Resistance de l'Air et l'Aviation, Paris, 1912.

this may largely be accounted for by the effect of the walls.[1] M. Eiffel's tests in the open air with large plates dropped from the Eiffel Tower indicate still larger values of K, but here velocities were as great as 90 miles per hour. In addition to the extreme difficulty of obtaining precise measurements of resistance and speed with this method, a further complication is injected into the problem from the fact that the velocity of fall was accelerating.

It seems obvious that the resistance to accelerated motion should be greater than that due to uniform motion, since the energy system accompanying the disk is being built up.

In conclusion, then, within the limits of these tests, the resistance of a flat disk normal to the wind for speeds above a certain minimum may be correctly represented by

$$R = K\rho A V^2.$$

The existence of a critical velocity for disks has not been detected in previous experiments. M. Eiffel's tests were not run at sufficiently low velocity, Riabouchinski's tests were not very precise; but the experimental work of Stanton or Föppl might have been expected to bring out a critical velocity.

One explanation may lie in the fact that the critical velocity at which eddy making begins to become a stable phenomenon depends largely on the quality of the wind in the tunnel. It is reasonable to suppose that a turbulent wind would cause the critical velocity to be reached sooner than it would be with a more steady wind.

Experiments at Göttingen on spheres showed a marked critical velocity at a certain point. When the wind was made more turbulent by a fish net up stream, the critical velocity came at about two-thirds of the former value.[2]

The Göttingen tunnel in which Föppl's tests were made is a closed circuit in which the wind is guided around the corners by vanes, strained through diaphragms and wire mesh and forced to flow in horizontal lines where the model is placed. The result of such forcing the air to take unnatural lines of flow must introduce a degree of

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[1] Lord Rayleigh, "On the Resistance Experienced by Small Plates Exposed to a Stream of Fluid," Phil. Mag., July, 1915. A simple yet delicate experiment showed that the resistance of two small plates was equal to that of a single large plate of the same area for the same speed. This is in agreement with the results presented above for disks, and indicates that if there be a critical velocity, this velocity is the same for both large and small areas. Lord Rayleigh used disks cut from cardboard, which had presumably a square edge resembling that of the brass disks.

[2] Zeitschrift für Flugtechnik und Motorluftschiffahrt, May 16, 1914.

turbulence into the translational motion that is finally realized. The turbulence of the wind may make the flow about the model turbulent at all speeds used. Hence no critical velocity would be found.

Stanton's tests were made with a Pitot tube placed 1 foot up stream from the model. The tube was made up of two ⅛-inch tubes. The disks were then placed in the wake of these tubes and we might expect the critical velocity to be affected. That the effect of such an arrangement is appreciable was determined from a simple test. In the course of our test on the 5-inch disk above, a glass tube 3/16 inch in diameter was run across the tunnel 1 foot up stream from the disk. The resistance of the disk was found to be reduced by about 6 per cent. Increasing Stanton's value of K by 6 per cent brings it into agreement with our mean value of K. The following table summarizes the values of K for flat plates normal to the wind given by the several experimenters. The notation and units used are as as above.

Authority	Diameter Plate Inches	Velocity Miles	K	Remarks
Eiffel	4	25	.000240	4.9 ft. wind tunnel.
"	6	25	.000243	4.9 ft. " "
"	10	25	.000247	4.9 ft. " "
"	15	40–90	.000263	Open air.
"	20	40–90	.000275	"
"	28	40–90	.000284	"
"	39	40–90	.000290	"
Riabouchinski..	0.5	3–15	.000255	3.9 ft. wind tunnel.
" ..	1	3–15	.000252	3.9 ft. " "
" ..	2	3–15	.000252	3.9 ft. " "
Föppl..	7.9	15–30	.000255	6.7 ft. " "
Author........	2	35	.000265	4.0 ft. " "
"	3	35	.000265	4.0 ft. " "
"	4	35	.000265	4.0 ft. " "
"	5	35	.000265	4.0 ft. " "
"	6	35	.000268	4.0 ft. " " corrected for influence of walls.
Stanton........	5	13	.000243	2.0 ft. wind tunnel.
"	1	13	.000250	2.0 ft. " "
"	1.5	13	.000248	2.0 ft. " "
"	2	13	.000246	2.0 ft " "
"	3	13	.000268	2.0 ft. " "

It has been suggested that the critical velocity found by us is not that of the disks but a critical velocity for the tunnel. It is well known that there is a marked critical velocity for the flow of air or water in pipes where the fluid becomes suddenly turbulent.

From the experiments of Stanton and Pannel[1] on the flow of air, oil, and water through pipes, it was concluded that turbulence began

[1] Phil. Trans. Roy. Soc., 1914, p. 200.

for a value of the "Reynolds Number" $\frac{VD}{\nu}=2500$. Here D is diameter of pipe, V is velocity, and ν the coefficient of kinematic viscosity. A very rough calculation for the square 4-foot wind tunnel, using the above figure, indicates a critical velocity of 1/16 mile per hour. For the 3-inch pipes of the honeycomb, the critical velocity is about 1 mile per hour. Our experiments were conducted well above these speeds.

Similarly there is a limiting velocity for the flow of air at atmospheric pressure to be deduced from St. Venant's equation for motion of a compressible fluid.

For air at ordinary temperatures this limiting velocity is about 770 miles per hour.

A more practical condition which may occur at ordinary speeds may account for the existence of an apparent critical velocity at which the fluid refuses to turn a corner due to its inertia. Thus, for true stream line motion about a disk, the air is required to turn sharply over the edge and close in on the back of the disk. The stream line has a finite radius of curvature and a finite velocity at any point, and there is consequently a centrifugal force on each particle of the fluid. Unless the pressure gradient is sufficient to balance this centrifugal force, the curvature of the stream line cannot be maintained. A dead water then forms at the rear of the disk which is dragged away by the viscosity of the moving air in contact with it, thus setting up a turbulent wake. A considerable increase in resistance might be expected to take place when turbulence is set up.

Our experiments show an abrupt change near 13 feet per second for thin disks, with unstable flow for velocities between 10 and 20 feet per second. This range is of the same order of magnitude as the critical velocity for a flat ellipsoid deduced by Mr. E. B. Wilson in the prefatory note above.

NOTE ON AN EMPIRICAL EQUATION TO EXPRESS THE EXPERIMENTAL RESULTS

Within the limits of these experiments the normal resistance of a thin disk may be represented by the expression

$$R=.0018\,D^2V+.00103\,D^2V^2,$$

in which

R is total force on disk in pounds,
D, diameter in feet, and
V, wind velocity in feet per second.

This leads to the equation to a straight line drawn on figure 39,

$$\frac{R}{D^2V} = .0018 + .00103V.$$

It appears that for velocities above 20 feet per second the values of

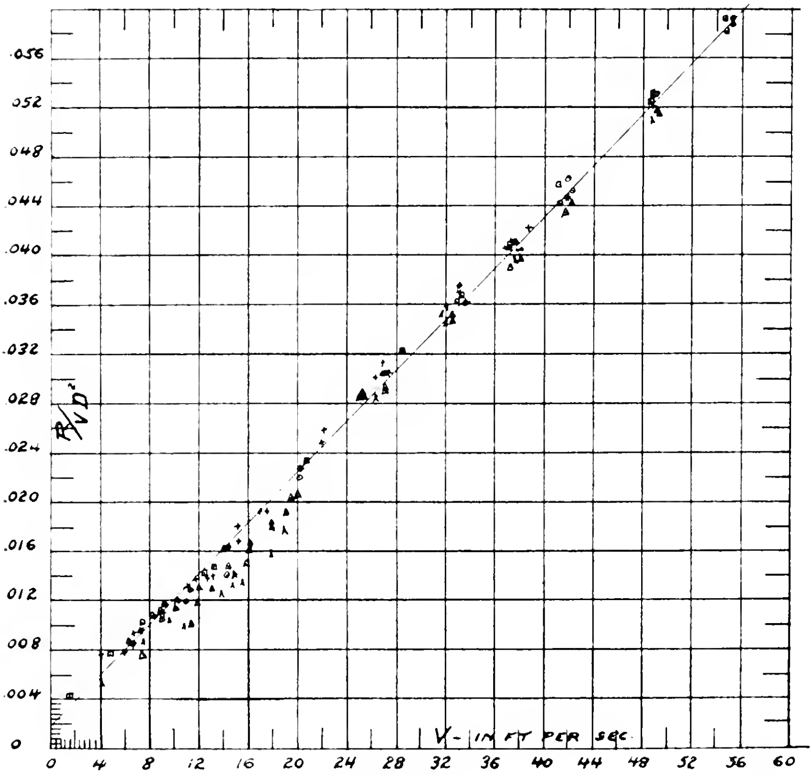

FIG. 39.—Units.

R = Pounds, force.
D = Feet, diameter.
V = Feet per second, velocity.

⊙ 6 ft. disk.
□ 5 " "
× 4 " "
Δ 3 " "
Λ 2 " "

$\frac{R}{D^2V}$ as observed for all disks lie near the line. It should be noted that the above expression is not dimensionally homogeneous, and it is therefore not safe for extrapolation. However, it was pointed out that the disks used were of uniform thickness and hence not geometrically similar.

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DYNAMICAL STABILITY OF AEROPLANES

(With Three Plates)

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PART I. LONGITUDINAL MOTION

§1. INTRODUCTION AND CONCLUSIONS

The present dynamical investigation of the stability of motion of aeroplanes is based upon the well-known theory of small oscillations of rigid dynamics as first applied by Bryan[1] to aeroplanes and extended by Bairstow.[2] The necessary coefficients for use in the equations of motion were determined by model tests in the wind tunnel of the Massachusetts Institute of Technology.

The application of model experiments to predict the performance of full-size aeroplanes is now so well established that no general discussion of the theory of models is undertaken. A great part of the actual experimental work was performed by Messrs. Huff and Douglas. The oscillating apparatus was designed by Mr. Chow under the direction of Professor E. B. Wilson of the Department of Mathematics. Captain V. E. Clark, U. S. A., while a student in aeronautical engineering, designed an aeroplane which was selected as one type for investigation.

It is necessary to acknowledge the cordial interest taken in the work by Professor C. H. Peabody, head of the Department of Naval Architecture. From the beginning of aeronautical research in his department, Professor Peabody has offered the warmest encouragement by countless arrangements to facilitate progress and to prevent interruptions.

Following the analysis of Clark's aeroplane, the work was repeated for a model of a military aeroplane known as Curtiss JN2.[3] The

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[1] G. H. Bryan, "Stability in Aviation."

[2] Technical Report of the Advisory Committee for Aeronautics, London, 1912-13.

[3] Plans and description given in "First Annual Report of the National Advisory Committee for Aeronautics" (Report No. 1, "Report on Behavior of Aeroplanes in Gusts," by J. C. Hunsaker and E. B. Wilson, Washington, D. C., 1916). Senate Document No. 268, 64th Cong., 1st Sess.

Curtiss Aeroplane Company gave their full cooperation with a desire to learn what improvements in the design might be suggested by our stability calculations. Dr. A. F. Zahm of the research department of that company made careful tests to locate the center of gravity and to determine the moments of inertia of the actual aeroplane.

The Curtiss machine is a practical aeroplane with powerful controls, which does not pretend to possess any particular degree of stability. The Clark aeroplane, on the other hand, was designed to be inherently stable while departing as little as possible from the lines of the ordinary military aeroplane as typified by the Curtiss JN2.

The comparison of these two aeroplanes is interesting and leads to the conclusion that inherent dynamical stability, both longitudinal and lateral, may be secured in an aeroplane of current type by careful adjustment of its surfaces and without material effect on controllability or performance.

The discussion in detail is confined to the Clark model, for brevity of presentation, and the results only of the parallel calculations for the Curtiss model are introduced where a comparison is suggested.

In Part I the general equations of motion are deduced in a simplified form which applies to horizontal flight in still air. The longitudinal motion is then considered separately and the necessary aerodynamical constants determined from wind tunnel tests. It is found that the longitudinal motion, if disturbed by any accidental cause, is a slow undulation involving a rising and sinking of the aeroplane as well as a pitching motion. This undulation is stable for high aeroplane speeds since it is rapidly damped out. At lower speeds, the undulation is less heavily damped until at a certain critical low speed the damping vanishes. For speeds below this critical speed, the undulations tend to increase in amplitude with each swing and the longitudinal motion is, therefore, unstable.

Similar calculations for the Curtiss aeroplane show a similar critical speed below which the longitudinal motion is unstable. It is believed that the existence of instability at low speeds has not been indicated before, and it is hoped that the recommendations made to reduce the critical speed may be of assistance to designers.

It appears a simple matter to secure any desired degree of longitudinal stability by the use of properly inclined tail surface, and by the use of light wing loading. It is pointed out that excessive *statical* stability, as indicated by strong restoring moments, is undesirable and may cause the motion to become violent in gusty air. This vio-

lence of motion may seriously impair the pilot's control and the aeroplane may "take charge" at a critical time.

However, the longitudinal motion for any particular speed of flight may be made dynamically stable, while at the same time only slightly stable in the static sense, by the use of a large tail surface which lies very nearly in the relative wind. If the minimum of statical stability be combined with the maximum of damping, the pitching will be very slow and heavily damped. The longitudinal motion can then be dynamically stable and yet be without violence of motion in gusty air.

The general prejudice among pilots against "very stable" aeroplanes is believed to be justified. It cannot be too strongly insisted upon that true dynamical stability is better given by damping than by stiffness.

Experience with rolling of vessels has led to the design of vessels of small metacentric height (a measure of statical stability) fitted with generous bilge-keels (damping surface) for passenger carrying. An effort is made to get away from the violence of motion associated with stiffness.

In Part II, the lateral or asymmetrical motion is investigated. The necessary aerodynamical constants are determined by wind tunnel tests wherever practicable and two coefficients which cannot readily be found experimentally are calculated by a simple approximate method. The character of the motion as indicated by the solution of the determinant formed from the equations is then discussed.

It is found that the lateral motion is a combination of a roll, yaw, and side slip or "skidding." Using approximate methods, the combined motion is resolved into three components, two of which are important.

One type of motion is a spiral subsidence if stable or divergence if unstable. The Clark aeroplane becomes spirally unstable at low speed. The motion is a "spiral dive" due to an overbank and a side slip inwards. The aeroplane makes a rapid turn with rapidly increasing bank accompanied by side slipping inwards. The instability is such that an initial deviation from course will double itself in about 7 seconds.

It is shown that the spiral motion may be made stable by adequate fin surface above the center of gravity or upturned wings and by reduction in "weather helm" due to too much rudder or fin surface aft.

The Curtiss aeroplane shows the same sort of spiral instability at high speeds. This aeroplane had no dihedral angle of wings and had a large rudder and deep body.

The second type of motion has been called a "Dutch roll" from analogy to a figure in ice skating. The aeroplane takes up an oscillation in yaw and roll simultaneously. It swings to the right banking for a right turn, then swings back to the left banking for a left turn. The combined yaw and roll has a fairly rapid period. The Clark model at all speeds shows that this motion is heavily damped and hence stable. At high speed, the period is 6 seconds and an initial amplitude is damped to half value in less than 2 seconds. At low speed the period is 12 seconds, damped to half amplitude in 6 seconds.

It appears from an approximate calculation that the "Dutch roll" may become unstable if an aeroplane has too much high fin surface and if there be not sufficient "weather helm" or rear fin surface. It is seen that these conditions are the reverse of those for spiral instability. The conflicting nature of the requirements for stability in these two kinds of motion suggests that an aeroplane is unlikely ever to be unstable in each sense. It also indicates the difficulty of obtaining lateral stability by raised wing tips.

In confirmation of theory, we found the Curtiss spirally unstable at high speed and stable in the "Dutch roll," while at low speed the spiral motion was stable and the "Dutch roll" unstable. The period was 6 seconds and an initial amplitude doubled itself in 8 seconds.

It is believed that the majority of modern aeroplanes of ordinary type are spirally unstable because of excess of fin surface aft. When attempts have been made to remedy this fault by use of a large dihedral angle upwards for the wings, matters have been made worse. It is only to be expected that in overcorrecting spiral instability a "Dutch roll" of more or less violence may be introduced. Especially in gusty air would one expect high fin surface to produce violent rolling.

The Clark aeroplane has very slight rise of wings, about 1°6, and a small rudder. It is shown that at ordinary speeds this aeroplane is stable in every sense, both longitudinally and laterally. Whether this stability is excessive can only be determined by actual flight experience in turbulent air. However, it has appeared possible to secure a degree of stability in every sense in an aeroplane of conventional type.

The object of the research has been to show how various features of design may affect the motion of the aeroplane and only incidentally

to produce a stable aeroplane. The discussion has been confined to motion in still air. If an aeroplane be unstable in still air it is obviously worse off in gusts. The converse is, unfortunately, not true, for an aeroplane which is very stable in still air may be so stiff that in turbulent air it will be violently tossed about.

It is conservative to conclude that aeroplanes should not be unstable and that they need not be, since slight changes in the nature of adjustments suffice to correct such instability of motion.

In view of the military use of aeroplanes inside the zone of fire the probability of controls becoming inoperative is ever present. An inherently stable aeroplane, with controls abandoned or shot away, could still be operated by a skilful pilot by manipulation of the motor power alone.

Any sort of automatic (or gyroscopic) stabilizer which operates on the controls is of no use when those controls fail, and it should be judged as an accessory to assist a pilot rather than as a cure-all for the inherent instability of an aeroplane's motion.

The ordinary type of aeroplane readily lends itself to adjustments which make for inherent stability of motion and there is no reason to seek radical changes of type to insure stability. Freak aeroplanes of great "stability" may be excessively stable in some ways and frankly unstable in others. It is likely that the most satisfactory aeroplane will be only slightly stable and that this aeroplane will in any possible attitude be easily controlled by the pilot.

Controllability and statical stability are to some extent incompatible. Dynamical stability requires some amount of statical stability and considerable damping. It appears to be of advantage to provide the minimum of statical stability and the maximum of damping. Then the aeroplane's motion will be of very long period but heavily damped.

It is believed that the methods of investigation here described may be applied to any type of aeroplane, and, by the systematic variation of one feature of design at a time, a full understanding may be had of the effect on the motion of each change. The process is of necessity laborious, but compared with the difficulty of full-scale experiment in the open air, the model method is rapid and inexpensive. It is rarely possible in actual flying to obtain any idea of the effect of slight changes in design. Weather conditions, motor troubles, personal peculiarities of pilots, etc., tend to add to the complexity of an otherwise very simple problem.

Furthermore, experimental flying is dangerous. For example, I knew a pilot who, to determine whether a new aeroplane was spirally

unstable, took his machine up to a good altitude and allowed it to get into a spiral dive. The machine made five turns of a rapidly winding and contracting helix before he could bring it out on a horizontal path. If the controls had been only a little less powerful the machine would surely have crashed to the ground. That the controls were adequate was purely a matter of good fortune. The experiment was a success in that spiral instability was demonstrated. Only a few minutes of time was required. However, no information was obtained as to the degree of instability present nor as to what particular changes would remedy matters. To complete the experiment, it would be necessary to repeat the dangerous feat for every change which suggested itself. Naturally, a designer will be very economical in his suggestions under such conditions.

§2. TYPE DESIGN

The type aeroplane selected for the first investigation is a two-place biplane tractor designed by Captain V. E. Clark, U. S. A., while a student in the graduate course in aeronautical engineering at the Massachusetts Institute of Technology. This aeroplane is considered to be representative of modern practice in aeroplane design. Its principal dimensions are as follows:

Wing area, including ailerons	464	sq. ft.
Span, feet	41	max., 40.2 mean.
Aspect ratio	7	
Gap	6.37	ft.
Dihedral of wings, degrees	176.75	
Area, stabilizer	16.1	sq. ft.
Area, elevators	16.0	sq. ft.
Area, rudder	9.35	sq. ft.
Length, body	24.5	ft.
Depth, body, maximum	3.2	ft.
Width, body, maximum	3.3	ft.
Weight, bare	1,070	lbs.
Weight, personnel	320	lbs.
Weight, fuel and oil	415	lbs.
Weight, full load	1,805	lbs.
Radii of gyration	5.2	ft., in roll.
	4.65	ft., in pitch.
	6.975	ft., in yaw.
Brake horse-power	110	
Fuel and oil per B. H. P., hour	0.73	lb.

Maximum speed	87	miles per hour.
Minimum speed	35	miles per hour.
Initial rate of climb..................	900	ft. per min.
Best glide	1 in 9	
Endurance, full power.................	5.6	hours.
Endurance, reduced power, 14 hours at..	47	miles per hour.

§3. MODEL

A model, $\frac{1}{26}$ scale, was made by Edward Tighe, model maker, giving a span of 1.58 feet. The size of the model was limited by the size of the wind tunnel which is 16 square feet in section. The model is shown in figure 1 (see pp. 8 and 9). Note that wires are omitted and struts are made round instead of "stream line" in section. It is believed that the effects of these changes on total resistance largely counterbalance each other. This model was carefully shellacked and polished to minimize skin friction. The model is, of course, much more smooth than the full-size aeroplane, as it should be, in order that the surfaces may remain geometrically similar. Model work was to the nearest hundredth of an inch. No propeller was fitted, but in the design account was taken of the propeller race in augmenting resistance.

For simplicity, the model was made with trailing ailerons or wing flaps integral with the wings. This somewhat increases the effective supporting area. The stabilizer and elevator were made in one, corresponding to the elevator flaps in neutral position. These points are made clear on figure 1.

§4. WING COEFFICIENTS

In the course of the design, a wing section was devised by Clark which showed a low resistance at high speed and small angle of attack and at the same time was thick enough to permit the use of robust wing spars. A model of this wing was made, of 18 inches span by 3 inches chord, and tested in the wind tunnel. For various angles of wing chord to wind, the lift L, drift D in pounds, and pitching moment M in pounds-inches were observed for a wind of 30 miles per hour; air of density .07608 pound per cubic foot.

The wind tunnel and balance are duplicates of the 4-foot installation at the National Physical Laboratory, England, and reference may be made to the technical report of the Advisory Committee for Aeronautics, year 1912-13, for a description of the apparatus and method of operation.

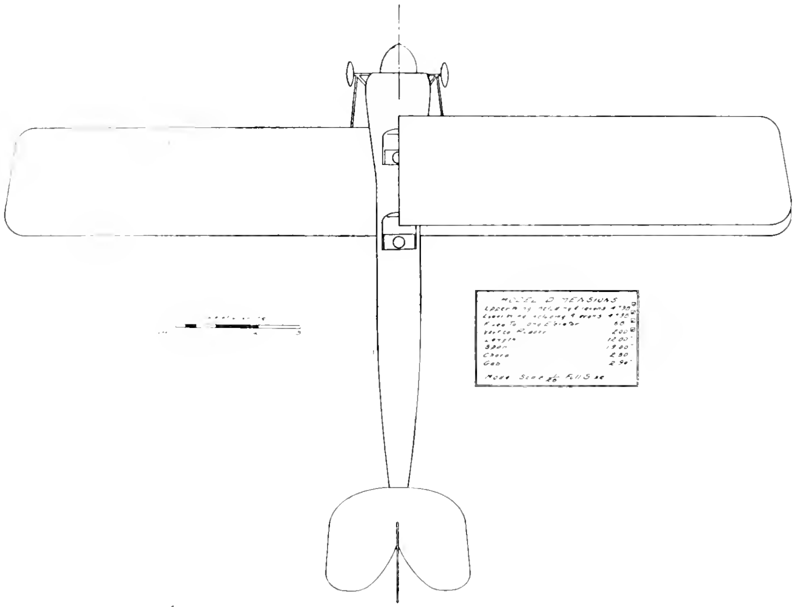


FIG. 1A.

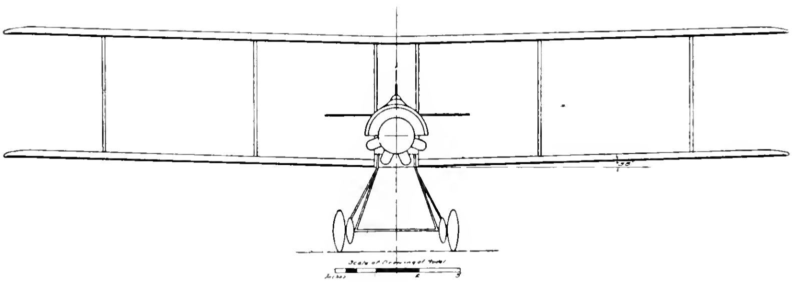


FIG. 1B.

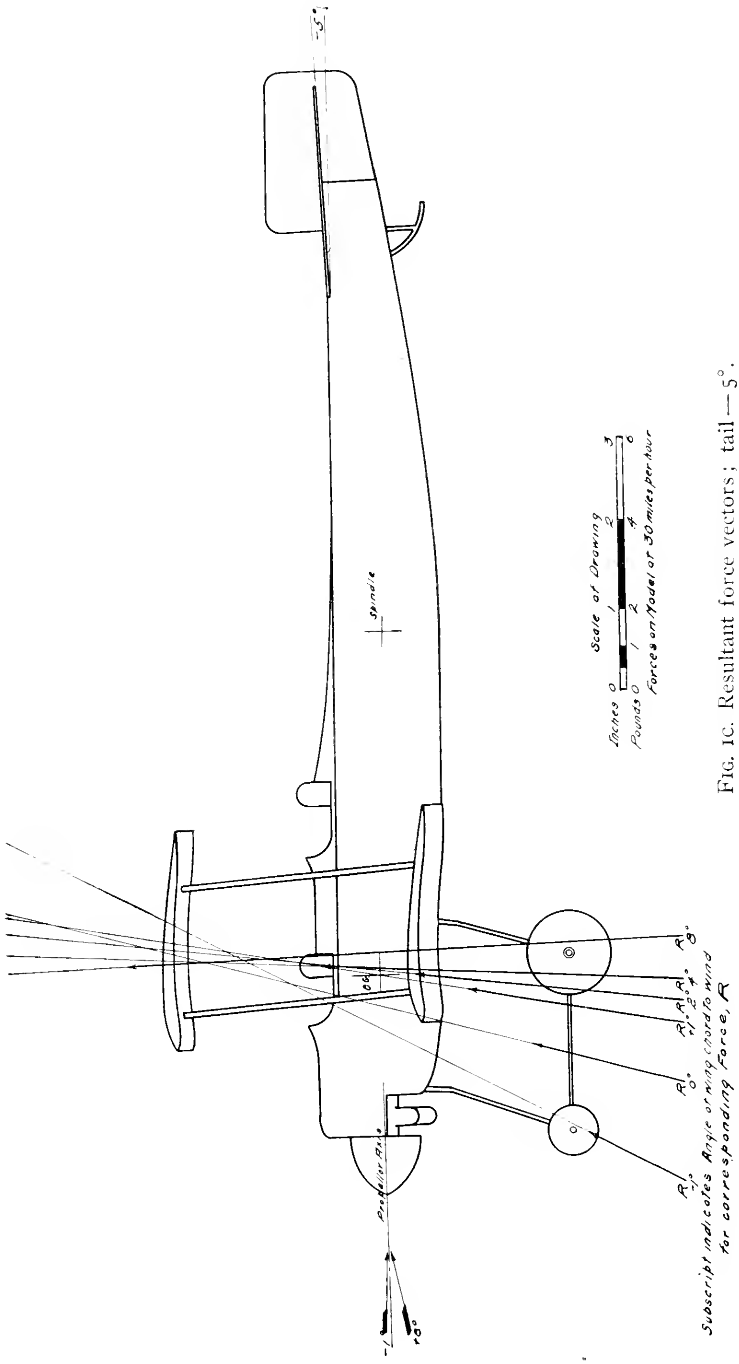


FIG. 1C. Resultant force vectors; tail —5°.

The lift and drift coefficients Ky and Kx were computed from the observed L and D, using such units that the coefficient is pounds force per square foot area per mile hour velocity. Curves of coefficients are given on figure 2, which also shows the ratio L/D, a measure of

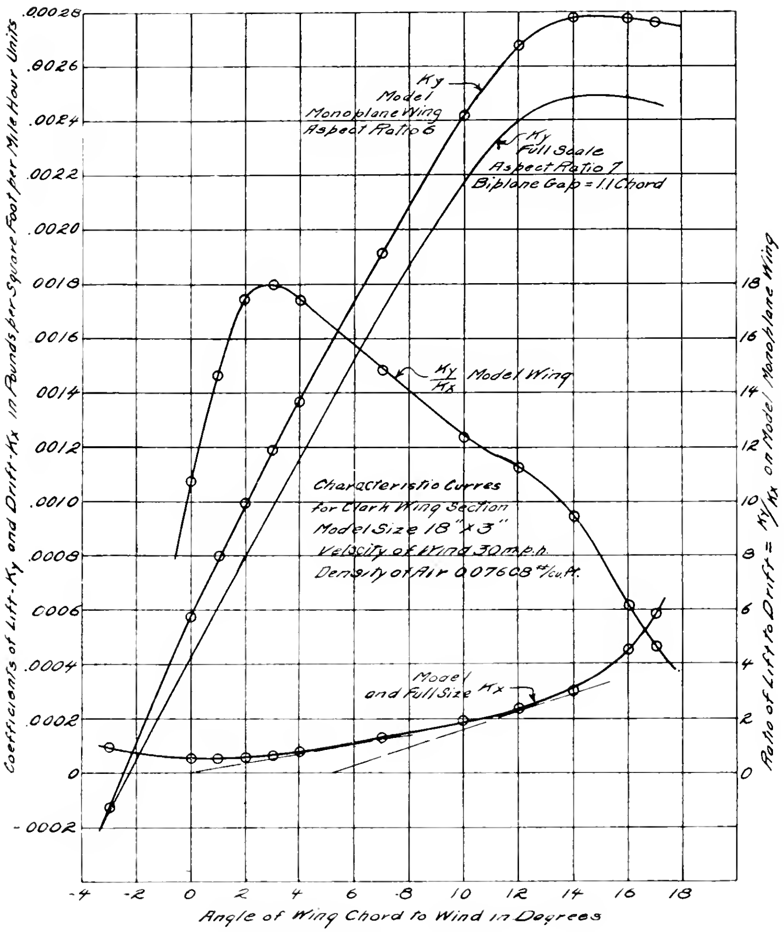
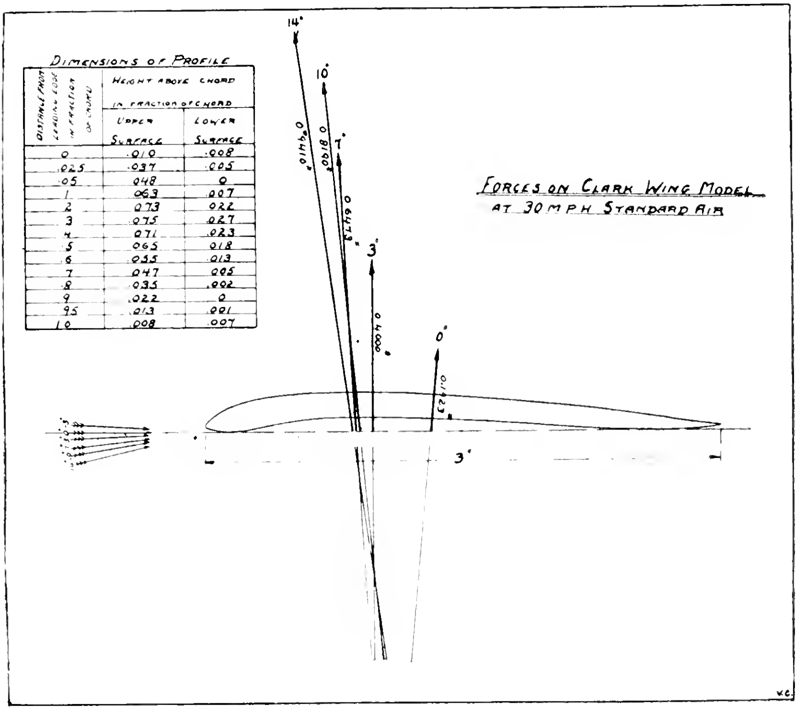


FIG. 2.—Wing coefficients.

the effectiveness of the wing. A maximum L/D ratio of 18 was found for an angle of attack of 3°. For a 41-foot wing at 70 miles per hour, it is believed that the lift coefficient is not greatly different, but that the drift coefficient at small angles is materially reduced. The effect is to increase the ratio L/D. Results of tests at the National Physical Laboratory (Tech. Rept. Adv. Comm. Aero., 1912-

13, p. 81) were applied to the L/D curve for our model to obtain an approximate curve of L/D to apply to the full-size wing. As a monoplane surface, we get a maximum value of L/D of about 20. The particular design is a biplane of aspect ratio 7. Well-known corrections for biplane interference loss and aspect ratio gain were applied to get a corrected curve for use in the design.



Distance from leading edge in fraction of chord	Height above chord in fraction of chord — Upper surface	Lower surface
0	.010	.008
.025	.037	.005
.05	.048	0
.1	.063	.007
.2	.073	.022
.3	.075	.027
.4	.071	.023
.5	.065	.018
.6	.055	.013
.7	.047	.005
.8	.035	.002
.9	.022	0
.95	.013	.001
1.0	.008	.007

FIG. 3.—Wing section dimensions and resultant force vectors.

The center of pressure for this wing is shown by figure 3, as well as the contour of the section. Center of pressure is defined as the intersection of the resultant force on the wing (represented as a vector) with the plane of the chord. It is seen that the wing section is unstable longitudinally at small angles. That is, if the wing heads down so that the angle of attack becomes $-3°$, the moment of the resultant force tends to turn it down still farther.

Applied to the aeroplane, it is necessary to balance and correct this tendency to dive by horizontal tail surfaces of proper size and attitude.

§5. LONGITUDINAL BALANCE

The complete model, using wings of the section described above and fitted with the tail shown in figure 1, was mounted in the wind tunnel on the balance with the wings vertical. A vertical spindle from the balance was driven into the side of the body at the point shown on figure 1. By swinging the model about the vertical axis passing through the spindle, the angle of the wind to the wing chord was varied from +20° to −8°. At each attitude the force across the wind or lift *L*, force down wind or drift *D*, and the pitching moment about the spindle were measured. The wind velocity was 30 miles per hour for all tests. The signs were taken so that an actual lift, actual drift, and a stalling moment are positive. Density of air is at 15° C., 776 mm. Hg., dry.

Test No. I was made with the horizontal tail surface making an angle of −2°.75 with the wing chord. That is to say, the rear edge of the tail was tilted up. Test No. II was a repetition but with the tail at −7°. Test No. III had the tail surface at −5°.

The lift and drift in pounds on the model at 30 miles per hour are given below, and on figure 4.

	Case I		Case II		Case III	
i	*L*	*D*	*L*	*D*	*L*	*D*
− 4	− .049	+.1147	− .172	+.1363	− .115	+.128
− 2	+ .18	+.1011	+ .035	+.1103	+ .112	+.108
− 1	+ .32	.0988	+ .143	.1047	+ .240	+.104
0	+ .454	.099	+ .298	.1014	.360	.101
+ 1	.569	.1016	.437	.1011	.490	.102
+ 2	.703	.106	.572	.1028	.625	.105
4	.927	.121	.807	.1135	.872	.115
6	1.131	.143	....	....	....	...
8	1.32	.167	1.224	.1381	1.305	.153
10	1.484	.195	....	....	....	...
12	1.604	.236	1.537	.2213	1.568	.213
14	1.653	.313	....	....	....	...
16	....	...	1.640	.391	1.64	.370
18	1.606	.547	1.614	.509	1.58	.498

The lift and drift at first sight appear to differ for the three cases, but it will be observed that the maximum lift is 1.65, 1.64, and 1.64, and the minimum drift is .099, .101, and .101 for the three cases

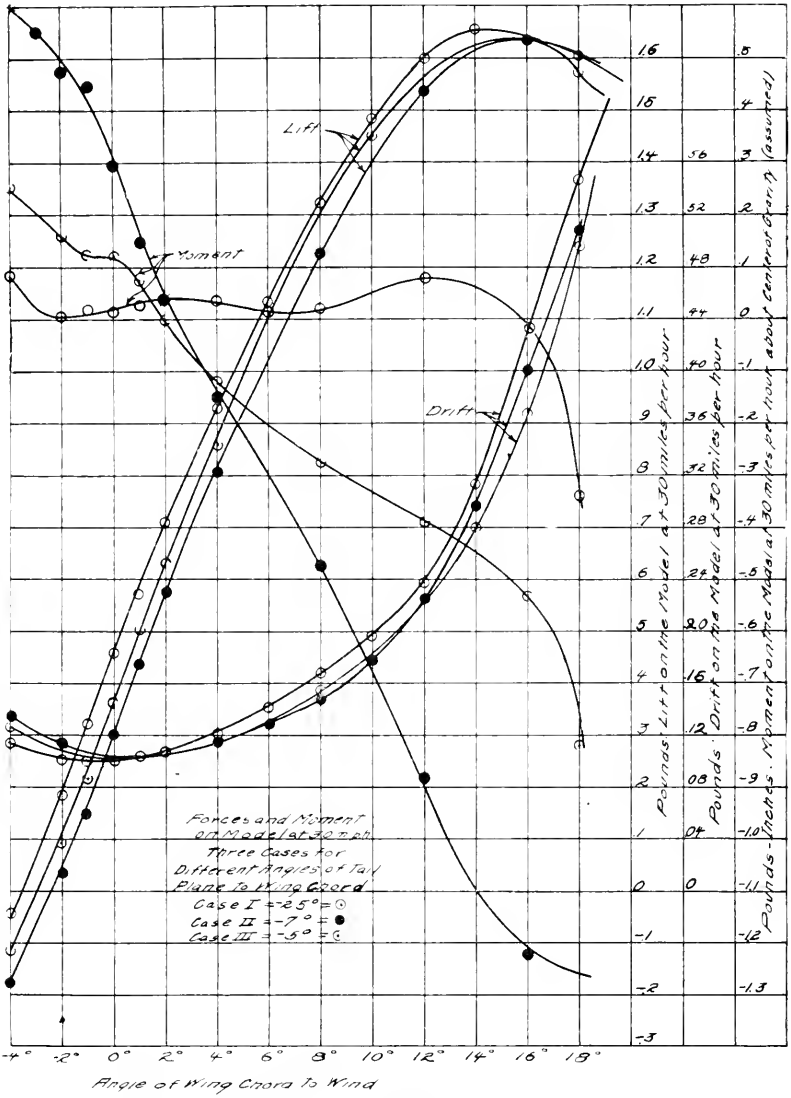


FIG. 4.—Curves L, D, and M for three tail settings.

respectively. The discrepancy is 1 per cent only and is about the precision of the measurements. The comparison is best brought out by eliminating reference to angle of attack as the effect of the change in tail angle appears to be mainly to move the curves of L and D, plotted on i, to the right or left.

Figure 5 shows the ratio L/D for the model for cases I, II, and III, plotted on L in pounds as abscissæ. For small values of L and angles of incidence between $-2°$ and $+2°$, corresponding in practice to high-flight velocity, the curves are practically identical. For angles

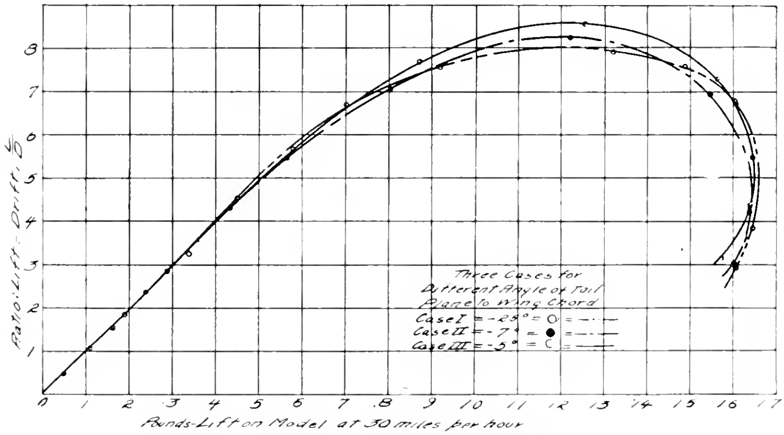


FIG. 5.—Curves of L/D plotted on L for three tail settings.

of incidence near 8°, the L/D ratio for case III is 8.6, while it is 8.2 for case II, and 8.0 for case I.

It appears, therefore, that changing the angle of tail surface has but slight effect on the lift and drift of the aeroplane. The actual aeroplane should have the same maximum and minimum speeds in any case since the maximum lift and minimum drift are about the same regardless of angle of tail surfaces.

The statical stability against longitudinal pitching is, however, very different for the three cases. Thus the pitching moments (observed about the spindle and converted to pitching moments about the assumed center of gravity) are as follows, in pounds-inches on the model at 30 miles per hour. Positive angles and positive moments are stalling angles and stalling moments respectively.

PITCHING MOMENTS, POUNDS-INCHES

i	Case I	Case II	Case III
−4	+.089	+ .599	+.26
−2	+.008	+ .473	+.16
−1	+.022	+ .454	+.12
0	+.016	+ .292	+.12
+1	+.030	+ .143	+.07
+2	+.037	+ .037	−.01
+4	+.039	− .159	−.12
6	+.016	...	..
8	+.023	− .476	−.28
10	...	...	..
12	+.086	− .884	−.39
14	...	...	..
16	−.013	−1.328	−.53
18	−.336	−1.378	−.82

Case I, with tail at −2°75, shows very small pitching moments and may be said to be neutral for ordinary angles of incidence. Thus if the aeroplane be flown at +2° incidence, in order to maintain balance at this attitude the pilot must impress a diving moment of −.037 pound-inch (on the model) to overcome the stalling moment +.037 given above. Then if the aeroplane be accidentally tilted up to +12° by a wind gust or other cause, in the new attitude the net pitching moment is still positive, and hence tends to tilt the machine still more. It is, therefore, unstable unless the pilot intervenes with the horizontal rudder.

For case II, tail at −7°, there is a strong righting moment always acting to prevent stalling or diving. The machine is very stable, in fact excessively so. For instance, flying at 2° incidence, the moment to be held by the pilot is very small. Suppose, however, he wishes to fly at +12° corresponding in the full-scale aeroplane to about 36 miles per hour. To maintain a balance at +12° incidence, he must exert a stalling moment by use of the horizontal rudder equal to about $\frac{.884}{12}\times(26)^3\left(\frac{36}{30}\right)^2=1{,}970$ pounds-feet. The arm of the elevator is about 20 feet (distance aft of center of gravity), requiring a lift of 100 pounds, on the elevator flaps. The elevator is able to exert this force if turned up about 10°. The elevator motion available for control in gusty air is thus largely used up in maintaining balance. The

drift on this elevator flap may be over 20 pounds, making a waste of 3.5 propeller horse-power, or about 6 brake horse-power.

It is preferable to balance a machine at high speed by placing the center of gravity well forward. Then the pilot will have to carry his elevator turned up when flying at low speed. But at low speed, he is most in need of the full elevator motion for control of pitching. We, therefore, conclude that case II, with fixed stabilizer at $-7°$, is very much too stable or stiff longitudinally, and case I, with stabilizer at 2°.75, is not stable enough.

Case III, with stabilizer at $-5°$, appears to balance longitudinally at $+2°$ incidence, and at $+12°$ incidence to have (full size) a natural diving moment which could be held by a negative lift on the elevator of only about 44 pounds, corresponding to about 4° elevator angle. Consequently, it was decided to adopt the arrangement of case III for the subsequent stability investigation.

§6. VECTOR REPRESENTATION

A clearer conception of longitudinal balance is obtained by representing the resultant forces acting on the model as vectors. Thus, for case II, we observed on the balance the lift L and drift D. The resultant force acting was then of magnitude $R=\sqrt{L^2+D^2}$. This resultant force lay in a direction making an angle θ given by $\theta=\tan^{-1} L/D$. The line of action of this resultant was at a perpendicular distance from the spindle axis given by $d=M_s/R$, where M_s is the observed pitching moment about the spindle. The resultant force, R, is thus defined in magnitude, direction, and line of application, and may be represented graphically as a vector. In figure 1, the resultant force vectors for case III are drawn on the side elevation of the model. The model is considered to be fixed and the wind direction to change so that the angle of incidence varies from $-1°$ to $+8°$. The vectors are, therefore, drawn relative to the aeroplane.

The vector for 2° passes near the center of gravity. If it were desired to balance the machine at some other attitude, 6° for example, the center of gravity should be located at some point on the vector for 6°.

Note that on figure 1, for angles greater than 2°, the vectors pass to the rear of the center of gravity indicating diving moments and *vice versa*. Thus the machine is in stable equilibrium at 2°, and if deviated from this angle, righting moments are at once created which tend to restore the normal attitude.

Such stability is "inherent" in the design of the aeroplane and depends wholly on the location of the center of gravity and setting of the stabilizer. No automatic devices are required which may or may not function in an emergency. The inherent stability here shown is static only. Later we will investigate the effects of inertia and damping involved in dynamical inherent stability. However, dynamical stability is impossible unless there be statical stability, and before undertaking a study of the former property, we were obliged to provide a reasonable righting moment to oppose diving and stalling.

§7. PERFORMANCE CURVES

In the design of this aeroplane, the resistance, and hence the speed for given power, was estimated from tests on wings, body, struts, wires, etc., considered separately. The test results were corrected and expanded to full speed full size, using reasonable corrective factors. As is well known, the resistance of many parts does not increase so rapidly as the square of the speed, on account of skin friction. Making all allowances a speed of over 85 miles per hour was predicted for 110 brake horse-power.

If we use the lift and drift observed on the model $\left(\frac{1}{26} \text{ full size}\right)$ at 30 miles per hour and convert to full size by assuming the "law of squares," the performance is not quite so favorable and a maximum speed of but 75 miles per hour is indicated.

For a stability investigation we are little concerned with the exact speed, and for simplicity, the L and D from the wind tunnel test on the complete model of figure 1 are converted to full size by multiplying by the squares of speed and scale.

A total weight of 1600 pounds is assumed, corresponding to tanks half full. For any speed V the lift is a function of speed and attitude and must equal the weight W.

By the "law of squares"

$$\frac{\text{Force on Model}}{\text{Force on Aeroplane}} = \left(\frac{30}{26V}\right)^2.$$

hence:

$$V = \frac{30}{26}\sqrt{\frac{W}{L}},$$

where L is lift on model at 30 miles per hour.

For a series of values of L, corresponding to a series of attitudes or angles of incidence, the required speed V was computed. The

head resistance of the aeroplane moving at these attitudes and with these speeds was computed from:

$$T = D\left(\frac{26V}{30}\right)^2,$$

where D is drift on model at 30 miles per hour, and T total thrust required.

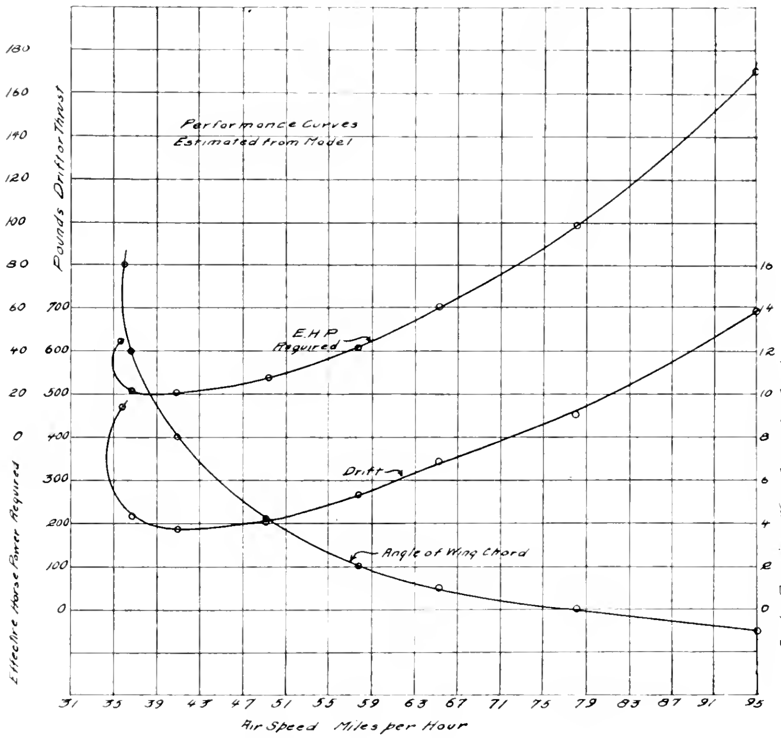


FIG. 6.—Characteristic performance curves.

The effective horse-power required, angle of wing chord to wind and thrust required are plotted as "characteristic performance curves" on figure 6.

§8. AXES AND NOTATION

We shall adopt a notation similar to Bairstow's for the study of dynamical stability. The normal attitude of the aeroplane is its position when in steady flight in a straight line. We select rectangular axes with origin at the center of gravity and fixed in the aero-

plane and moving with it in space. In the normal attitude, the axis of x is tangent to the trajectory of the center of gravity with its positive direction toward the rear. The axis of z is normal to x and

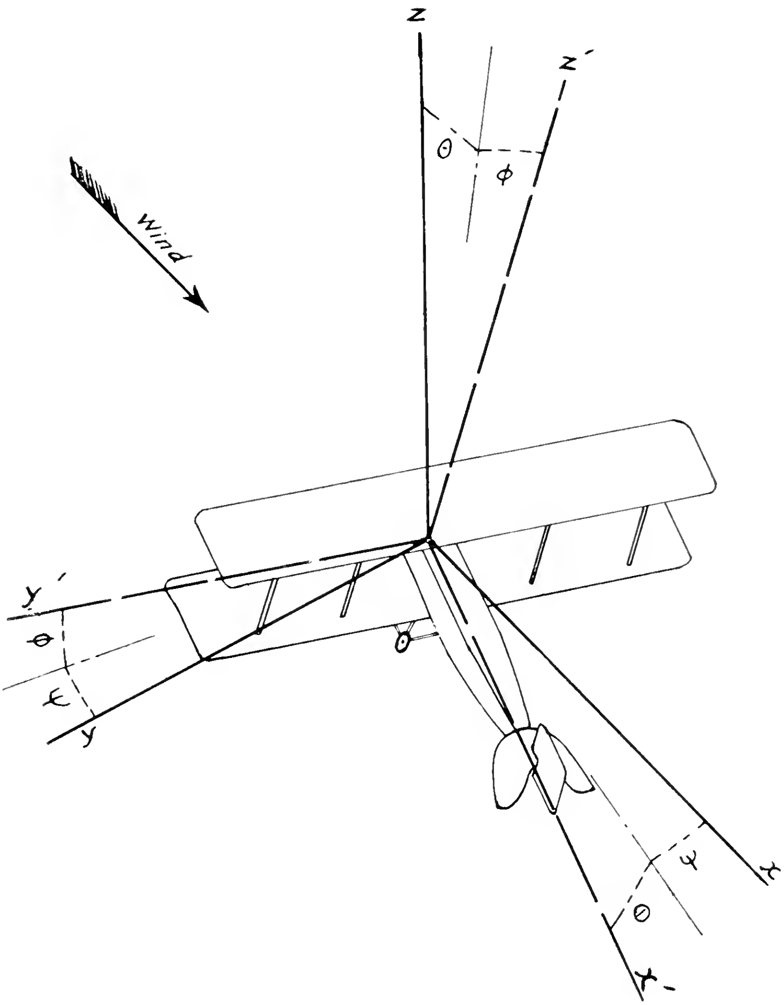


FIG. 7.—Coordinate axes, x, y, z.

y in the vertical plane, and the axis of y horizontal and directed to the left. The axes are shown in figure 7. As the aeroplane rolls, yaws, and pitches these axes move with it, so that z is no longer in the vertical plane of x, nor y horizontal.

Let the aerodynamical forces along the axes x, y, z be denoted by X, Y, Z and expressed in pounds force per unit mass.[1] The moments about these axes are L, M, N in pounds-feet per unit mass. Angular velocities about the axis are p, q, r in radians per second. Let angles of pitch, roll, and yaw away from the normal attitude be θ, ϕ, ψ in radians. Signs are positive in the directions xy, yz, and zx.

The radii of gyration about the axes x, y, z are K_A, K_B, K_C in feet. The mass of the aeroplane is m in slugs. The products of inertia are D, E, F. Two are zero for reasons of symmetry, and one is small in ordinary aeroplanes.

In normal flight in still air, the apparent wind blows in the positive direction of the axis of x. Let this velocity be produced by the forward velocity U of the aeroplane in normal flight. U is a negative number of feet per second.

Let small changes in velocity components along the axes x, y, z be u, v, w when any departure is made from the normal flying attitude.

In normal flight it is assumed that the power available maintains the aeroplane at such a speed that the weight is sustained and also that the normal attitude is that proper for the speed.

§6. EQUILIBRIUM CONDITIONS AND DYNAMICAL EQUATIONS OF MOTION

Let the inclination[2] of the flight path to the horizontal be θ_0. Since normal flight takes place in a straight line, $\psi_0 = \phi_0 = 0$. There is no oscillation and $p_0 = q_0 = r_0 = 0$, and $L_0 = N_0 = 0$.

If the propeller thrust T_0 be exerted in a line above or below the center of gravity h feet, then

$$M_0 = -T_0 h,$$
$$T_0 = -g \sin \theta_0 - X_0,$$
$$Z_0 = g \cos \theta_0.$$

In this aeroplane $h = 0$, and hence $M_0 = 0$.

If any accidental cause slightly disturbs the normal attitude of the aeroplane, the relative wind is no longer symmetrical and the aerodynamical forces and moments are X, Y, Z, L, M, N.

In general, the aerodynamical forces and moments caused by the deviation from "normal attitude" depend upon the relative motion of the aeroplane through the air, which motion is defined by U, u, v, w, p, q, r. Thus $X = f(U, u, v, w, p, q, r)$ where the form of the function f is not known: and five similar expressions for Y, Z, L, M, N.

---

[1] Unit mass is the slug of 32.2 pounds weight.

[2] Consider θ_0 positive for an upwardly inclined path as when climbing.

In the theory of small oscillations u, v, w, p, q, r are small by hypothesis and we may expand X by Maclaurin's theorem, neglecting squares and products of these small quantities. Hence,

$$X = X_0 + uX_u + vX_v + wX_w + pX_p + qX_q + rX_r,$$
$$Y = Y_0 + uY_u + vY_v + wY_w + pY_p + qY_q + rY_r,$$

and similar equations for Z, L, M, N.

Here X_u, X_v, etc., are the partial derivatives of X with respect to u, v, etc., and are the rates of change of X with u, v, etc. That is,

$$X_u = \frac{\partial X}{\partial U} = \frac{\partial X}{u}.$$

There are, therefore, 36 "resistance derivatives" involved which are constants for the aeroplane and depend upon the arrangement of surfaces and their presentation to the relative wind.

Fortunately, for reasons of symmetry, 18 of these derivatives vanish, for example: X_v, X_p, X_r. We then write:

$$X = X_0 + uX_u + wX_w + qX_q,$$
$$M = M_0 + uM_u + wM_w + qZ_q,$$
$$Z = Z_0 + uZ_u + wZ_w + qM_q,$$
$$Y = Y_0 + vY_v + pY_p + rY_r,$$
$$L = L_0 + vL_v + pL_p + rL_r,$$
$$N = N_0 + vN_v + pN_p + rN_r.$$

The above expressions are only approximate if u, v, w, etc., are not small.

The equations of motion for a rigid body having all degrees of freedom, are:

$$\frac{du}{dt} + wq - vr = X + T_0 + g\sin(\theta_0 + \theta),$$
$$\frac{dv}{dt} + (U + u)r - wp = Y - g\sin\phi,$$
$$\frac{dw}{dt} + vp - (U + u)q = Z - g\cos(\theta_0 + \theta),$$
$$\frac{dh_1}{dt} - rh_2 + qh_3 = mL,$$
$$\frac{dh_2}{dt} - ph_3 + rh_1 = mM + hT_0,$$
$$\frac{dh_3}{dt} - qh_1 + ph_2 = mN,$$

where

$$h_1 = pK_A^2 m - qF - rE,$$
$$h_2 = qK_B^2 m - rD - pF,$$
$$h_3 = rK_C^2 m - pE - qD.$$

But the products of inertia (relative to moving axes fixed in the body) $D=F=0$, because the aeroplane is symmetrical about the xz plane. Substituting the above expressions for h_1, h_2, h_3, in the equations of motion, and neglecting products of small quantities, we have:

$$\frac{du}{dt}=X+T_0+g\sin(\theta_0+\theta),\quad K_2\frac{dp}{dt}-\frac{E}{m}\frac{dr}{dt}=L,$$

$$\frac{dv}{dt}+Ur=Y+g\sin\psi\sin(\theta_0+\theta)-g\sin\phi\cos(\theta_0+\theta).$$

$$K_B^2\frac{dq}{dt}=M+hT_0.$$

$$\frac{dw}{dt}-Uq=Z-g\cos(\theta_0+\theta).\quad K_C^2\frac{dr}{dt}-\frac{E}{m}\frac{dp}{dt}=N.$$

If we substitute for X, Y, etc., their values from the expansion in terms of the first powers of u, v, w, etc., and observing that from the conditions of equilibrium,

$$M_0+T_0h=T_0+X_0+g\sin\theta_0=Z_0-g\cos\theta_0=0,$$

we will have, making $\sin\phi=\phi$, $\sin\psi=\psi$, $\sin\theta=\theta$, and $\cos\theta=1$.

$$\frac{du}{dt}=uX_u+wX_w+qX_q+g\theta\cos\theta_0,$$

$$\frac{dw}{dt}=qU+uZ_u+wZ_w+qZ_q+g\theta\,sin\,\theta_0,$$

$$\frac{dv}{dt}=-rU+vY_v+pY_p+rY_r+g\psi\sin\theta_0-g\phi\cos\theta_0.$$

$$K_B^2\frac{dq}{dt}=uM_u+wM_w+qM_q,$$

$$K_A^2\frac{dp}{dt}-\frac{E}{M}\frac{dr}{dt}=vL_v+pL_p+rL_r,$$

$$K_C^2\frac{dr}{dt}-\frac{E}{M}\frac{dp}{dt}=vN_v+pN_p+rN_r.$$

We here assume T_0 a constant, or that there is no change of propeller thrust with small change in forward speed. With a motor in "free route," if the machine speeds up, the propeller tends to race or to speed up so that the slip shall be about constant, and hence the thrust is not materially changed. Since the forward speed $(U\pm u)$ is approximately equal to U, the thrust is approximately constant and equal to T_0.

We have also assumed that T_0 lies parallel to the axis of x. At very slow speed this is not exactly the case and T_0 has a small vertical component assisting in sustaining the weight of the aeroplane. At high speeds, T_0 is, however, usually parallel to x and the assumption

that it always is so parallel is here made for simplicity. In any case T_0 is eliminated by the conditions of equilibrium.

In the present investigation the normal flight path is assumed horizontal, or $\theta_0=0$. The product of inertia E is small for ordinary aeroplanes with the heavy weights fairly symmetrical above and below the axis of x. In view of the probable insignificance of E and the fact that E cannot easily be determined for an aeroplane by simple experiments, it is here neglected. In the simplified form the equations of motion then are:

$$\frac{du}{dt}=uX_u+wX_w+qX_q+g\theta, \tag{1a}$$

$$\frac{dw}{dt}=qU+uZ_u+wZ_w+qZ_q, \tag{1a}$$

$$\frac{dv}{dt}=-g\phi-rU+vY_v+pY_p+rY_r. \tag{1b}$$

$$K_A^2\frac{dp}{dt}=vL_v+pL_p+rL_r. \tag{1b}$$

$$K_B^2\frac{dq}{dt}=uM_u+wM_w+qM_q, \tag{1a}$$

$$K_C^2\frac{dr}{dt}=vN_v+pN_p+rN_r. \tag{1b}$$

It is seen that equations (1a) involve only the longitudinal motion or motion in the plane of symmetry xz of the aeroplane, since p, r, v, and ϕ do not appear. Likewise, equations (1b) involve only the asymmetrical motion, lateral and directional, and do not contain θ, u, w, and q. The two sets may then be considered separately, the former on integration giving the "symmetrical motion" and the latter the "asymmetrical motion."

Since $\dfrac{d\theta}{dt}=q$, equations (1a) may be written in terms of three variables u, w, and θ and their first derivatives. The "resistance derivatives" X_u, X_v, X_q, etc., are constant coefficients. The three variables are each functions of the time, and the three equations at any instant of time must be satisfied by a concordant set of values of u, w, and θ. The equations are, therefore, simultaneous and are linear differential equations with constant coefficients.

Writing the operator D to indicate differentiation with regard to time or $\dfrac{d}{dt}$,

$$\left.\begin{aligned}(D-X_u)u-X_w w-(X_qD+g)\theta=0,\\ -Z_u u+(D-Z_w)w-(Z_q+U)D\theta=0,\\ -M_u u-M_w w+(K_B^2D^2-M_qD)\theta=0.\end{aligned}\right\} \tag{2a}$$

The right-hand members of these equations are no longer zero if any wind gusts are assumed.[1] The complementary function may be found by the well-known "operational method" by algebraic solution for D. (See: Wilson's "Advanced Calculus," p. 223.)

The physical condition that the three equations shall be simultaneous is expressed mathematically by equating to zero the determinant Δ formed by the coefficients of the variables u, w, and θ. Thus:

$$\Delta = \begin{vmatrix} D+X_u, & -X_w, & -(X_qD+q) \\ -Z_u, & D-Z_w, & -(Z_q+U)D \\ -M_u, & -M_w, & (K_B^2D^2-M_qD) \end{vmatrix} = 0.$$

Expanding the determinant we obtain:

$$A_1D^4+B_1D^3+C_1D^2+D_1D+E_1=0,$$

where for abbreviation:

$$A_1=K_B^2,$$

$$B_1=-(M_q+X_uK_B^2+Z_wK_B^2),$$

$$C_1=\begin{vmatrix} Z_w, & U+Z_q \\ M_w, & M_q \end{vmatrix} + \begin{vmatrix} X_u, & X_q \\ M_w, & M_q \end{vmatrix} + K_B^2 \begin{vmatrix} X_u, & X_w \\ Z_u, & Z_w \end{vmatrix}$$

$$D_1=-\begin{vmatrix} X_u, & X_w, & X_q \\ Z_u, & Z_w, & U+Z_q \\ M_u, & M_w, & M_q \end{vmatrix} - g\begin{vmatrix} M_u, & (-)\sin\theta_0 \\ M_w, & \cos\theta_0 \end{vmatrix}.$$

$$E_1=-g\begin{vmatrix} X_u, & X_w, & \cos\theta_0 \\ Z_u, & Z_w, & \sin\theta_0 \\ M_u, & M_w, & 0 \end{vmatrix}.$$

The solution of the biquadratic Δ for D is of the form:

$$D=a,\ b,\ c, \text{ or } d,$$

$$\theta=K_1e^{at}+K_2e^{bt}+K_3e^{ct}+K_4e^{dt},$$

where K_1, K_2, K_3, K_4, K_5, ... K_{12} are constants determined by initial conditions. Solutions for u and w are similar.

The condition for stability of motion is that θ, u, and w shall diminish as time goes on. Hence, each of the roots of the biquadratic must be negative if real, or, if imaginary, must have its real part negative. This condition for stability may be applied without finding the constants K_1 to K_{12}, by solving only the biquadratic for a, b, c, d. Indeed, Bryan has shown that by use of Routh's discriminant the biquadratic need not be solved. The condition that a biquadratic equation have negative real roots or imaginary roots with real parts negative, is that A_1, B_1, C_1, D_1, E_1 and $B_1C_1D_1-A_1D_1^2-B_1^2E_1$ be each positive.

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[1] Loc. cit., p. 1, §1, footnote 3.

In a similar manner the equations (1b) defining the asymmetric motion may be expressed as linear differential equations with constant coefficients.

Substitute $D^2\phi$ for $\frac{dp}{dt}$ and $D\phi$ for p.[1] Then:

$$(D-Y_v)v+(U-Y_r)r+(g-Y_pD)\phi=0,$$
$$-L_vv-L_rr+(K_A^2D^2-L_pD)\phi=0,$$
$$-N_vv+(K_C^2D-N_r)r-N_pD\phi=0,$$
$$\Delta_2=A_2D^4+B_2D^3+C_2D^2+D_2D+E_2=0,$$

where:

$$A_2=K_C^2K_A^2,$$
$$B_2=-Y_vK_C^2K_A^2-K_C^2L_p-N_rK_A^2,$$
$$C_2=-L_rN_p+N_rL_p+K_C^2L_pY_v+N_rY_vK_A^2+N_vUK_A^2$$
$$-(L_vY_pK_C^2+N_vY_rK_A^2),$$
$$D_2=Y_v(L_rN_p-N_rL_p)+L_v(UN_p+gK_C^2)-UL_pN_v$$
$$+(N_vY_rL_p-L_rY_vN_p+L_vY_pN_r-N_vY_pL_r),$$
$$E_2=g(N_vL_r-L_vN_r).$$

As before, the condition for stability is that the real roots and real parts of imaginary roots of the biquadratic be negative.

§10. CONVERSION TO MOVING AXES, LONGITUDINAL DATA

Horizontal flight at 0° incidence i of wing chord requires a speed of 112.5 feet per second, or about 77 miles per hour (see the characteristic performance curves). The normal attitude then has the axis of x parallel to the wing chord and horizontal. The axis z is vertical. For slow speed with an angle of incidence i of 12°, a speed of 54 feet per second, or about 37 miles per hour, must be maintained. In this case, the normal attitude has the axes x horizontal and z vertical, but the axes are entirely different from those used for the high-speed condition if they are considered with reference to the aeroplane. The axis of y is, however, the same in both cases.

---

[1] Since we consider only the small oscillations, ϕ and ψ are of the nature of infinitesimals, and hence compound vectorially as do p and r. Professor E. B. Wilson suggests the important simplification of the treatment given by Bryan or Bairstow due to making $\frac{d\phi}{dt}=p$ and $\frac{d\psi}{dt}=r$. They used angular coordinates giving expressions for $\frac{d\phi}{dt}$ and $\frac{d\psi}{dt}$ in terms of p and r and the angles which are initially cumbersome but ultimately reduce to the simple form here given.

The aeroplane may pitch about its normal attitude. At any instant the angle of pitch is the angle θ between the normal attitude axis of x and the new position of x. The axes, of course, pitch with the aeroplane. The axes are fixed by the equilibrium conditions and differ for each speed since each speed requires a different attitude.

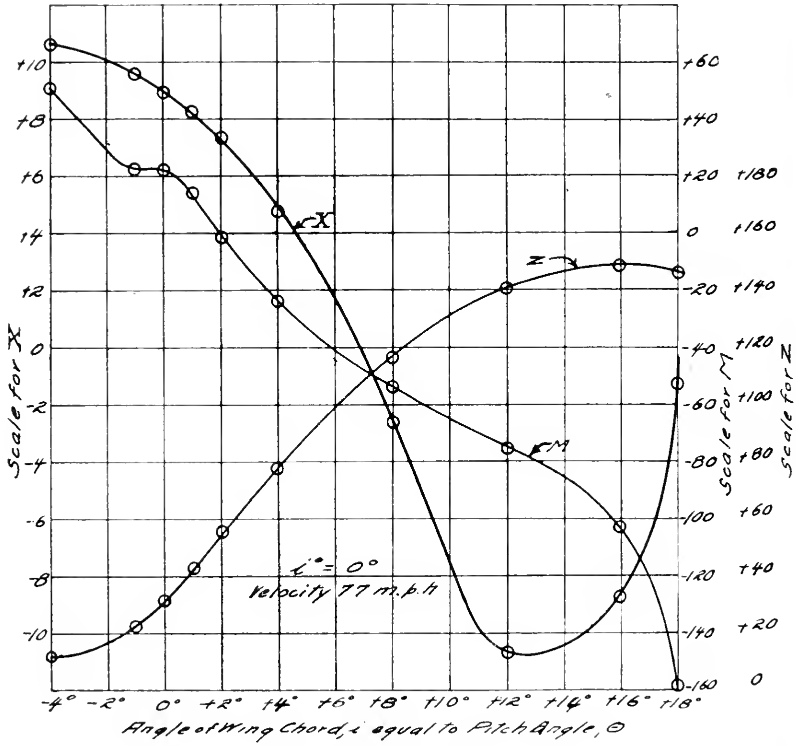

FIG. 8.—X, Z, and M for $i = 0°$.

It was convenient to measure in the wind tunnel the lift and drift on the model referred to axes always vertical and horizontal. The corresponding forces along the moving axes x and z are readily obtained from:

$$Z' = L \cos \theta + D \sin \theta,$$
$$X' = D \cos \theta - L \sin \theta.$$

Here L and D are pounds on model, θ is angle of pitch, and Z' and X' are pounds force along the moving axes. X' and Z' are then con-

verted to full-speed full scale as usual and divided by the mass m in slugs to obtain X and Z in pounds per unit mass on the full-size aeroplane at the proper speed.

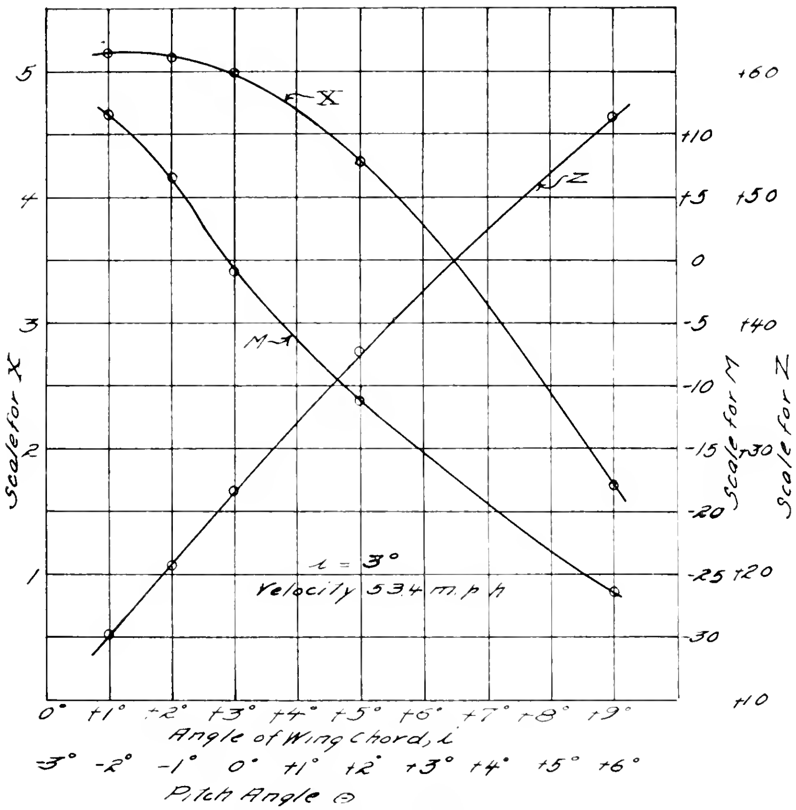

FIG. 9.—X, Z, and M for $i = 3°$.

The pitching moment full size is obtained from the observed model pitching moment about the center of gravity by an obvious manipulation. The moment is expressed in pounds-feet per unit mass and lettered M.

For this aeroplane we have, for example,

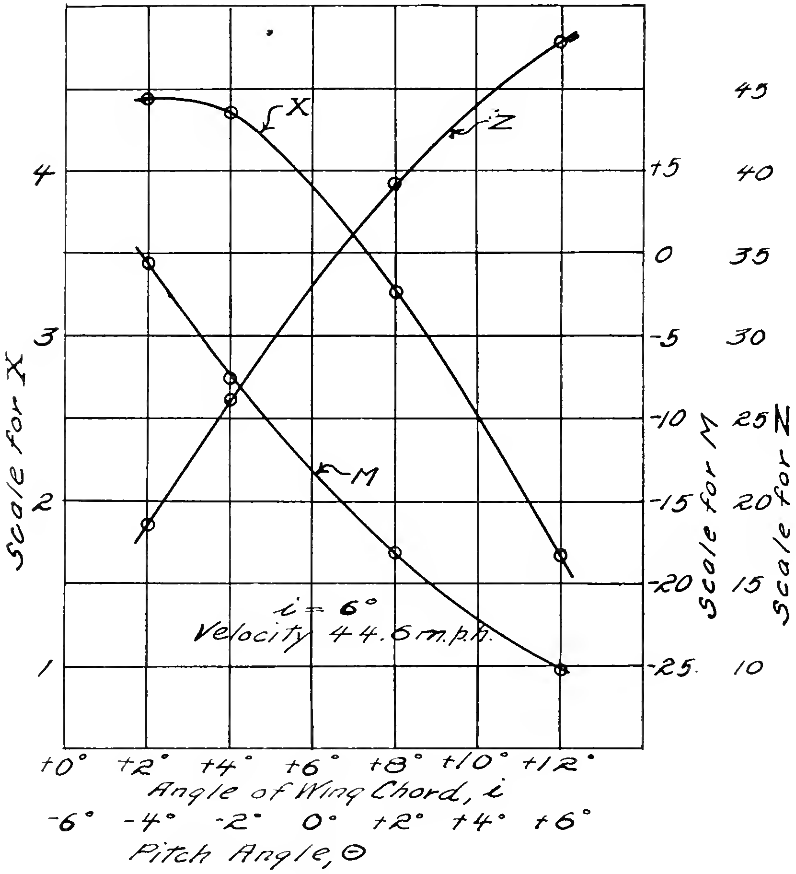

FIG. 10.—X, Z, and M for $i = 6°$.

For this aeroplane we have, for example:

$m = 50$ slugs, $\theta_0 = 0$,

$U = -112.5$ foot-seconds (high speed, 76.9 miles per hour),

$i = 0$, normal attitude.

$i°$	$\theta°$	X	Z	M
−4	−4	+10.66	+ 11.00	+ 50.2
−1	−1	+ 9.62	+ 21.18	+ 23.2
0	0	+ 8.98	+ 32.06	+ 23.0
+1	+1	+ 8.28	+ 43.74	+ 13.5
2	2	+ 7.38	+ 56.00	− 1.93
4	4	+ 4.80	+ 78.26	− 23.2
8	8	− 2.58	117.00	− 54.0
12	12	−10.68	140.6	− 75.3
16	16	− 8.72	149.00	−102.3
18	18	− 1.25	147.00	−158.2

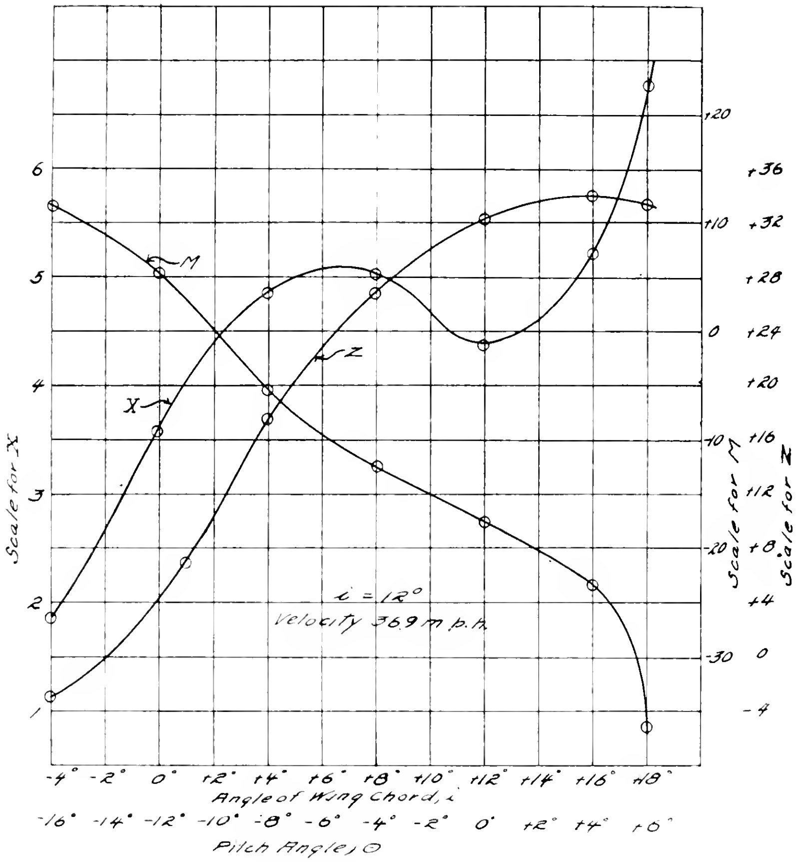

FIG. 11.—X, Z, and M for $i = 12°$.

$U = -54$ foot-seconds (low speed, 36.9 miles per hour), $\theta_0 = 0$, $i = 12°$, normal attitude.

$i°$	$\theta°$	X	Z	M
−4	−16	+1.87	− 3.0	+11.6
0	−12	+3.58	+ 6.8	+ 5.30
+4	− 8	+4.84	+17.4	− 5.32
8	− 4	+5.02	+26.6	−12.5
12	0	+4.38	+32.3	−17.4
16	4	+5.24	+34.2	−23.6
18	6	+6.78	+33.4	−36.5

When $\theta = 0$, note that Z_0 should equal 32.2 or g, a check on the table.

Curves of X, Z, M for the four speed conditions are given on figures 8, 9, 10, and 11. These curves are not "faired," but drawn

through the experimental points to show the consistency of the measurements and calculations.

§11. RESISTANCE DERIVATIVES, LONGITUDINAL

The longitudinal oscillations of the aeroplane are given by three equations of motion of §9, in which certain "resistance derivatives" are required.

The quantity X_u is the rate of change of X with change of forward speed u. Since X varies as the square of the speed, $X_0 = CU^2$ where C is some constant.

Then $\frac{\partial X}{\partial U} = 2CU = \frac{2X_0}{U} = X_u$ and $Z_u = \frac{2Z_0}{U} = \frac{2g}{U}$, so that these coefficients are readily calculated.

The derivatives X_w, Z_w, M_w represent the effect of a vertical component of velocity w. The vertical component of velocity w acts with the horizontal velocity U to cause the resultant wind to have an inclination to the horizontal

$$\Delta\theta = \tan^{-1}\frac{w}{U} = 57.3\,\frac{w}{U},$$

when $\Delta\theta$ is a small angle measured in degrees.

Hence

$$X_w = \frac{\Delta X}{w} = \frac{57.3}{U}\cdot\frac{\Delta X}{\Delta\theta}.$$

$$Z_w = \frac{57.3}{U}\cdot\frac{\Delta Z}{\Delta\theta}.$$

$$M_w = \frac{57.3}{U}\cdot\frac{\Delta M}{\Delta\theta}.$$

The method practically substitutes the slopes $\frac{\Delta X}{\Delta\theta}$, $\frac{\Delta Z}{\Delta\theta}$, $\frac{\Delta M}{\Delta\theta}$ of the tangents to curves of X, Z, M, at $\theta = 0$, for the actual curves. We have assumed $\Delta\theta$ small. If a curve be nearly a straight line, we may substitute the tangent for the curve without great error. Thus it may not always be necessary to assume $\Delta\theta$ *very* small. In fact, a range of from 5° to 8° is tolerable.

Since we assume $M_0 = 0$, the balance should be undisturbed by change of forward speed. Therefore, $M_u = 0$ in all cases.

Note that a positive value of M_w corresponds to a curve of pitching moments giving statical stability or a righting moment. If M_w is positive it does not necessarily follow that the aeroplane will be dynamically stable, but if M_w is negative, instability is of course certain. X_u should be negative to indicate increased resistance for increase of forward speed $-u$. For stability, Z_w should be large and negative, indicating increase lift for larger angles of incidence and *vice versa*. At stalling angles, Z_w tends to approach zero.

§12. DAMPING

The derivative M_q is the rate of change of pitching moment due to angular velocity, or rapidity of pitching q. For a pitch of velocity $\frac{d\theta}{dt}=q$, there is a moment of qM_q tending to resist such pitching. This is the damping due to the horizontal stabilizer, elevator flaps, body, and all parts forward and aft of the center of gravity. The pitching takes place about the center of gravity. The damping is increased by a large tail and a long body.

The damping of a surface should depend on the area of the surface, the moment arm of that surface, the linear velocity with which it swings through the air (which varies also as the moment arm), and with the velocity of advance. Thus: $qM_q \sim ql^4U$, where l is a linear dimension.

If we can measure M_q for the model at any wind speed, we may convert it to M_q for the full-scale aeroplane at its proper speed by multiplying by the fourth power of the scale and the ratio of aeroplane speed to wind speed. Naturally this is an approximate method, but it is the best available since full-scale tests for M_q are not practicable.

Similarly N_r and L_p may be obtained from model tests. These refer to the damping of a yaw and a roll respectively.

In order to measure M_q, N_r, and L_p a special oscillator was designed, shown in the photograph in figure 12. By setting the apparatus to oscillate in pitch, roll, or yaw the corresponding damping coefficients can be computed from the observed decrement. The photograph (pl. 1) shows the apparatus with model as used for pitching oscillations.

Let:

I = moment of inertia of all oscillating parts in slug foot units,
m' = mass of all oscillating parts in slugs,
M_0 = moment of air forces on model at rest,
M_s = moment of springs at rest,
$K\theta$ = additional moment of springs when deflected,
c = center of gravity of entire apparatus above pivot, feet,
θ = angle of pitch from normal attitude in radians,
$\mu_0 \frac{d\theta}{dt}$ = damping moment due to friction,
$\mu_w \frac{d\theta}{dt}$ = damping moment due to wind on apparatus,
$\mu_m \frac{d\theta}{dt}$ = damping moment due to wind on model,
$cm'\theta$ = static moment due to gravity.

The equation of motion then is:

$$I\frac{d^2\theta}{dt^2}+(\mu_0+\mu_w+\mu_m)\frac{d\theta}{dt}+(K-cm')\theta+M_0-M_s=0.$$

But $M_0=M_s$ by the initial condition of equilibrium. Let

$$\mu=\mu_0+\mu_w+\mu_m;$$

then

$$I\frac{d^2\theta}{dt^2}+\mu\frac{d\theta}{dt}+(K-cm')\theta=0.$$

The solution of this equation is well known to be:

$$\theta=Ce^{\frac{\mu t}{2I}}\cos\left\{t\sqrt{(K-cm')\frac{1}{I}-\frac{\mu^2}{4I^2}}+a\right\},$$

where C and a are arbitrary constants. If time be counted when the amplitude of swing is a maximum, then $\cos\{\ \}=1$, and $\theta=\theta_0$, the initial displacement. Also if the number of beats be counted by observing the times for succeeding maxima, a plot of amplitude on time will have for its equation the simple form:

$$\theta=\theta_0 e^{-\frac{\mu t}{2I}}.$$

The coefficient μ is the logarithmic decrement of the oscillation and must be numerically positive to insure that the oscillation dies out with time.

The apparatus was fitted with a small reflecting prism by which a pencil of light was deflected toward a ground glass plate set in the roof of the tunnel. Nine lines spaced 0.2 inch were ruled on this plate. With the model at rest the beam of light was brought to a sharp focus on the line marked zero. By means of a trigger the observer started an oscillation of the model, and the spot of light was observed to oscillate across the scale. The time t was observed in which an oscillation was damped from an amplitude of 9 to an amplitude of 1, for example.

Then: $\log_e\frac{\theta_0}{\theta}=\frac{\mu}{2I}t=\log_e 9$, and knowing I and t, μ is calculated.

Preliminary tests showed that the same value of μ was obtained whether the timing stopped at $\theta=5$, 4, 3, 2, or 1.

Oscillation tests were made at five wind velocities varying from 5 to 35 miles per hour. The coefficient μ appeared to vary approximately as the first power of the velocity.

Similar tests were made with the model for no wind to determine μ_0, which may be said to be due almost wholly to friction and very slightly to the damping of apparatus and model moving through the air.

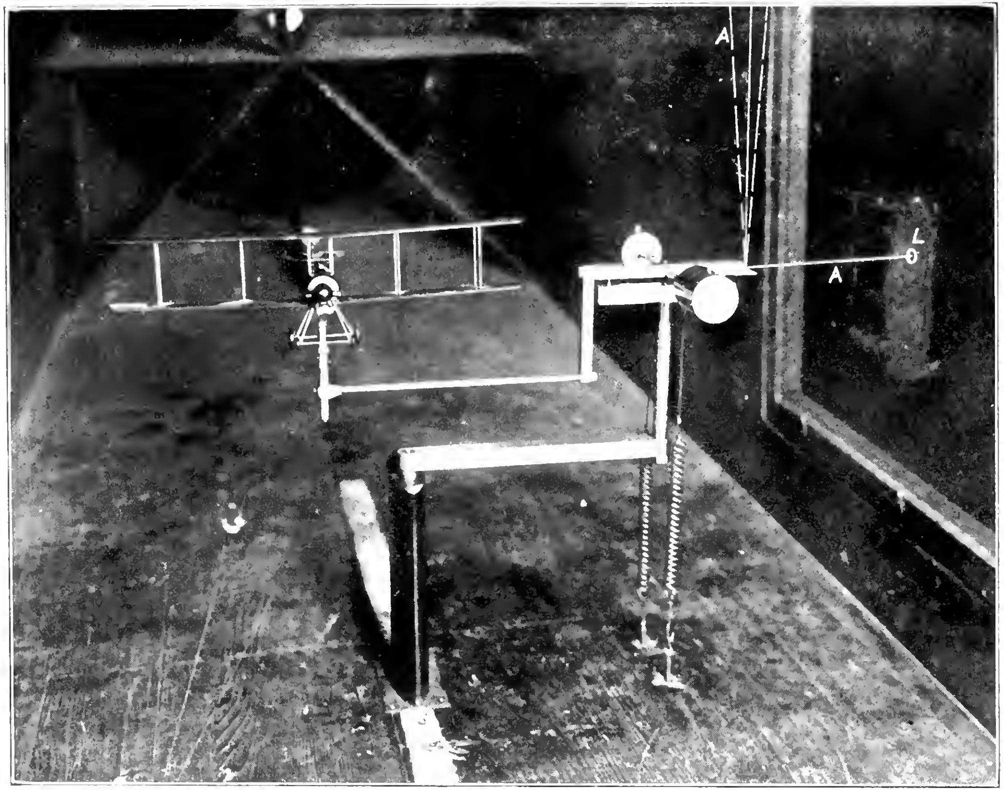

FIG. 12.—MODEL IN POSITION FOR PITCHING OSCILLATIONS ABOUT CENTER OF GRAVITY. L SPECTACLE LENS A, PENCIL OF LIGHT DEFLECTED TO SCALE ON ROOF

Likewise μ_w was obtained by oscillating the apparatus without model in winds from 5 to 35 miles per hour.

The coefficient μ_m has the dimensions $\rho l^4 V$, where ρ is density of air, l a linear dimension, and V the velocity of the wind. To convert μ_m to M_q for the full-size machine at full speed, multiply by the fourth power of 24, the scale, and by the ratio of full speed to model speed.

The model is mounted in such a manner that the axis of oscillation through the two steel pivot points passes through the assumed center of gravity location for the aeroplane. The actual center of gravity of the model is not considered.

Transverse arms carry counter weights by which the natural period may be adjusted. The springs insure that the motion shall be oscillatory. Knife-edged shackles bearing in notches in the transverse arms carry the pull of the springs. The springs are not calibrated as the calculation eliminates the spring coefficient.

Friction is kept small by careful design. All pivots are glass-hard tool-steel points bearing inside polished conical depressions of tool steel. A convenient period for observation is ½ second. In still air, the apparatus will oscillate over 300 times before the amplitude is diminished to ⅑ the initial displacement. The latter is about 3°.

Numerical results for the pitching oscillation follow:

§13. OSCILLATIONS IN PITCH

Inertia, model and apparatus = .03945
Inertia, apparatus = .03680

APPARATUS

Wind velocity, miles per hour	30	20	0
t, seconds	94.0	96.2	105
μ	.00172	.00168	.00154
μ_w (less zero)	.00018	.00014	0

APPARATUS AND MODEL, INCIDENCE OF WING, 0°

Velocity, miles per hour	35	30	24	18	8
t, seconds	15.5	17.5	21.0	26.5	50
μ	.0112	.00994	.00828	.00656	.00348
μ_0	.0015	.00154	.00154	.00154	.00154
μ_w	.0002	.0002	.00016	.00012	.00005
μ_m (net)	.00950	.00820	.00658	.00490	.00189

APPARATUS AND MODEL, INCIDENCE OF WING, 6°

Velocity	30	24
t	20	24
μ	.00870	.00725
μ_0	.00160	.00156
μ_w	.0002	.0002
μ_m (net)	.00690	.00550

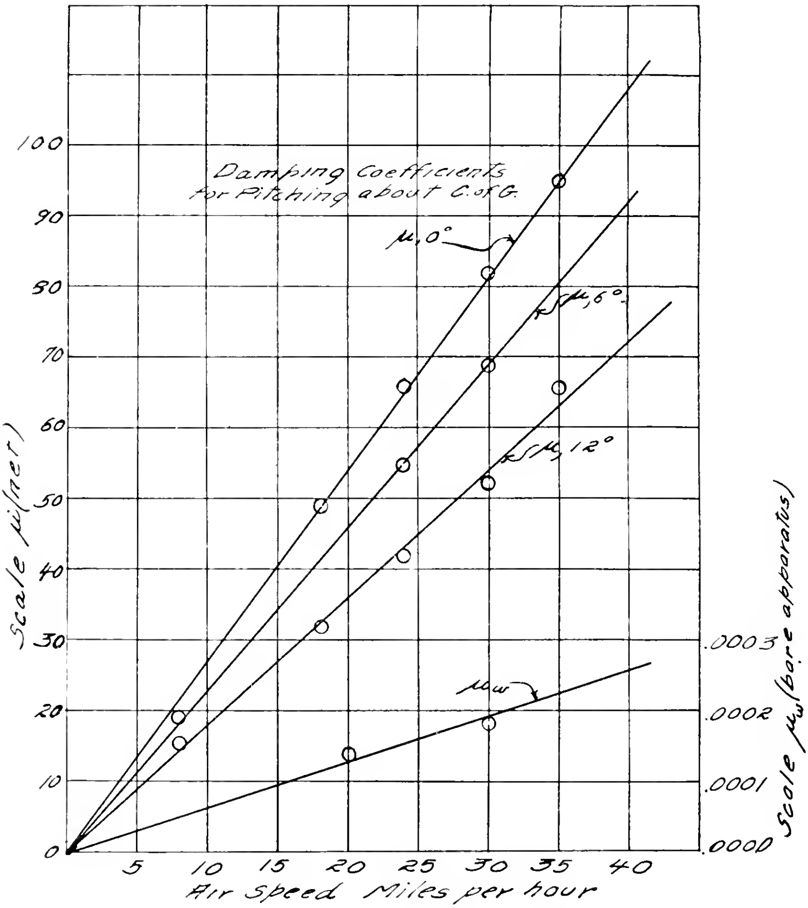

FIG. 13.—Curves for μ (net) and μ_w for oscillations in pitch.

Apparatus and Model, Incidence of Wing, 12°

Velocity	35	30	24	18	8	0
t	23.5	25.0	29.0	35.5	55.5	112
μ	.0074	.00696	.0060	.0049	.00314	.00156
μ_0	.0016	.00156	.0016	.0016	.00156	.00156
μ_w	.0002	.0002	.0002	.0001	.00005	.00000
μ_m (net)	.0066	.0052	.0042	.0032	.00153	.00000

Values computed as above for μ_m, net, for the three cases are plotted in figure 13. The points appear to lie along straight lines in justification of the assumption that the damping coefficient varies as the first power of the velocity of flight. To convert to full-speed full-scale, we use the formula,

$$M_q = \frac{\mu_m}{m} (26)^4 \left[\frac{\text{Velocity aeroplane}}{\text{Velocity model}}\right],$$

$$\begin{aligned} \text{for } i=0°, \quad & M_q = (-)192.0 = 1.71U, \\ i=6°, \quad & M_q = (-)\ 93.7 = 1.43U, \\ i=12°, \quad & M_q = (-)\ 60.5 = 1.12U. \end{aligned}$$

The marked decrease in damping at slow speed must impair stability. For the Curtiss Tractor JN2, with a somewhat shorter tail, we found $M_q = 1.32U$ at $i=2°$, and $M_q = 1.66U$ at $i = 15°.5$. Bairstow found for the Blériot, $M_q = 1.84U$ at $i=6°$. We should expect greater damping to be shown there, since the horizontal tail surface is very large.

§14. LONGITUDINAL STABILITY, DYNAMICAL

We have now determined the resistance derivatives needed for the three equations of the longitudinal motion in the plane of symmetry with the exception of X_q and Z_q. From a consideration of various terms in the *criteria* for stability it is concluded that both X_q and Z_q enter into products which are small and relatively unimportant. They are consequently neglected.

The biquadratic has been calculated, following the formulæ given above, for several speeds and attitudes of flight. The results are summarized in the following table. The curves of figures 8, 9, 10, and 11 were used to obtain the resistance derivatives.

V miles per hour	76.9	53.4	44.6	36.9
U feet per sec..	− 112.5	− 78.2	− 65.3	− 54.0
Normal incidence	0°	3°	6°	12°
X_u	− .158	− .12	− .1194	− .162
X_w	+ .356	+ .249	+ .245	0
Z_u	− .57	− .823	− .985	− 1.19
Z_w	− 5.62	− 3.77	− 2.92	− 1.0
M_w	+ 3.2	+ 3.99	+ 2.25	+ 1.41
M_q	− 192.0	− 123.0 [1]	− 93.7	− 60.5
A_1	21.6	21.6	21.6	21.6
B_1	317.0	207.0	159.3	85.1
C_1	1492.0	804.0	444.0	150.0
D_1	266.0	128.3	72.6	22.1
E_1	59.2	106.0	71.4	54.0
Routh's discr. ..	$+117\times10^6$	16.4×10^6	3.2×10^6	$-.12\times10^6$
m	50	50	50	50
Long period, sec.	34.7	17.6	15.8	10.56
Time to damp, 50%	8.1	11.0	13.1	....
Time to double.	....	....	....	24.7
Character......	Stable	Stable	Stable	Unstable

The coefficients of the biquadratic computed from the formulæ of §9 give for high speed

$$21.62D^4+317.0D^3+1492.0D^2+266.0D+59.2=0.$$

Each coefficient is positive and Routh's discriminant

$$B_1C_1D_1-A_1D_1^2-B_1^2E_1$$

is also positive and equal to 117×10^6. The motion is, therefore, stable. The aeroplane if set pitching will return in time to its normal attitude.

Bairstow has shown that, considering the usual values of the coefficients of the biquadratic, it may be factored approximately, giving:

$$\left(D^2+\frac{B_1}{A_1}D+\frac{C_1}{A_1}\right)\left(D^2+\left[\frac{D_1}{C_1}-\frac{B_1E_1}{C_1^2}\right]D+\frac{E_1}{C_1}\right)=0.$$

The first factor reduces to:

$$D^2+14.75D+69.0=0,$$

$$D=-7.38\pm3.83i \text{ where } i=\sqrt{-1}.$$

This is the well-known condition for a simple damped oscillation of period,

$$p=\frac{2\pi}{3.83}=1.64 \text{ seconds,}$$

[1] By interpolation.

and damped to one-half amplitude in time,

$$t=\frac{0.69}{7.38}=0.094 \text{ second.}$$

For most aeroplanes, this first factor corresponds to a short oscillation so heavily damped that it is of no importance. Indeed, it could not be observed on the actual aeroplane in flight.

The second factor, similarly, reduces to:

$$D^2+.17D+.04=0,$$

$$D=-.085\pm.181i,$$

$$p=\frac{2\pi}{.181}=34.7 \text{ seconds,}$$

$$t=\frac{0.69}{.085}=8.1 \text{ seconds.}$$

This is a longer oscillation but heavily damped. The period of 34.7 seconds for the motion is great, and at high speed this aeroplane if left to itself after an accidental longitudinal disturbance should follow an undulating path with rising and sinking of the center of gravity, together with pitching and periodic changing of forward speed. There is an oscillation in u, w, and θ. In 34.7 seconds, the aeroplane runs 3900 feet, which is the distance from crest to crest of the flight path. In one period the amplitude of the undulation is almost completely damped. It is unlikely that this motion would be uncomfortable to the pilot even if the initial disturbance due to a gust or other cause were severe.

At high speed, this aeroplane is very stable compared with other machines which have been tested. The natural period of the Curtiss JN2 is about 34 seconds, damped 50 per cent in 11 seconds, according to calculations made by us. A Blériot monoplane model tested by Bairstow had a period of pitching of 25 seconds, damped 50 per cent in 15 seconds.

There is no other published data of this character. It appears that great statical stability or large M_w will give a stiff machine with a rapid period. Such a machine, though very stable, may be so violent in its motion as to lead the pilot to pronounce it unstable. The design tested here appears to have as easy a period as the Curtiss and Blériot, both considered very satisfactory in flight, together with greater damping.

High speed and a long tail tend to damp the pitching. What we aim to secure—namely, steadiness in flight—may better be obtained by large damping factors rather than by strong righting moments (statical stability). It is well known that the French monoplane pilots

demanded at one time a neutral aeroplane with no stability whatever against pitching, on the ground that "stable" aeroplanes were too violent in their motion in gusty air. Another disadvantage of excessive statical stability lies in the tendency of the machine to "take charge" and take a preferred attitude relative to the wind at a time when such a maneuver may embarrass the pilot, as when approaching a landing. However, it appears possible that a machine with the minimum of "statical" stability may be given the maximum of damping and so have a very slow period of pitching. The motion will be nearly dead beat.

This digression with regard to damping *vs.* "statical" stability applies with equal force to the rolling and yawing motions of the aeroplane to be considered under "lateral stability."

For low speed, 36.9 miles, similar calculations give for the longitudinal motion

$$21.6D^4+85.1D^3+149.8D^2+22.1D+54=0.$$

Routh's discriminant

$$B_1C_1D_1-A_1D_1^2-B_1^2E_1=-12\times10^4. \quad \text{Unstable.}$$

Short oscillation:

$$D^2+(B_1/A_1)D+C_1/A_1=D^2+3.9D+6.94=0,$$

$$D=-1.95\pm1.77i,$$

$$p=\frac{2\pi}{1.77}=3.58 \text{ seconds,}$$

$$t=\frac{0.69}{1.95}=.36 \text{ second to damp 50 per cent.} \quad \text{Stable.}$$

Long oscillation:

$$D^2+(D_1/C_1-B_1E_1/C_1^2)D+E_1/C_1=D^2-.056D+.36=0,$$

$$D=+.028\pm.594i,$$

$$p=\frac{2\pi}{.594}=10.56 \text{ seconds,}$$

$$t=\frac{0.69}{-.028}=-24.7 \text{ seconds,}$$

or +24.7 seconds will double the initial amplitude. Unstable.

At this speed Routh's discriminant is negative, indicating that the motion is unstable. The instability is seen to appear when the real parts of the roots corresponding to the long oscillation become positive. The motion is rapid: only 11 seconds' period compared with 35 seconds at high speed, and any initial displacement will double itself in two periods. The damping of the motion has vanished and although the increase of amplitude is not so rapid that there is danger

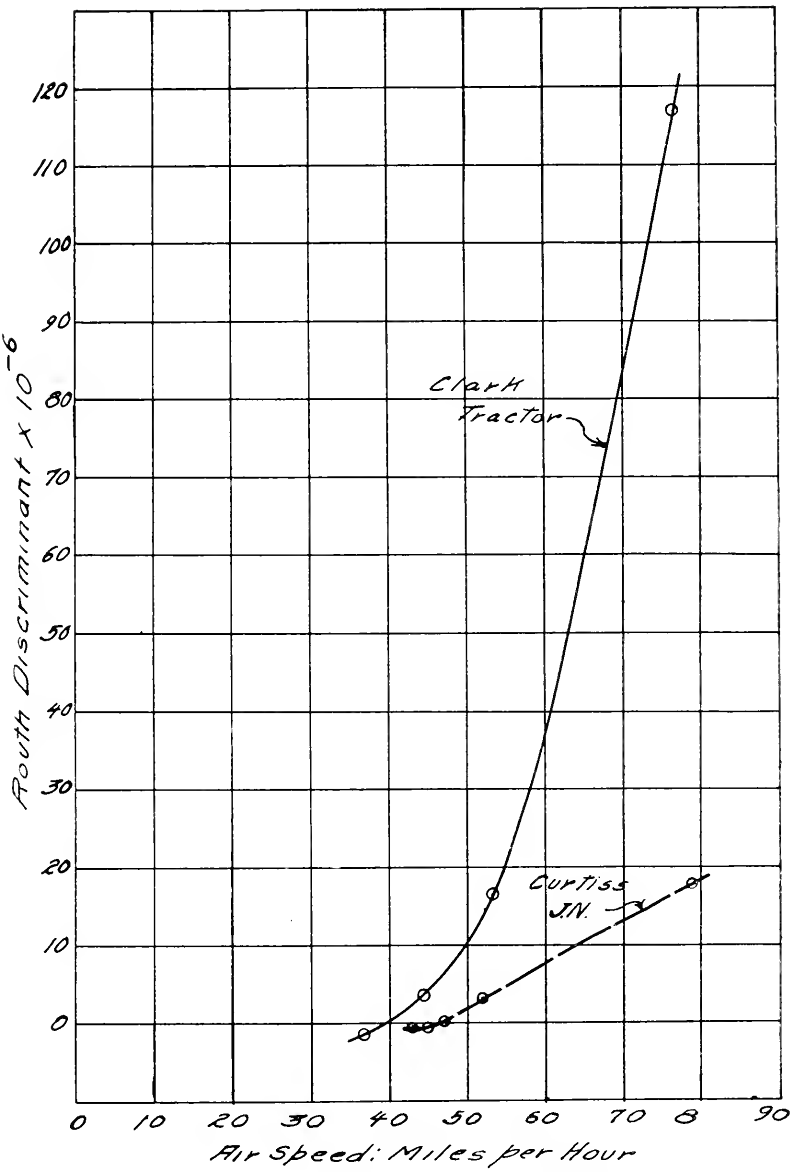

FIG. 14.—Routh's discriminant, variation with velocity.

of the pilot's losing control, yet it is clear that he cannot fly at this speed unless he is alert.

Taking Routh's discriminant as a measure of dynamical stability we have its value $+117 \times 10^6$ at high speed and -0.12×10^6 at low speed. Compared with the high-speed value, the latter is insignificant and we may conclude that the instability at low speeds is of relatively slight danger. Indeed, we may say that the aeroplane is stable at high speed and about neutral at low speed.

The progressive change in Routh's discriminant with speed is more clearly shown on figure 14. On the same plot, we give a similar curve for a Curtiss type tractor. The "critical velocity" for the Clark type is about 40 miles per hour and 47 miles per hour for the Curtiss type.

All aeroplanes of normal type are probably longitudinally stable at high speeds but lose this stability for all speeds below a certain critical speed where Routh's discriminant becomes zero or changes sign.

The examination of the longitudinal stability of the Blériot mentioned above applied only to high speed. The importance of investigating stability at low speeds has, it is believed, never before been shown.

The reason the stability of the longitudinal motion vanishes at a critical velocity must be found in the approximate factor representing the long oscillation.

$$D^2 + \left[\frac{D_1}{C_1} - \frac{B_1 E_1}{C_1^2} \right] D + \frac{E_1}{C_1} = 0.$$

Stability vanishes where $D_1/C_1 = E_1 B_1 / C_1^2$, or where $D_1 C_1 = E_1 B_1$. In other words, stability is reduced as $E_1 B_1$ is made large or $D_1 C_1$ small. At high speed we have $266 \times 1492 > 59.2 \times 317$, but at low speed $22.1 \times 149.8 < 54 \times 85.1$. It appears that B_1 is smaller at low speeds, which is desired, but D_1 and C_1 are reduced to a greater degree, which is not desired.

The cause of the reduction in the magnitude of D_1 from 266 to 22.1 can be shown in the effect of change in resistance derivatives in:

$$D_1 = - \begin{vmatrix} X_u, & X_w, & X_q \\ Z_u, & Z_w, & U + Z_q \\ M_u, & M_w, & M_q \end{vmatrix} - g \begin{vmatrix} M_u, & \sin\theta_0 \\ M_w, & \cos\theta_0 \end{vmatrix}.$$

For $\theta_0 = 0$, $X_q = Z_q = M_u = 0$, we have

$$D_1 = -X_u Z_w M_q + X_u U M_w + Z_u X_w M_q.$$

The first term is reduced at low speed because Z_w is less than $\frac{1}{5}$ and M_q $\frac{1}{3}$ of their values at high speed. Since U and M_w are smaller, the

second term is but $\frac{1}{6}$ of its high-speed value. The third term is unimportant.

From

$$C_1 = \begin{vmatrix} Z_w, & U \\ M_w, & M_q \end{vmatrix} + X_u M_q + K_B^2 (X_u Z_w - X_w Z_u)$$

we see by inspection that the principal reduction in C_1 at low speed is due to smaller values U, M_w, Z_w, and M_q which greatly reduce the terms $Z_w M_q$ and UM_w. These two terms are the principal numerical ones in the expression for C_1.

In general, $E_1 = -gZ_u M_w$ will increase in value due to increase in Z_u and M_w, but the effect on the motion is not great. On the other hand, $B_1 = -M_q - K_B^2(X_u + Z_w)$ will drop rapidly for large angles of incidence due to drop in M_q and in Z_w. This is favorable to stability.

It is seen that the quantities U, Z_w, and M_q preponderate in the numerical values of the coefficients D_1C_1 and E_1B_1. For ordinary speeds, or speeds above the speed of minimum power, we have, approximately,

$$D_1 = -X_u(Z_w M_q - UM_w) + Z_u X_w M_q = -X_u(Z_w M_q - UM_w),$$
$$C_1 = (Z_w M_q - UM_w) + X_u M_q + K_B^2(X_u Z_w - X_w Z_u) = (Z_w M_q - UM_w),$$
$$B_1 = -M_q - K_B^2(X_u + Z_w) = -M_q - K_B^2 Z_w,$$
$$E_1 = -gZ_u M_w.$$

The condition for damped motion then becomes:

$$D_1C_1 > E_1B_1 \text{ or } (Z_w M_q - UM_w)^2 > -\frac{gZ_u}{X_u} M_w (M_q + K_B^2 Z_w).$$

where $\dfrac{Z_u}{X_u} = \dfrac{Z_0}{X_0}$ and M_w are nearly constant. Damping of the long oscillation is then favored by large values of Z_w, M_q, and U. That is, by light wing loading, large damping surfaces, and high velocity. As speed is reduced these quantities become smaller and the oscillation is less strongly damped.

For very low speeds, including those below the speed for minimum power, the value of Z_w nearly vanishes and M_q becomes small. Here the approximate expressions would be written,

$$D_1 = X_u UM_w + Z_u X_w M_q,$$
$$C_1 = -UM_w,$$
$$B_1 = -M_q - K_B^2(X_u + Z_w),$$
$$E_1 = -gZ_u M_w,$$

and

$$-\left(\frac{X_0}{Z_0} M_w U + M_q X_w\right) > \frac{g}{U} \left(M_q + K_B^2(X_u + Z_w)\right).$$

For very low speeds, the quantity X_w is often found to change sign; therefore, the two terms on the left may be of opposite sign and a large value for M_q diminishes D_1C_1 and increases B_1E_1. In a "stalling" attitude the aeroplane should have M_q small, M_w large, and, if possible, the radius of gyration in pitch K_B small.

The attitude of a "stalled" aeroplane is not ordinarily considered a "normal" attitude of flight, but, unfortunately, an aeroplane is frequently "stalled" by an inexperienced pilot. The longitudinal motion of an aeroplane if held in a "stall" would be, in general, unstable, but under favorable circumstances with Z_q, Z_w, K_B small and M_w large it is possible to have a stable motion. For example, in an extreme case with Z_w zero, if the aeroplane head up higher due to large X_w it slows down, loses lift and sinks. In sinking, M_w, if large, will head the machine down, speed will be gained on the dive and the resultant gain in lift causes the aeroplane to rise again. The oscillation will not increase in amplitude with time if the machine is able to respond quickly to the righting moment M_w. The damping M_q and radius of gyration K_B must not be too large. If M_q and K_B are too large, the machine is dynamically unstable by having $D_1C_1 < E_1B_1$.

The question of safe flight at a stalling attitude is complicated by the fact that the lateral controls become ineffective, but by manipulation of the power delivered by the motor, combined with skilful use of the rudder, an expert can land an aeroplane at surprisingly low speed.

The period is given by the imaginary part of the roots, or

$$p = \frac{2\pi}{\frac{1}{2}\sqrt{\frac{4E_1}{C_1} - \left(\frac{D_1C_1 - B_1E_1}{C_1^2}\right)^2}}.$$

Since $\left(\frac{D_1C_1 - B_1E_1}{C_1^2}\right)^2$ is usually small, we may write approximately,

$$p = 2\pi\sqrt{\frac{C_1}{E_1}},$$

$$= 2\pi\sqrt{\frac{Z_wM_q - UM_w}{-gZ_uM_w}}, \quad \text{but } Z_u = \frac{2g}{U},$$

then

$$p = \frac{\pi}{g}\sqrt{2U\left(U - \frac{Z_wM_q}{M_w}\right)}.$$

At low speed, U as well as Z_w and M_q are reduced and the period becomes short. A stiff machine with large M_w would have a rapid period. For given speed, if we make M_q large in order to provide heavy damping, care must be taken that M_w shall be small in order to

secure a slow motion in pitch. It will be remembered that M_w is a measure of statical stability or "stiffness" and was mentioned as somewhat analogous to metacentric height for a ship.

By adjustment of Z_w, M_q, and M_w it appears possible to combine heavy damping with a fairly long period and so obtain great steadiness in normal flight.

§15. CONCLUSIONS (LONGITUDINAL DYNAMICAL STABILITY)

Stability calculations are of greater interest when they can be compared for different aeroplanes. At present, information is scanty but we may obtain by inference some general conclusions by comparing the Clark type aeroplane just described with a Curtiss type aeroplane previously tested by us.

The two aeroplanes are designed to have about the same performance. The principal difference at first sight is the greater wing area of the Clark—about 3.45 pounds per square foot against about 4.7 pounds per square foot for the Curtiss. In consequence of the lighter wing loading, the Clark type should have a steeper curve of Z giving Z_w large, which is favorable to stability.

The Clark aeroplane has a smaller horizontal tail area than the Curtiss, but the fixed part is inclined at $-5°$ to the wing chord against $-3°5$ in the Curtiss. The Clark tail is only a trifle longer than the Curtiss and we may conclude that the pitching moment due to air pressure on the tail surfaces is about the same in the two machines. However, the Clark model uses a wing section on which the center of pressure motion for small angular changes is very slight. The Curtiss has a section described as R. A. F. 6 [1] in which this motion is considerable. For equal tail moments we may then expect M_w to be larger for the Clark machine. This is favorable to stability.

Due to the smaller tail, the damping of the pitching for the Clark model might be less than for the Curtiss. However, we find M_q at high speed -150 for the Curtiss against -192 for the Clark model. The increase must be due to the greater wing area of the latter since a calculation of the damping due to the tail alone gives a result less than one-half that observed for the whole machine.

The greater stability of the Clark model at high speeds is then due principally to greater values of Z_w and M_w. At low speeds, the resistance derivatives of these two aeroplanes are not greatly different. Both become very slightly unstable in their longitudinal motion.

[1] See Technical Report Advisory Committee for Aeronautics, 1912-13.

The following table facilitates comparison:

	Stable High Speed		Unstable Low Speed	
	Curtiss	Clark	Curtiss	Clark
i	1°	0°	14°	12°
X_u	− .128	− .158	− .223	− .162
X_w	+ .162	+ .356	− .132	0
Z_u	− .557	− .57	− .993	− 1.19
Z_w	− 3.95	− 5.62	− .555	− 1.0
M_w	+ 1.74	+ 3.2	+ 1.99	+ 1.41
M_q	− 150.0	− 192.0	− 108.0	− 60.5
K_B^2	34.0	21.6	34.0	21.6
A_1	34.0	21.6	34.0	21.6
B_1	289.0	317.0	134.0	85.1
C_1	834.0	1492.0	213.0	150.0
D_1	115.0	266.0	28.0	22.1
E_1	31.2	59.2	63.6	54.0
Routh's discr.	18×10^6	117×10^6	$-.37 \times 10^6$	$-.12 \times 10^6$
p sec.	34.0	34.7	11.5	10.6
t sec.	11.0	8.1	− 24.7	− 24.7
U, ft.-sec.	− 115.5	− 112.5	− 64.8	− 54.0

We may infer in general that:

1. Any ordinary aeroplane is likely to be unstable longitudinally below a certain critical speed.

2. Stability is improved by large wing area, *i. e.,* light load per square foot.

3. Stability is improved by large horizontal tail surfaces.

4. Stability is improved by high speed.

5. Stability is improved by great head resistance or a poor lift drift ratio.

6. Stability is improved by a small longitudinal moment of inertia.

7. Stability is improved by wings with slight center of pressure motion.[1]

There appears to be no reason to depart from the normal type of aeroplane in a search for longitudinal stability. A steady motion in flight is to be obtained by careful adjustment of surfaces in the ordinary type aeroplane, and the invention of freak types to accomplish great stability at the expense of speed or climb is to be discouraged.

Furthermore, the ordinary type of aeroplane may be made dynamically stable longitudinally without material sacrifice of desirable

[1] For a biplane combination giving a stationary center of pressure without material loss in other desirable features, see "Stable Biplane Arrangements," by J. C. Hunsaker, *Engineering*, London, Jan. 7 and 14, 1916.

flying qualities, such as ease of control. In this connection it is important not to give too great statical stability. Safety in flight may well depend more upon ease of control than upon stability. The almost universal prejudice among accomplished flyers against so-called "stable aeroplanes" appears to have a rational foundation.

PART II. LATERAL MOTION

§1. LATERAL OR ASYMMETRICAL TESTS

When the aeroplane is yawed to right or left of its course through an angle of yaw ψ, the wind blows through the wings obliquely and gives rise to a lateral force Y at right angles to the longitudinal axis x of the aeroplane, a rolling moment L tending to roll the aeroplane about the x axis, and a yawing moment N tending to yaw the machine about the z axis.

To measure the force Y and moments L and N as the aeroplane yaws, the model was mounted in the wind tunnel and held at various angles of yaw to the direction of the wind. At each position measurements were made from which the component forces X, Y, Z and moments L, M, N could be calculated.

The details of the method are given in the Technical Report of the Advisory Committee for Aeronautics, 1912-13, p. 128, where a description is found of the special apparatus required.

Briefly stated, the balance is arranged to measure the moments of the air forces about axes parallel to those axes used for calculation, whose origin is at the center of gravity of the aeroplane. A yawing moment is measured about a vertical axis passing through the main pivot of the balance. The moments of the drift and cross-wind forces are measured about horizontal axes parallel and at right angles to the tunnel axis and passing through the same point. In order completely to determine all forces and moments, a special fitting is provided on which three more measurements may be made. This moment device measures the pitching and rolling moments about horizontal axes passing through the pivot of the attachment. In addition, the total lift or vertical force is measured on the balance. We then have five moment observations and one force observation, as follows:

V_F, measured on vertical force lever (a lift),

M_Z, measured on torsion wire (a yawing moment),

V_P, pitching moment about a high point o_1,

V_R, rolling moment about a high point o_1,

M_D, moment of drift force about a low point o_2,

M_C, moment of cross-wind force about o_2.

We first reduce to the origin o_1 abont which V_P and V_R are measured, which is l inches vertically above o_2.

Denote by primes forces and moments in pounds and pound-inches on the model for 30 miles per hour wind velocity referred to axes through the point o_2. Then:

$$L' = V_R \cos\theta - M_Z \sin\theta,$$
$$M' = V_P,$$
$$N' = V_R \sin\theta + M_Z \cos\theta,$$
$$X' = -V_F \sin\theta + \{M_D \cos\psi - M_C \sin\psi - V_P\}\frac{\cos\theta}{l},$$
$$Y' = \frac{V_R - M_C \cos\psi - M_D \sin\psi}{l},$$
$$Z' = V_F \cos\theta + \{M_D \cos\psi - M_C \sin\psi - V_P\}\frac{\sin\theta}{l}.$$

If the center of gravity of the aeroplane (model) be arranged to have the y coordinate zero, and its x and z coordinates a and b (in inches) referred to o_1, we have for the axes passing through the center of gravity:

$$X_1 = X',$$
$$Y_1 = Y',$$
$$Z_1 = Z',$$
$$L_1 = L' + cY',$$
$$M_1 = M' - cX' + aZ',$$
$$N_1 = N' - aY',$$

where X_1, Y_1, Z_1, L_1, M_1, N_1 are the quantities expressed in pounds and inch-pounds on the model at 30 miles per hour. Converting to full-speed full-scale and to units of pounds and pounds-feet per unit mass, we obtain the required X, Y, Z, L, M, N.

The model was first set at an angle of wing chord to wind of 0° corresponding to high speed. Measurements were then made as above for angles of yaw of ±25°, ±15°, ±10°, ±5°, 0°, keeping the incidence constant. In reducing the observations, values for left- and right-hand angles of yaw were averaged to eliminate errors due to lack of symmetry in the model. In the first test the angle of pitch θ is zero, and the axis of x horizontal. The test was repeated with the model at angles of incidence of 6° and 12°, corresponding to the intermediate and slow speed conditions. Here, again, θ in the formulæ of reduction is zero, since each new axis of x is also horizontal.

It is apparent that the labor involved in the complete solution for X, Y, Z, etc., is considerable and, unfortunately, the method requires

the use of formulæ in which the difference between products of observed quantities is involved. Naturally, the precision of the result is poor when we are left with a small difference between large quantities.

The measurements V_F, M_Z, V_P, V_R, M_D, M_C are probably correct within 2 per cent. L involves no difference and may be taken as equally precise.

Since $N_1 = N' - aY'$ we may make the distance a very small in setting up the apparatus and so keep the precision of N about 2 per cent.

From

$$Y' = \frac{V_R - M_C \cos\psi - M_D \sin\psi}{l},$$

we note that $(M_C \cos\psi + M_D \sin\psi)$ is from three to five times as large as V_R. The precision of Y should then be between 2 and 6 per cent.

From similar reasoning, we may expect Z and X to be precise within 10 per cent, but in special cases, where we must take the difference of quantities of nearly equal magnitude, the precision is not so good.

The quantity M is a small moment which should be nearly zero if the aeroplane is balanced properly. Obviously, no estimate of the precision of M as a per cent can be given in such a case. Where M is large, as in the 12° condition, the measurement is precise to about 10 per cent.

Fortunately, for a study of lateral stability, we are concerned with Y, L, and N only, and these quantities are determined with fair precision.

The values computed for X, Z, and M for zero yaw may be compared with X, Z, M, determined independently in the tests on lift and drift discussed in Part I. The latter are probably precise within 2 per cent. Consequently the computed X, Z, and M obtained from the asymmetrical tests have been adjusted to make them agree with X, Z, M obtained from the symmetrical tests.

The change of X, Z, and M with ψ is not important, and X, Z, and M are not used in the theory of asymmetrical or lateral stability. Since by our equilibrium conditions, the pitching moment M_0 must be zero for normal flight, we must assume that the pilot makes M_0 zero by slight adjustment of his elevator flaps. In the tables below, the small value of M_0 observed when the angle of yaw ψ is zero has been

subtracted from the observed M for each angle of yaw. This adjustment is required to give longitudinal equilibrium to the aeroplane when in its normal attitude.

The following tables summarize the data upon which the subsequent calculations are based:

High-speed attitude, $i=0°$, $l=26$ inches, $c=6.37$ inches, $a=-2.41$ inches.

OBSERVED

ψ	V_R	V_P	M_Z	M_D	M_C	V_F
0	0	1.74	0	4.25	0	.463
5	.284	1.74	.0333	4.41	.511	.453
10	.518	1.72	.0474	4.52	1.011	.450
15	.764	1.61	.0676	4.73	1.566	.445
25	1.222	1.52	.0851	5.43	2.650	.409

CALCULATED

ψ	X	Z	M	Y	L	N
0	9.00	32.2	0	0	0	0
5	9.99	31.5	+ 1.92	− 2.06	25.9	− 4.42
10	9.72	31.2	+ 1.92	− 4.31	40.2	−13.45
15	9.66	30.8	−15.37	− 6.74	54.0	−22.1
25	8.69	28.3	− 3.84	−12.23	66.9	−47.4

Intermediate-speed attitude, $i=6°$, $l=26$ inches, $c=6.67$ inches, $a=-2.06$ inches.

OBSERVED

ψ	V_R	V_P	M_Z	M_D	M_C	V_F
0	0	3.84	0	6.26	0	1.061
5	.416	3.80	.0041	6.39	.309	1.046
10	.776	3.72	.0090	6.50	.633	1.021
15	1.118	3.22	.0054	6.62	1.173	.993
25	1.455	2.69	−.0128	7.11	2.11	.891

CALCULATED

ψ	X	Z	M	Y	L	N
0	3.89	32.2	0	0	0	0
5	4.03	32.1	− 2.6	− .516	19.55	− 2.04
10	4.07	31.0	− 4.6	− 1.124	34.1	− 4.43
15	4.43	30.3	−37.6	− 1.99	43.8	− 8.54
25	4.40	27.2	−59.0	− 3.97	36.9	−18.6

Low-speed attitude, $i = 12°$, $l = 26$ inches,
$c = 6.91$ inches, $a = -1.71$ inch.

OBSERVED

ψ	V_R	V_P	M_Z	M_D	M_C	V_F
0	0	3.73	0	9.395	0	1.483
5	.348	3.71	−.0195	9.53	.094	1.464
10	.718	3.67	−.0541	9.61	.259	1.441
15	1.059	3.52	−.0847	9.68	.464	1.402
25	1.705	3.27	−.1455	9.86	.817	1.270

CALCULATED

ψ	X	Z	M	Y	L	N
0	4.4	32.2	0	0	0	0
5	4.5	31.7	0	− .45	8.65	− 2.53
10	4.46	31.2	0	− .95	17.6	− 5.95
15	4.43	29.8	− 3.5	− 1.51	24.7	− 9.35
25	4.14	27.1	0	− 2.45	37.8	−15.85

The variation, with angle of yaw, ψ, of the rolling moment L, yawing moment N, and lateral force Y, are shown by the curves of figure 15. In its symmetrical position, the aeroplane has no tendency to roll, yaw, or slide slip, and L_0, N_0, and Y_0 are zero, as stated in connection with the discussion of equilibrium conditions.

As the aeroplane yaws from its course, the plane of symmetry swings through an angle ψ, measured positive to the pilot's right hand. The momentum tends to carry the center of gravity forward in its original direction of motion. As a result, the apparent wind seems to strike the left cheek of the pilot. The curves of N show that, if this aeroplane yaw to the right, a negative yawing moment is produced which tends to turn the aeroplane to the left and hence to put it back on its course. The aeroplane is hence "directionally" stable, having a preponderance of fin surface behind the center of gravity, and the pilot need not use his rudder to stop the yaw. Numerically, we see that for a yaw of 10° at high speed, the value of N is -13.5 units, or about 670 pounds-feet. For a perfectly neutral aeroplane, to produce an equal yawing moment the pilot must exert a force of about 34 pounds on a vertical rudder 20 feet to the rear of the center of gravity.

When flying straight ahead, if the direction of the wind suddenly shifts so as to bring the apparent wind 10° to the left of the fore and aft axis of the aeroplane, the aeroplane tends to head over into the wind. An excessive amount of "directional" stability, indicated by

a steep curve of yawing moments N, may cause the aeroplane to be unmanageable in gusty air. It may "take charge" and, due to excessive "weather helm," be difficult to keep on any desired course.

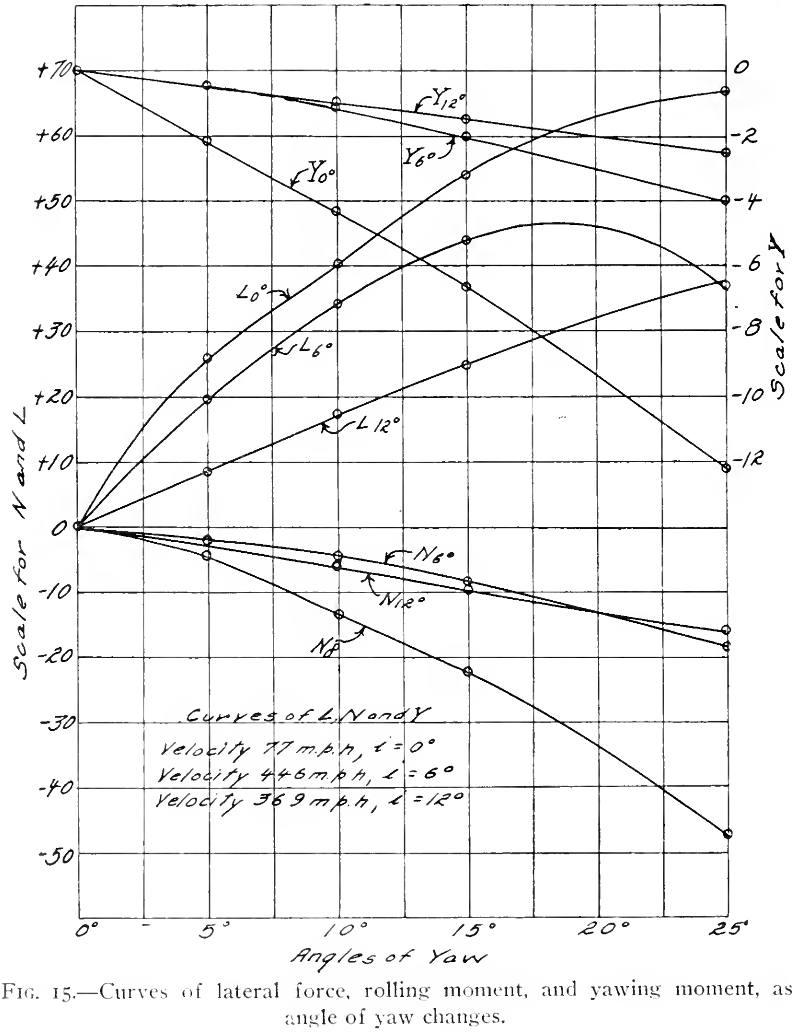

FIG. 15.—Curves of lateral force, rolling moment, and yawing moment, as angle of yaw changes.

It will be shown later that the so-called "directional" stability is not only undesirable in gusty air, but is the determining factor in "spiral instability." Indeed, "directional stability" is very nearly incompatible with inherent dynamical stability in roll, yaw, and side slip considered together.

If the aeroplane yaw to the right, it is practically starting off on a turn to the right. As is well known, to make such a turn safely an aeroplane should be "banked" to such an angle of roll that the centrifugal force, acting to the left, is about balanced by the horizontal component of the normal force Z acting to the right. In other words, the bank proper to a right turn requires a positive angle of roll ϕ given by a positive rolling moment L. The curves of L in the figure show that for this aeroplane the natural rolling or banking moments are positive for a positive yaw, and hence tend to bank the aeroplane suitably for the turn. This property is extremely valuable in preventing capsizing.

As in the case of the yawing moments, an excessive amount of natural banking may be uncomfortable, especially in gusty air. Thus, if the wind shifts to the left, the relative angle of yaw is positive, the aeroplane tends to turn to the left due to its "directional" stability and to bank for a turn to the right due to the natural banking or rolling moment L. The result may be to throw the aeroplane about in a somewhat violent manner, or it may capsize. This motion is discussed later under the heading "Dutch roll."

Large banking moments L can be given by vertical fin surface above the center of gravity, by a dihedral angle upwards or a "retreat" or sweep back of the wings. All these arrangements are probably equivalent and, though tending to give a stable motion in still air, tend toward violence in gusty air.

The model under test has, as is shown by the drawings, a dihedral angle upwards of the wings made by raising each wing tip 1.6°. This amount of dihedral has been found in practice to be not excessive on ordinary aeroplanes.

The curves of lateral force Y are negative for a positive yaw. This means that if the aeroplane yaws to the right in still air, it is pushed to the right and started off on a right turn. We saw above that the natural banking is suitable for the turn. In gusty air, if the apparent wind shifts 10° to the left the lateral force pushes the aeroplane to the right.

Numerical values are interesting. Suppose a plus yaw of 10° in still air. The rolling moment at high speed is 2,000 pounds-feet. This is equivalent to a down load of 55 pounds on the right aileron and an up load of 55 pounds on the left aileron. The pilot with his aileron control can, if he wish, produce a rolling moment over three times this magnitude, so that he can prevent the aeroplane taking charge and hold it level. Approaching a landing, it is most important

that the aileron control shall be very powerful compared with the natural banking tendency. Excessively stable aeroplanes may be really dangerous to land in gusty air. In any aeroplane design, the relative magnitudes of the natural rolling moment and the aileron control available should be carefully considered.

If the aeroplane side slip with a lateral velocity v, the resultant velocity of the center of gravity of the aeroplane is obtained by combining v as a vector with the forward speed U. The apparent wind in still air is then inclined to the axis of the aeroplane as it would be were the aeroplane yawed from her course by an angle

$$-\psi = \tan^{-1} \frac{v}{U} .$$

A side slip to the left is equivalent aerodynamically to a positive or right-hand yaw. The sign of the lateral force Y is negative for a plus yaw and hence resists the side slip, as is desired.

The asymmetrical motion is a combination of rolling, yawing, and side slipping as is indicated by the qualitative discussion given above and by the equations of motion in Part I, §9. In order that, under the influence of N, L, and Y, acting in concert, the disturbed motion shall be stable, the aeroplane must tend to return in time to its original attitude. It is impossible to determine whether the aeroplane is thus stable from a consideration of N, L, and Y separately. The term "directional stability," frequently used, means very little with regard to the probable motion of the aeroplane.

The quantitative determination of the stability of the motion can be made only after we have found the numerical values of the coefficients needed in the equations of motion in Part I, §9.

§2. RESISTANCE DERIVATIVES

The rates of change of N, L, and Y with velocity of side slip v are the partial derivatives N_v, L_v, Y_v. The side slip velocity v is equivalent to an angle of yaw ψ given by:

$$\tan \psi = -\frac{v}{U} .$$

If ψ is small and measured in degrees, the tangent is equal approximately to the circular measure of the angle, or

$$\psi = -\frac{1}{57.3} \frac{v}{U} ,$$

and

$$N_v = \frac{\partial N}{\partial v} = -\frac{57.3}{U} \cdot \frac{\Delta N}{\Delta \psi} .$$

The fraction $\frac{\Delta N}{\Delta\psi}$ is the slope of the curve of N plotted on angle of yaw ψ as abscissæ.

Similarly:

$$L_v = -\frac{57.3}{l}\cdot\frac{\Delta L}{\Delta\psi},$$

and

$$Y_v = -\frac{57.3}{U}\cdot\frac{\Delta Y}{\Delta\psi}.$$

Taking the slopes of the curves of L, N, Y at $\psi=0$ from figure 15, we obtain the following "resistance derivatives" needed in the lateral equations of motion.

High speed:

$$i=0^\circ\begin{cases} Y_v=-.204, \\ L_v=+3.06, \\ N_v=-.449. \end{cases}$$

Intermediate speed:

$$i=6^\circ\begin{cases} Y_v=-.0878, \\ L_v=+3.44, \\ N_v=-.351. \end{cases}$$

Slow speed:

$$i=12^\circ\begin{cases} Y_v=-.106, \\ L_v=+1.91, \\ N_v=-.53. \end{cases}$$

Note that these derivatives do not change greatly with speed. In the longitudinal motion the effect of change of speed (attitude) was more marked.

§3. ROLLING MOMENT DUE TO YAWING, L_r

It is obvious that if an aeroplane yaws quickly, the outer wing tip moves through the air more rapidly than the inner wing tip and, hence, due to the spin, the lift on the outer wing is the greater. The resultant rolling moment tends to bank the aeroplane suitably for the turn. The magnitude of this rolling moment was in dispute in the recent Curtiss-Wright patent litigation. The following calculation leads to a simple formula to determine the roll due to angular velocity in yaw.

In our notation, a rolling moment L is expressed in pounds-feet per unit mass. In pounds-feet on the aeroplane, the moment is mL, where m is the mass W/g in slugs.

The derivative L_r is the rate of change of rolling moment with an angular velocity in yaw of r radians per second, or

$$\frac{\partial L}{\partial r} = L_r.$$

Let U = the velocity of advance of the center of gravity of the aeroplane in feet per second. U is a negative number.

S = span of the aeroplane (one plane) in feet.

b = chord of one plane in feet.

$W/g = m$ = mass of aeroplane in slugs.

r = angular velocity of yaw in radians per second, positive for a right-hand turn.

Consider an element of wing area on the left wing of width dy in the y axis and depth b in x axis. The distance from the center of gravity of the aeroplane to the center of this element is y feet, positive for the left wing.

The velocity through the air of this element is $U - yr$, since the increase of air speed due to spin is yr.

If we assume that the lift of the wings is equal to the weight of the aeroplane, we neglect the small vertical forces on body and tail only.

The lift in pounds per square foot per foot-second velocity is the usual " lift coefficient " for the wing, which can be computed from the model tests for Z. Thus:

$$K = \frac{Z_0 m}{AU^2}.$$

Where:

$A = 265$, the total area of both wings.

Then the lift in pounds on the elementary strip of wing of area bdy is

$$Kbdy(U - yr)^2.$$

The rolling moment on the aeroplane of this elementary lift force is

$$Kbydy(U^2 - 2Uyr + y^2r^2),$$

and the total rolling moment on one whole plane is,

$$Kb\int_{-\frac{S}{2}}^{+\frac{S}{2}} (U^2 - 2Uyr + y^2r^2)ydy.$$

But $\int_{-\frac{S}{2}}^{\frac{S}{2}} by^2dy = I$, the moment of inertia of the area of one plane, and

$$\int_{-\frac{S}{2}}^{\frac{S}{2}} U^2ydy = 0 = \int_{-\frac{S}{2}}^{\frac{S}{2}} y^3dy.$$

Hence the rolling moment on one plane is $-2UKIr$, and substituting for K its expression above,

$$-2\frac{Z_0 m}{4U}Ir.$$

For two identical wings of rectangular form, we have for our complete aeroplane a total rolling moment in pounds-feet per unit mass:

$$L = -\frac{1}{6}\frac{Z_0 S^2}{U}r, \text{ making } Z_0 = g,$$

$$L_r = -\frac{gS^2}{6U} \text{ for horizontal flight.}$$

It appears that L_r can be made small by short span and high speed. The sign of L_r is such that the bank is proper for the turn.

Numerically, we have, making the mean span $S = 40.2$ feet and $b = 5.62$ feet,

$$\begin{aligned} L_r &= -8660/U, \\ &= +77.0, \text{ high speed, } i = 0°, \\ &= +132.5, \text{ intermediate speed, } i = 6°, \\ &= +160.0, \text{ slow speed, } i = 12°. \end{aligned}$$

Note that L_r (which is unfavorable to "spiral" stability) becomes larger at low speed.

§4. YAWING MOMENT DUE TO ROLLING, N_p

When an aeroplane rolls with an angular velocity p radians per second (positive when right wing goes down), an elementary area of the right wing has its angle of incidence increased and a corresponding element of the left wing has its angle of incidence diminished by the same amount.

If p is small, the resultant air velocity at a point y feet from the center line is

$$\sqrt{U^2 + p^2y^2} = U, \text{ neglecting } p^2.$$

On the right wing, the angle of incidence at any point is increased by a small angle a, given by $\tan a = py/U$. Due to the greater angle of incidence, the head resistance of the element is increased.

On a curve of the "drift coefficient" for the wing shape (see fig. 3, Part I) we may draw a tangent line at the point on the curve corresponding to the angle of incidence for normal flight. For small changes in incidence from normal incidence, we may substitute this tangent line for the actual curve without material error. The value

of the drift coefficient in pounds per square foot per foot-second is then

$$K_{x_1} = K_{x_0} + \sigma a,$$

where K_{x_0} is the coefficient when i is the normal angle, σ is the slope of the tangent line and a the small change in incidence defined above. The slope σ is conveniently measured in units of K_x change per degree angle. If the subtangent or projection of the tangent line is j degrees,

$$\sigma = \frac{K_{x_0}}{j},$$

and

$$K_x = K_{x_0} + K_{x_0}\frac{a}{j}.$$

The head resistance of an element of the right wing is

$$-bdyK_xU^2 = -\left(K_{x_0} + K_{x_0}\frac{a}{j}\right)bdyU^2,$$

and the yawing moment on the aeroplane due to it is

$$+\left(K_{x_0} + K_{x_0}\frac{a}{j}\right)bU^2ydy.$$

But $\tan a = \frac{py}{U}$ or $a = 57.3\frac{py}{U}$, if a is small. Then the total yawing moment on a single plane is

$$\int_{\frac{S}{2}}^{-\frac{S}{2}}\left(K_{x_0} + K_{x_0}\frac{a}{j}\right)bU^2ydy.$$

The integral of the first term is zero, and the second term reduces to

$$-\frac{57.3UK_{x_0}I}{j}p,$$

where I is moment of inertia of one plane. For a biplane of two rectangular wings, the total yawing moment in pounds-feet is

$$mN = \frac{-57.3UK_{x_0}bS^2}{6j}p.$$

Hence:

$$N_p = \frac{-57.3UK_{x_0}bS^2}{6jm}.$$

To calculate N_p, we have:

i	U	K_{x_0}	j	b	S	m	N_p
0	−112.5	....	∞	5.62	40.2	50	0
6	− 65.3	.0000443	6.0	5.62	40.2	50	+33.5
12	− 54.0	.0001047	6.9	5.62	40.2	50	+57.0

Since U is feet per second, K_{x_0} must be in pounds per square foot per foot-second velocity. Values for the drift coefficient were taken

from a curve corrected to apply to full-speed full-scale, aspect ratio 7, and biplane of gap 1.1 times chord.

Note that the positive sign of N_p indicates that for a positive roll (to the right) a yaw to the right is assisted. At high speed the aeroplane flies at a small angle of incidence where the drift curve plotted on incidence is about horizontal. N_p is, therefore, zero at this attitude.

§5. DAMPING OF ROLL, L_p

The wide spreading wings very effectively damp the rolling, and the resisting or damping moment in pounds-feet on the aeroplane is mpL_p for an angular velocity p radians per second in roll.

The method of oscillations previously used to determine the damping of the pitching M_q is applied to determine L_p. Figure 17 (pl. 2) shows the oscillating apparatus set up to impress an oscillation in roll about the center of gravity of the model.

Using a similar notation, the equation of motion of the complete apparatus with model is

$$I\frac{d^2\phi}{dt^2}+(\lambda_0+\lambda_w+\lambda_m)\frac{d\phi}{dt}+(K-Cm')\phi+M_0-M_s=0.$$

Where λ_0 represents damping due to friction, λ_w due to wind on apparatus, and λ_m due to wind on model. The moment of inertia of the entire oscillating mass I is found by a simple experiment.

The solution for points of maximum amplitude is of the form

$$\phi=\phi_0 e^{-\frac{\lambda t}{2I}},$$

or

$$\frac{\lambda t}{2I}=\log_e\frac{\phi_0}{\phi}=\log_e 9,$$

since the ratio $\frac{\phi_0}{\phi}$ is arranged to be as 9 to 1 on the scale for the pencil of light.

The numerical work follows:

Oscillation in Roll

I model and apparatus $=.0399$, $\frac{\phi_0}{\phi}=9$

I apparatus $=.0373$

Test on Bare Apparatus

V, wind velocity, miles	30	20	0
t, seconds	78	98	197
λ	.0021	.00168	.00083
λ_w (less zero)	.0013	.00085	0

TEST ON APPARATUS WITH MODEL

INCIDENCE OF WINGS 0°

V	35	30	24	18	6.8	0
t	7.3?	6.0	8.3	12	30	175
λ	.024?	.0292	.0211	.0146	.0058	.001
λ_0	.001	.001	.001	.001	.001	.001
λ_w	.0014	.001	.001	.0007	.0003	0
λ_m	.022?	.027	.019	.013	.0045	0

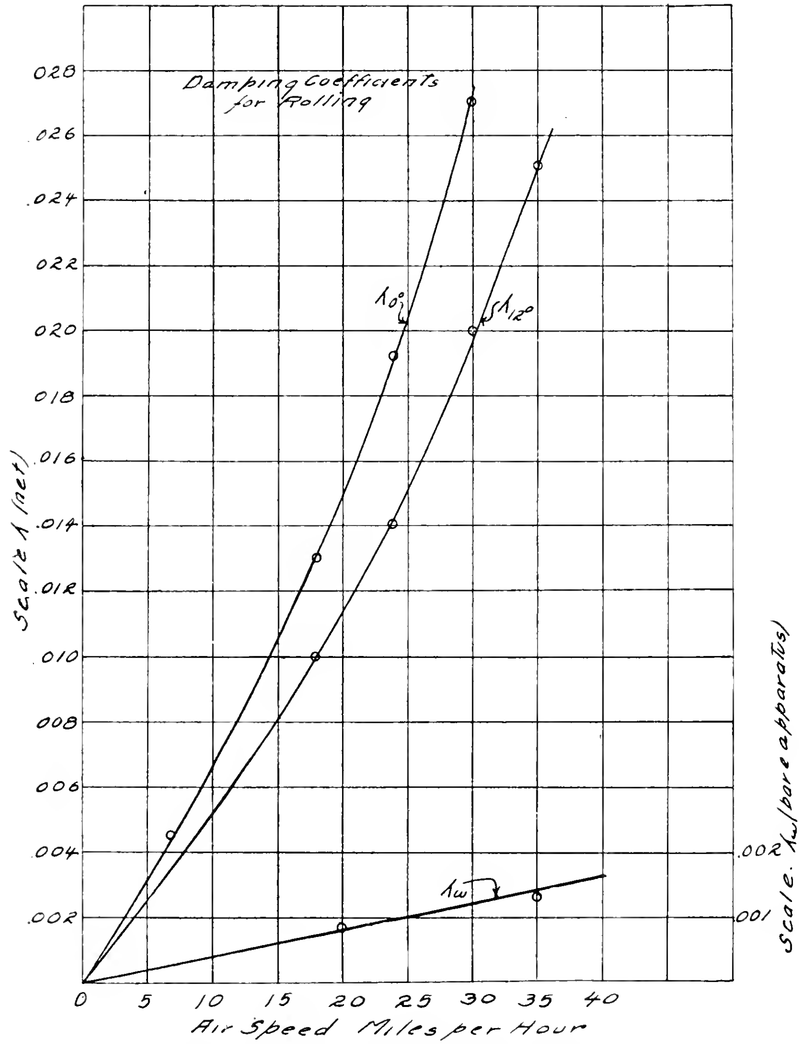

FIG. 16.—Curves of damping coefficient for rolling.

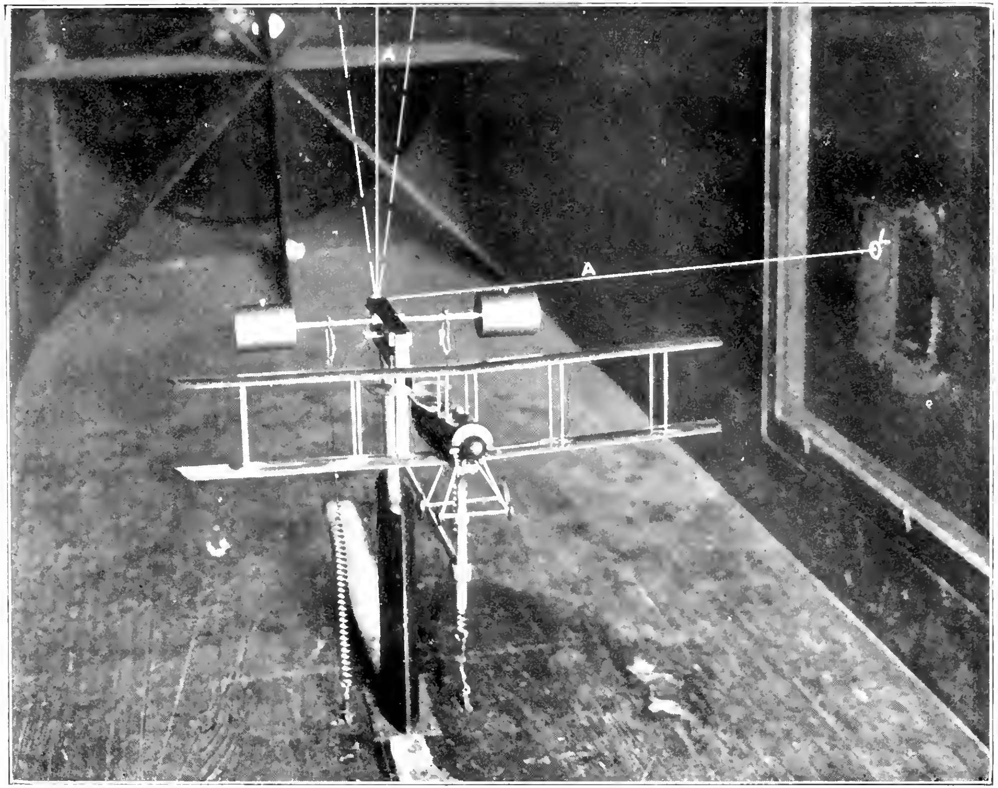

FIG. 17.—MODEL IN POSITION FOR ROLLING OSCILLATION. L, SPECTACLE LENS. AA, PENCIL OF LIGHT

INCIDENCE OF WINGS 12°

V	35	30	24	18	0
t	6.5	8	11	14.5	175
λ	.027	.022	.016	.0121	.001
λ_0	.001	.001	.001	.001	.001
λ_w	.001	.001	.001	.001	0
λ_m	.025	.020	.014	.010	0

The values of λ_w due to wind on apparatus are taken from the curve of λ_w on figure 16 and applied in the calculation to find λ_m net. Figure 16 shows the values of λ_m. It is obvious that the values of λ_m for $i=0°$ at 35 miles per hour is grossly in error. This point is, therefore, rejected.

The curves of λ_m appear to increase more rapidly than the velocity: in fact, a plot on logarithmic paper shows that over the range of wind tunnel speeds λ_m varies approximately as $V^{1.35}$.

Since this damping helps to stop violent rolling, we shall be on the safe side in our stability calculation if we assume that the damping varies directly as the velocity.

To convert λ_m to full scale, we have

$$L_p = \frac{-26^4}{m} \cdot \frac{V}{V_m} \cdot \lambda_m.$$

Where V_m is the speed at which λ_m was measured. Taking the scale factor 26, $m=50$ slugs, $V_m=30$ miles, $V=76.9$ miles for $i=0°$, and $V=36.9$ miles for $i=12°$, we have

$$L_p = -631 = 5.61U, \text{ for high speed,}$$
$$L_p = -224 = 4.15U, \text{ for low speed,}$$

and for the intermediate speed, by interpolation,

$$L_p = -319 = 4.88U.$$

§6. DAMPING OF YAW, N_r

The damping of an oscillation in yaw is probably due to the long body and vertical surfaces at the tail, as well as to the wings. It is not practicable to compute this, and we have employed the same apparatus as before to determine the damping in yaw by the method of oscillations. The model set for the oscillation in yaw is shown on figure 18 (pl. 3).

The equation of motion is similar to that for roll and pitch, thus:

$$I\frac{d^2\psi}{dt^2} + (\nu_0 + \nu_w + \nu_m)\frac{d\psi}{dt} + (K - cm')\psi + M_0 - M_s = 0,$$

and

$$\psi = \psi_0 e^{-\frac{\nu t}{2I}}, \text{ or } \frac{\nu t}{2I} = \log_e \frac{\psi_0}{\psi} = \log_e 9.$$

Oscillation in Yaw

I model and apparatus $= .0396$
I apparatus $= .0343$

Test on Bare Apparatus

V	35	20	0
t	108	115	120
ν	.0014	.00131	.00126
ν_0	.0013	.00126	.00126
ν_w	.0001	.00005	0

Test on Apparatus with Model

INCIDENCE OF WINGS 0°

V	35	30	24	12
t	52	57	64	105?
ν	.00335	.00306	.00272	.00166?
ν_0	.00126	.00126	.00126	.00126
ν_w	.00013	.00011	.00009	.00004
ν_m	.00196	.00169	.00137	.00036?

INCIDENCE OF WINGS 12°

V	35	30	18	8
t	33	36	47	73
ν	.00528	.00484	.00371	.00239
ν_0	.00126	.00126	.00126	.00126
ν_w	.00013	.00011	.00006	.00003
ν_m	.00389	.00347	.00239	.00110

INCIDENCE OF WINGS 6°

V	35	30	20
t	46	53	71
ν	.00379	.00329	.00245
ν_0	.00126	.00126	.00126
ν_w	.00013	.00011	.00007
ν_m	.00240	.00192	.00112

$$N_r = \frac{26^4}{50} \cdot \frac{V}{V_m} \cdot \nu_w.$$

$N_r = .35U \;= -39.4,\quad$ for $i = 0°$,
$N_r = .398U = -26.0,\quad$ for $i = 6°$,
$N_r = .72U \;= -38.9,\quad$ for $i = 12°$.

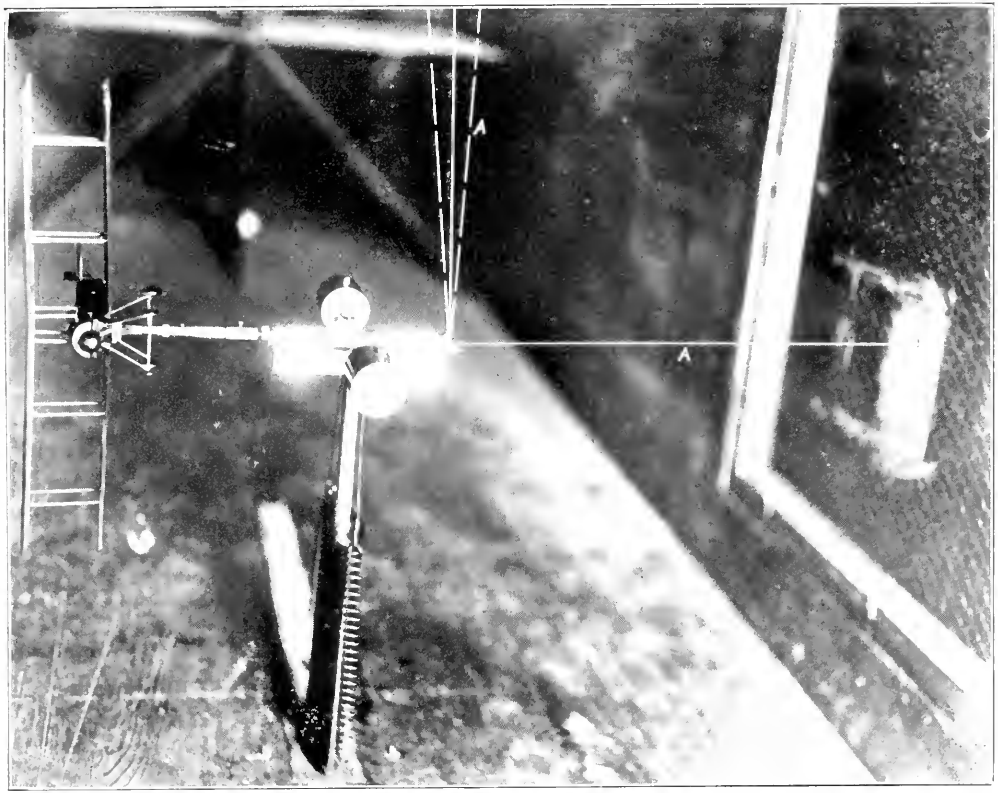

FIG. 18.—MODEL IN POSITION FOR YAWING OSCILLATION. L, SPECTACLE LENS. AA, PENCIL OF LIGHT

The curves of v_m of figure 19 show that the damping of the yaw increases with speed approximately as the first power. The damping of yaw N_r is in magnitude only about $\frac{1}{10}$ the damping of roll L_p. Consequently, the precise determination of N_r is attended with some experimental difficulty.

It is to be noted that N_r diminishes with the velocity, while at the same time it increases with the angle of attack. The value of N_r at

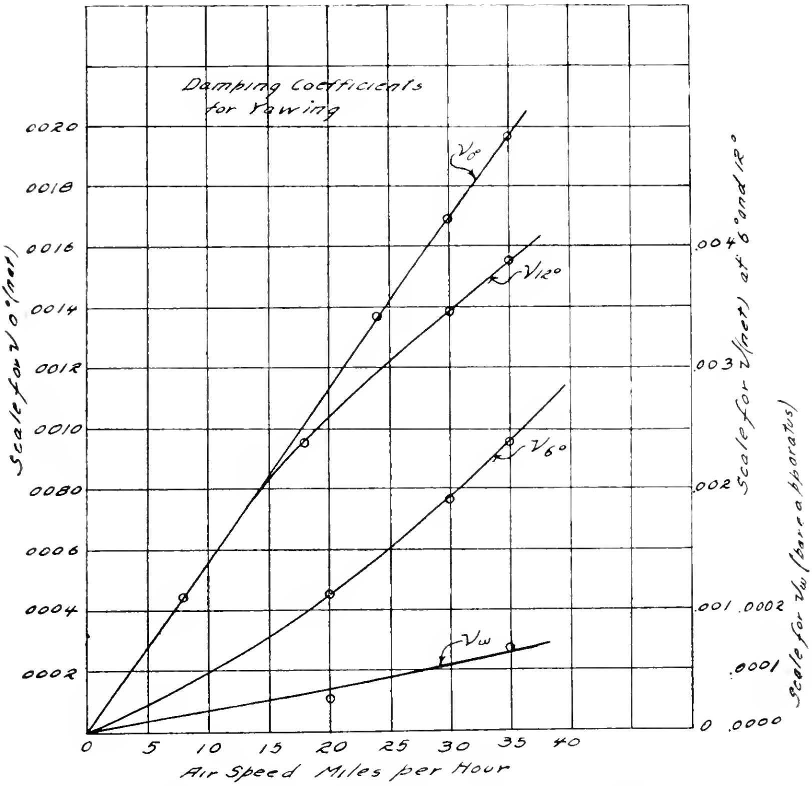

FIG. 19.—Curves of damping coefficient for yawing.

high speed $.35U$ is practically equal to its value at low speed $.72U$. It seems reasonable to expect that at large angles of incidence the damping of yaw due to the wings would be much greater than at small angles were the speed the same.

For the intermediate speed $i=6°$ the coefficient N_r is least. This is due to the fact that from 0° to 6°, U drops from -117.5 to -65.3 feet per second, while from 6° to 12° U drops very little more: only from -65.3 to -54 feet per second.

§7. NEGLECTED COEFFICIENTS

The changes in lateral force Y due to angular velocity of roll and yaw, represented by the coefficients Y_p and Y_r, are neglected as unimportant. The surface of the aeroplane is fairly symmetrical about the center of gravity and it is unlikely that any appreciable lateral force could be created by any small angular velocity p or r. In the calculations to follow Y_p and Y_r are made zero.

The products of inertia are also neglected as not important and difficult to estimate for an actual machine.

§8. INDEPENDENCE OF THE LONGITUDINAL AND LATERAL MOTION

It is seen on figure 20 that the values of X, Z, and M are somewhat changed as the aeroplane yaws, and to this extent it is not strictly correct to consider the lateral motion separately. We may imagine that if there be set up a combined oscillation about the flight path in roll, yaw, and side slip, the aeroplane will be influenced to take up an oscillation in pitch of the nature of a forced oscillation. However, any oscillation in pitch has already been shown to die out rapidly (since the longitudinal motion is stable and strongly damped). We may then consider the pitching induced by yawing, etc., as of the same nature as that caused by any accidental disturbance of longitudinal equilibrium, such as might result from gusty winds, shifting of weights, or the firing of a gun. If the longitudinal motion be stable, that stability should be quite independent of the nature of any disturbing agent which gives the initial amplitude to the oscillation, provided the phenomenon of resonance is not present. That is, if the natural period of the lateral motion, if oscillatory, happen by some remote chance to be equal to the natural period of the longitudinal oscillation, it may be possible for a machine which is unstable laterally to seriously compromise its longitudinal stability.

If the lateral motion be stable and, if oscillatory, damp out quickly, it is difficult to see how any marked disturbance of the longitudinal motion can be induced by the lateral motion.

In circling flight, there is a constant angular velocity of yaw and probably some side slip. In this case, the lateral and longitudinal motions are interdependent, and the methods of calculation of this paper will not apply. Indeed, we should have to combine the six general equations of motion giving rise to a single equation of the eighth order, which must then be solved for all the roots. In the

present state of our knowledge, the calculation of the stability of circling flight appears impracticable.

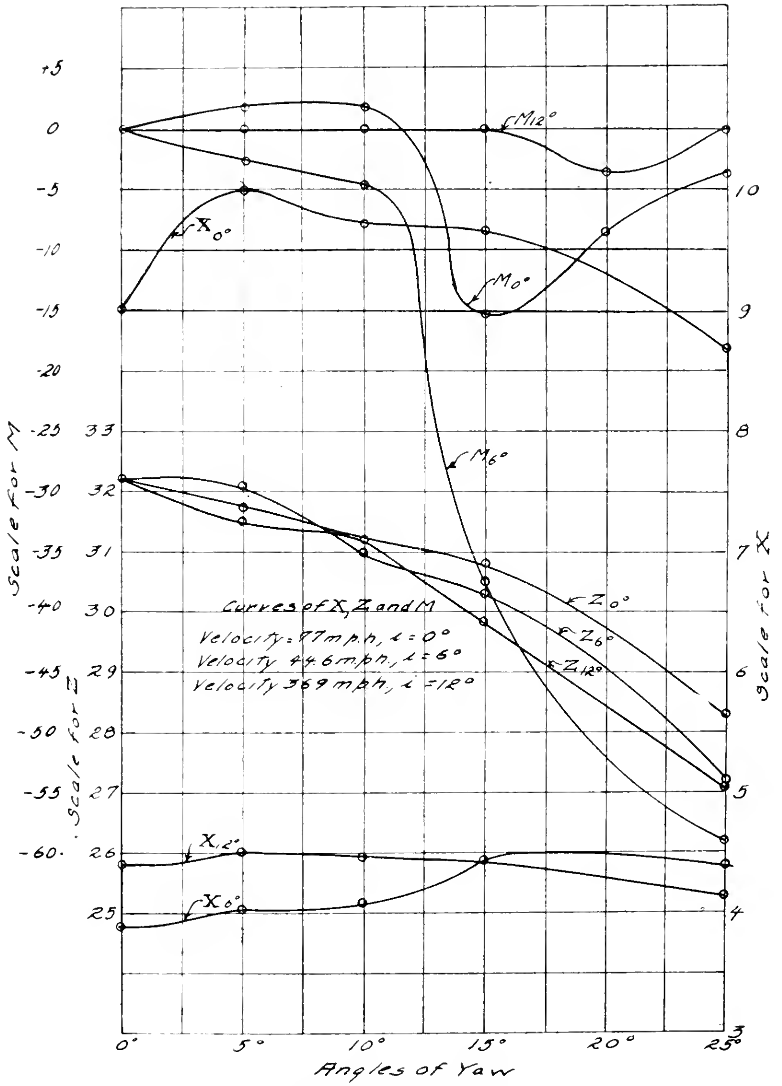

FIG. 20.—Curves of normal force, longitudinal force, and pitching moment as angle of yaw changes.

For flight in a straight line, we may reasonably conclude that if the lateral motion be stable it will not compromise the stability of the longitudinal motion, and *vice versa*. Such a machine should, in still

air, follow its trajectory without the aid of the pilot. In gusty air, it would roll and pitch and yaw as well as side slip and rise and sink, but, if the altitude be great, there should be no danger. The machine would not follow a fixed course, if controls were abandoned, but would adjust its trajectory constantly to the changing conditions of the air in an effort to maintain the same relative velocity through the air and the same angle of incidence.

On the other hand, if the lateral motion be unstable and the angle of yaw become as great as 10°, the curves of figure 16 show that the head resistance X is not greatly changed for slow-speed attitudes and increases but 10 per cent at high speed. This should tend to slow down the aeroplane very little.

The change in Z, or lift, is insignificant.

However, the change in M is most interesting. For $i=12°$ no change in M is produced by yaw, but for $i=6°$ a small diving moment is induced. For an angle of yaw of 15° or more, this diving moment is enormously increased. For $i=6°$, $\psi=15°$, $mM=37\times 50=1{,}850$ pounds-feet, corresponding to a force on the elevator of nearly 100 pounds.

If the pilot attempt to turn without banking he may side slip so rapidly that he has the relative wind making an angle of 15° to the longitudinal axis of the aeroplane. The aeroplane will then tend to dive sharply. Similarly, an excessive bank may induce a side slip inwards and the same tendency to nose dive. The cause of this tendency to nose dive shown here is not understood, but it is significant that many accidents have occurred to inexperienced pilots in turning.

§9. LATERAL STABILITY, DYNAMICAL

The combined asymmetrical motion in roll, yaw, and side slip will be called "lateral." For simplicity we will consider horizontal flight in a straight line in still air, and for this condition investigate the character of the disturbed motion.

From the detail plans, the radii of gyration K_A and K_C have been calculated. It is assumed that these values are not appreciably changed by change of axes corresponding to the changed attitudes proper for different speeds. K_A and K_C as given are referred to the axes used at high speed. The products of inertia are neglected as unimportant.

From Part I, §9, we obtain the following simplified formulæ for the coefficients of the biquadratic equation which is characteristic of

the lateral motion. The quantities Y_r, Y_p, θ are made equal to zero. Then:

$$A_2D^4+B_2D^3+C_2D^2+D_2D+E_2=0.$$

Where:

$$A_2=K_A^2K_C^2,$$
$$B_2=-Y_vK_A^2K_C^2-K_C^2L_p-K_A^2N_r,$$
$$C_2=(N_rL_p-L_rN_p)+Y_vL_pK_C^2+K_A^2(N_vU+N_rY_v),$$
$$D_2=-Y_v(N_rL_p-L_rN_p)+U(N_pL_v-N_vL_p)+gK_C^2L_v,$$
$$E_2=g(N_rL_v-L_rN_v).$$

These coefficients may now be calculated from the known constants of the aeroplane, and Routh's discriminant, $B_2C_2D_2-A_2D_2^2-B_2^2E_2$, found. The condition that the motion shall be stable is that A_2, B_2, C_2, D_2, E_2 shall each be positive as well as Routh's discriminant.

The numerical work is laborious and the results only are given in the table.

COEFFICIENTS AFFECTING LATERAL MOTION

	High speed	Intermediate speed	Low speed
Angle of incidence, i	0°	6°	12°
Velocity, ft.-sec., U	− 112.5	− 65.3	− 54.0
Mass, slugs, m	50.0	50.0	50.0
K_A	5.2	5.2	5.2
K_C	6.975	6.975	6.975
Y_v	− .204	− .0878	− .106
L_v	+ 3.06	+ 3.44	+ 1.91
N_v	− .449	− .351	− .53
Y_p	0	0	0
L_p	−631.0	−319.0	−224.0
N_p	0	+ 33.5	+ 57.0
Y_r	0	0	0
L_r	+ 77.0	+132.5	+160.0
N_r	− 39.4	− 26.0	− 38.9
A_2	1310.0	1310.0	1310.0
B_2	31830.0	16350.0	12090.0
C_2	3274.0	5910.0	1630.0
D_2	41780.0	5490.0	3490.0
E_2	2770.0	1386.0	−335.0
$B_2C_2D_2-A_2D_2^2-B_2^2E_2$	37400×10^9	123×10^9	3.7×10^9
Character of motion	Stable	Stable	Unstable

It is seen that, for the particular aeroplane under consideration, Routh's discriminant and the coefficients of the biquadratic are all positive at high and intermediate speeds. The motion in these two cases is, therefore, stable.

At low speed, however, we observe that E_2 becomes negative, indicating that the lateral motion is unstable. That is to say, one at least of the roots of the biquadratic increases with time. In this case Routh's discriminant continues to be positive, but is small compared with its value at high speed.

It is unfortunate that this lateral instability is associated with the longitudinal instability which was found in Part I to be present at low speed.

§10. CHARACTER OF LATERAL MOTION

Bairstow has shown that for the usual values of the coefficients of the biquadratic equation for the lateral motion, the equation in question may be factored approximately, giving:

$$\left(D+\frac{E_2}{D_2}\right)\left(D+\frac{B_2^2-A_2C_2}{A_2B_2}\right)\left(D^2+\left(\frac{C_2}{B_2}-\frac{E_2}{D_2}\right)D+\frac{B_2D_2}{B_2^2-A_2C_2}\right)=0,$$

provided E_2 is small compared with B_2 or D_2, and $B_2D_2-C_2$ is small compared with C_2^2.

In our cases, the second condition is not satisfied but the error made is found by trial solutions to be unimportant.

High Speed.

Thus for the high-speed condition:

$$\text{First factor, } D=-\frac{E_2}{D_2}=-.0665.$$

This is a subsidence which tends to reduce the amplitude of an initial disturbance to half value in $t=\dfrac{0.69}{.0665}=10.4$ seconds. We may consider this motion fairly stable.

For the second factor we have another subsidence given by

$$D=-\frac{B_2^2-A_2C_2}{A_2B_2}=-23.2,$$

which reduces to half value in $t=\dfrac{0.69}{23.2}=.03$ second. Such motion is so heavily damped that it would never be observed on the aeroplane.

The third factor gives upon substitution:

$$D^2+\left(\frac{C_2}{B_2}-\frac{E_2}{D_2}\right)D+\frac{B_2D_2}{B_2^2-A_2C_2}=D^2+.967D+1.375=0,$$

or

$$D=-.484\pm1.07i.$$

This is a pair of imaginary roots indicating an oscillation of natural period $p=\frac{2\pi}{1.07}=5.9$ seconds, which is damped to half the initial amplitude in $t=\frac{0.69}{.484}=1.4$ second. The motion is so heavily damped as to be of no consequence. The period is fairly rapid, and if the damping were not great, the oscillation might become uncomfortable.

For the high-speed case, it appears that the lateral motion is quite stable.

Intermediate Speed.

At the intermediate speed, where $i=6°$, we have for the first factor:

$$D=-.252,$$

a subsidence which damps to half amplitude in

$$t=\frac{0.69}{.254}=2.72 \text{ seconds.}$$

This motion is very strongly damped, even more than at the high speed.

Similarly, the second factor gives an enormously damped subsidence.

$$D=-12.1,$$

$$t=\frac{0.69}{12.1}=.057 \text{ second.}$$

The oscillation corresponding to the third factor is of fairly slow period, but so strongly damped that it is of slight importance. Thus:

$$D^2+.11D+.346=0,$$

$$D=-.55\pm.586i,$$

$$p=\frac{2\pi}{.586}=10.7 \text{ seconds' period,}$$

$$t=\frac{0.69}{.55}=1.25 \text{ second to damp 50 per cent.}$$

Slow Speed.

For the slow-speed condition, $i=12°$, we observed that the coefficient E_2 is negative indicating instability of motion. Mathematically, that is to say, the real root corresponding to the first factor of Bairstow's approximate method,

$$D=-\frac{E_2}{D_2}=.096,$$

is now no longer a subsidence, but a divergence which doubles itself in $t=\frac{0.69}{.096}=7.2$ seconds. This is not an alarming rate of increase, since 7 seconds should be ample time for a pilot to observe a devia-

tion from normal attitude and to correct it by use of his controls. However, the aeroplane could only be flown at this speed even in still air provided the pilot were alert.

The second factor is a strongly damped subsidence $D = -9.12$, which damps to half amplitude in .08 second.

The third factor is an oscillation,

$$D^2 + .231D + .292 = 0,$$
$$D = -.116 \pm .528i,$$

having a period of $\frac{2\pi}{.528} = 12$ seconds, which is damped to half amplitude in $t = \frac{0.69}{.116} = 6$ seconds. This oscillation is stable, but the damping is only moderate, and it may well be felt on the aeroplane in flight. In some types of aeroplane, it is likely that this motion may be undamped and hence the amplitude of successive oscillations will be increasing, giving rise to instability of a new character.

§11. THE "SPIRAL DIVE"

The motion found corresponding to E_2 negative, as at slow speed, may be traced to the resistance derivatives involved in the expression for E_2. Thus:

$$E_2 = g(N_v L_r - L_v N_r),$$

and E_2 will be positive only when L_v/N_v is greater than L_r/N_r. For stability, or E_2 positive, L_v and N_r should be large and N_v and L_r small.

The derivative L_v depends on the rolling moment due to side slip and can be made large and positive by an upward dihedral angle to the wings or by vertical fin surface above the center of gravity of the aeroplane. At low speed and high angle of incidence we see that L_v is diminished. Thus, at 6° and 44.6 miles, $L_v = 3.44$, while at 12° and 36.9 miles, $L_v = 1.91$. The drop in speed is only about 18 per cent. Hence the drop in L_v cannot be due to the lower speed, but must be due to the greater angle of incidence.

Let i be the angle between the wind direction and the center line of the wings where yaw ψ is zero. Let ϕ be the angle through which each wing tip is raised, and let the angle between the wind direction for a yaw ψ and the plane of the chord of the up wind wing be i'. Then it can easily be shown by geometry that approximately

$$i' = i_0 \pm \beta\psi,$$

when i, ψ, and β are small[1] and expressed in circular measure.

[1] A. Fage, "The Aeroplane," p. 82, Griffin, London, 1915.

In our case $\beta = 1.^\circ6$, then for $i = 12^\circ$ and $\psi = 10^\circ$, $i' = 12.^\circ3$, while for $i = 6^\circ$, $i' = 6.^\circ3$. This is an increase of incidence with 10° yaw of but 2.5 per cent at low speed, and 5 per cent at intermediate speed.

Since a side slip is equivalent to a yaw, and since the rolling moment due to side slip is largely caused by greater lift on the wing which is toward the wind, it appears reasonable to conclude that this greater lift is a consequence of the greater angle of incidence. But we see above, by a rough calculation, that the relative increase in incidence on a dihedral wing for given angle of yaw is much greater for the 6° attitude than for the 12° attitude. The falling off of L_v observed experimentally is, therefore, to be expected for an aeroplane with raised wing tips.

A discussion might be opened here as to whether it would not be preferable to use vertical fin surfaces above the center of gravity or a swept back wing (" retreat ") to obtain the desired righting moment L_v on side slip, rather than the dihedral arrangement. Until further experiments have been made, it is not profitable to speculate on this question, but one would see no reason *a priori* to expect the coefficient L_v, given by vertical fins, to depend in any way upon the angle of incidence of the normal flight attitude.

To preserve stability, we must make N_r large also. This coefficient is a measure of the damping of angular velocity in yaw, and can be made great by vertical surface forward and aft of the center of gravity. A rectangular body with flat sides, vertical fin surface at the tail (rudder), and the increased drift on the forward moving wing all combine to resist or damp the spin in yaw. The designer can, at his pleasure, increase both L_v and N_r by proper fin disposition. Note that N_r is not different at different speeds.

On the other hand, it is necessary to make N_v or the yaw due to side slip small. A preponderance of fin surface aft will make N_v large and is, therefore, dangerous. A machine that shows strong " weather helm " or has great so-called directional stability is likely to be unstable because the large N_v may make E_2 negative. The vertical fin surface should be fairly well balanced fore and aft, and directional restoring moments should not be great. Note that N_v does not vary much with different speeds.

The derivative L_r is characteristic of the rolling moment due to velocity of yaw or spin and was shown to be caused by the greater air speed on the outer wing in turning. It is not generally possible for a designer to make L_r small, though a short span will help matters.

Note that L_r is greatest at low speed and high angle of incidence. It should be unaffected by dihedral angle of wings.

The instability corresponding to E_2 negative is, therefore, a tendency on side slip to the right, for example, to head to the right toward the relative wind on account of much fin surface aft. At the same time, due to the spin in yaw, the machine tends to overbank on account of the greater lift on the left wing. The increased bank, increases the side slip, the yaw becomes more rapid and in turn the overbanking tendency is magnified. The aeroplane starts off on a spiral dive and will spin with greater and greater angular velocity. The term " spiral instability " has been given to this motion.

Spiral instability appears to be the most probable form of instability present in an ordinary aeroplane. It appears to be readily corrected by modification of fin surface and there appears to be no excuse for leaving it uncorrected. It is true that an alert pilot should have no trouble in keeping an aeroplane out of a spiral dive, but in case of breaking of a control wire disaster would be certain if the machine were spirally unstable.

§12. "ROLLING"

The second approximate factor

$$D+\frac{B_2{}^2-A_2C_2}{A_2B_2}=0,$$

when A_2C_2 is small compared with $B_2{}^2$, is seen to reduce to:

$$D+\frac{B_2}{A_2}=0,$$

or

$$D=-\frac{B_2}{A_2}=+Y_v+\frac{L_p}{K_A{}^2}+\frac{N_r}{K_B{}^2}.$$

Now Y_v, L_p, and N_r may be expected to be always negative in ordinary machines, and the radii of gyration K_A and K_B are essentially positive. Hence this root D will always be negative and the motion a damped subsidence. It will be observed that Y_v expresses resistance to side slip, L_p damping of an angular velocity in roll due to the wings, and N_r damping of an angular spin in yaw. In magnitude L_p is usually so great that Y_v and N_r may be neglected, giving roughly

$$D=\frac{L_p}{K_A{}^2}=-\frac{224}{27}=-8.3$$

at low speed, or a subsidence damped 50 per cent in $t=.08$ second.

The more exact calculation made in §11 showed $t=.076$ second. In a machine of very short span and great moment of inertia in roll, we might expect $\frac{L_p}{K_A^2}$ to become small, but never positive so long as forward speed is maintained.

When an aeroplane is at such an attitude that further increase in angle of incidence produces no more lift ("stalled"), the damping of a roll by the wings L_p may vanish. Then the downward moving wing, although its angle of incidence be increased, has no additional lift over the other and, hence, there is no resistance to rolling. In this critical attitude, pilots have reported that the lateral control by ailerons has no effect and the aeroplane is unmanageable.

In any reasonable attitude short of stalling, there appears to be no reason to fear instability in "rolling" corresponding to this second factor of the equation.

§13. THE "DUTCH ROLL"

In the approximate solution of the biquadratic, the third factor,

$$D^2+\left(\frac{C_2}{B_2}-\frac{E_2}{D_2}\right)D+\frac{B_2D_2}{B_2^2-A_2C_2}=0,$$

for most machines will have A_2C_2 small compared with B_2^2, and we may write:

$$D^2+\left(\frac{C_2}{B_2}-\frac{E_2}{D_2}\right)D+\frac{D_2}{B_2}=0.$$

Considering the usual magnitudes of the derivatives entering in B_2, C_2, D_2, E_2, we may write very approximately:

$$\begin{aligned}B_2&=-K_C^2L_p,\\ C_2&=(N_rL_p-L_rN_p),\\ D_2&=+gK_C^2L_r,\\ E_2&=g(N_vL_r-L_vN_r).\end{aligned}$$

The motion is damped and stable, provided $\frac{C_2}{B_2}-\frac{E_2}{D_2}$ is positive, and the period

$$p=\frac{2\pi}{\frac{1}{2}\sqrt{\frac{4D_2}{B_2}-\left(\frac{C_2}{B_2}-\frac{E_2}{D_2}\right)^2}}.$$

or approximately $=p=2\pi\sqrt{\frac{B_2}{D_2}}$.

Since $\sqrt{\frac{B_2}{D_2}}$ is ordinarily of the order of 1 or 2 the period may be of the order of 6 or 12 seconds. This period is rapid compared with

that of the longitudinal motion and unless strongly damped, the motion may become so violent as to be uncomfortable. Note that since N_v, L_v, N_p, L_p, N_r, L_r are involved, the motion must consist of a combination of side slipping, rolling, and yawing.

The motion is stable and the oscillation tends to damp out in time and the aeroplane to return to her course if $\frac{C_2}{B_2} - \frac{E_2}{D_2}$ is positive. To damp to half amplitude requires $t = \dfrac{0.69}{\frac{1}{2}\left(\frac{C_2}{B_2} - \frac{E_2}{D_2}\right)}$ seconds.

Substituting approximate expressions we have

$$\frac{C_2}{B_2} - \frac{E_2}{D_2} = \frac{L_r}{K_C{}^2}\left(\frac{N_p}{L_p} - \frac{N_v}{L_v}\right).$$

Since L_r is positive, in order for the damping to be real, $-N_v/L_v$ must be greater than N_p/L_p and positive.

Stability of this motion is, therefore, assisted by:

1. Large negative yawing moment due to side slip ("weather cock" stability) N_v. This is incompatible with stability against a "spiral dive."

2. Large damping of the rolling due to rolling L_p.

3. Small positive rolling moment due to side slip L_v. This is also incompatible with stability against the "spiral dive."

4. Small yawing moment due to rolling N_p.

5. Large rolling moment due to yawing velocity L_r; another requirement incompatible with "spiral" stability.

6. Small radius of gyration K_c in yaw.

It does not appear practicable to make N_p small on account of the steepness of the drift curve at high angles of incidence. The drift of the downward moving wing when the aeroplane rolls is increased while the drift of the rising wing is decreased. The resultant yawing moment tends to swing the aeroplane away from her course. Note that at slow speed, near stalling angles, N_p becomes large. This is not desirable, but is unavoidable.

The rolling is heavily damped by the wings and L_p will always be large and negative. This assists stability.

To avoid "spiral" instability, we saw above that it was necessary to make the weather cock or "directional stability" small. That is, N_v was to be small and the preponderance of vertical fin surface aft slight. In the motion now under discussion, we wish to make N_v large. The two conditions imposed are unfortunately conflicting. We must compromise and make N_v numerically not too great, but still essentially negative.

In a similar manner, the rolling moment, due to side slip, or restoring moment, such as is given by high fins or raised wing tips, should be large to avoid " spiral " instability. In the present case, however, we wish to make L_v small.

Likewise the natural banking due to spin in yaw we wish small for " spiral " stability, but we now wish to have this coefficient large.

The conflicting nature of the requirements for stability is here shown by the use of rather drastic simplifications in the more exact formulæ. For the analysis of stability the exact formulæ are easily applied, and the present approximate forms are deduced only in order to trace the effect on the motion of such changes as the designer may be tempted to make on a machine.

It is believed that an excessive dihedral angle upwards is not a cure-all for stability problems. Indeed, in practice, aeroplanes with a large dihedral angle for the wings have been found so violent in their motion under certain circumstances that the average pilot has a firm prejudice against the use of such a wing arrangement. That this prejudice has some physical basis has been shown here. A dihedral angle machine is not likely to run into a " spiral dive," but it is very likely to be unstable on what we may term a " Dutch roll," from analogy to a well-known figure of fancy skating.

We may imagine an aeroplane to yaw to the right accidentally. Due to L_r and L_v the aeroplane banks in a manner proper for the turn, but the roll is retarded by the large damping due to L_p. The turn is assisted by the increased drift on the lower wing due to N_p, and were it not for the much discussed " weather helm " given by N_v, the aeroplane would run off on a right turn. However, N_v tends to turn the aeroplane back to her course. If N_v be sufficient, the machine will swing back to her course and the bank will flatten out. But since the moment of inertia in yaw is considerable, the machine will swing past her course and start on a turn to the left. This swinging to right and left of her course is accompanied by rolling outward and some side slipping.

The analogy to a " Dutch roll " on skates is obvious. If the skater lean too far out he may fall, and if the aeroplane roll too far on the side swings it may happen that the motion will become unstable. If the air be gusty it is very likely that such an aeroplane may be caught on the roll by a side gust and capsized.

The " Dutch roll " in ordinary aeroplanes (which are " spirally " unstable) is not likely to be present, since there is no dihedral and a large rudder. The average pilot would much prefer to deal with a

machine which tended to swing down into a "spiral dive" if left to itself because there is no oscillation of rapid period involved.

The production of a laterally stable aeroplane is attendant with many compromises, and it cannot be too strongly insisted upon that a freak type designed to be "very stable" is likely to be rapid and violent in its motion, and even if stable against a "spiral dive" to be frankly unstable against the "Dutch roll."

One may inquire whether a machine made directionally neutral can be made stable. In the notation here used N_v would be approximately zero. The condition that "spiral" instability be not present is:

$$L_v/N_v > L_r/N_r.$$

But for N_v zero, we need only make L_v slightly positive to insure stability in this motion. L_v may be made positive by a very slight preponderance of fin surface above the center of gravity, raised wing tips, etc.

However, in the approximate criterion for stability in the "Dutch roll," we have

$$-N_v/L_v > N_p/L_p,$$

and for N_v zero, the motion is clearly unstable unless the magnitude of the neglected terms is greater than N_p/L_p, which is unlikely.

Replacing neglected terms in C_2, we obtain as a more nearly exact expression:

$$\left(\frac{C_2}{B_2} - \frac{E_2}{D_2}\right) = \frac{L_r}{K_C^2}\left(\frac{N_p}{L_p} - \frac{N_v}{L_v}\right) - Y_v - \frac{K_A^2}{K_C^2}\frac{N_v}{L_p}U.$$

If we make N_v very small as in the case under analysis, the last term vanishes as well as the second, and we have as a condition for $\frac{C_2}{B_2} - \frac{E_2}{D_2}$ positive:

$$-Y_v > \frac{L_r}{K_C^2}\left(\frac{L_p}{N_p}\right).$$

Substituting numerical values for the derivatives, for the slow-speed condition, we find

$$-Y_v = +.106,$$

and

$$\frac{L_r}{K_C^2}\frac{N_p}{L_p} = -\frac{160 \times 57}{48.6 \times 224} = -.856.$$

The slow-speed motion would, therefore, be very unstable if N_v were zero. Consideration of the magnitude of the derivatives leads us to the conclusion that in any aeroplane, if N_v be made very small, the

motion called "Dutch roll" will probably be unstable at low speeds where N_p becomes great.

For high speed, if both N_r and N_p are zero, the lateral motion should be stable regardless of the magnitude of the other derivatives.

With the yawing moment due to rolling as measured by N_p increasing from zero at high speed to $+57$ at low speed, it would seem that, at the maximum speed, any reasonable aeroplane will be stable so far as the "Dutch roll" is concerned, but at low speed it may become unstable in this particular motion.

In general, for high speed, considering the two possible kinds of lateral instability, it is believed that very slight modifications in fin disposition will suffice to render any ordinary aeroplane laterally stable. Likewise, at high speed, longitudinal stability is easily secured. At low speed, the longitudinal motion tends to become unstable as well as one or the other kind of lateral motion.

§14. COMPARISON WITH OTHER AEROPLANES

Any stability discussion is much more suggestive if several aeroplanes can be analyzed in parallel. The only published information on lateral stability is Bairstow's investigation of the Blériot monoplane used above in connection with the longitudinal stability discussion. This monoplane had only a very small rolling moment due to side slip $L_v = .83$ as against $L_v = 3.06$ for the Clark aeroplane for high speed. The coefficient N_v, yawing moment due to side slip, is not greatly different in the two machines. The other coefficients are of the same order of magnitude, except L_p, the damping of a roll, which is small in the monoplane on account of the small wings of short span.

Without further knowledge, we should expect the Blériot to be stable on the "Dutch roll" on account of the small L_v. Bairstow finds a period of 6.5 seconds damped to half amplitude in 1.65 second.

On the other hand, the small L_v would lead us to suspect the stability of the spiral motion, especially as L_p is also small. In fact, the coefficient E_2 was found to be slightly negative and the aeroplane, in consequence, spirally slightly unstable. The motion is a slow divergence which doubles itself in 68 seconds. This is an extremely slow change and should give no trouble to a pilot. Indeed, the well-known steadiness in flight of this famous aeroplane is in full agreement with the theoretical conclusions. The Blériot makes no claim to lateral stability, but is essentially a steady aeroplane easily controlled. In the "Dutch roll" the Blériot is very strongly damped and hence very stable. The spiral motion is not damped, but is so slow that the stability may be called neutral. The aim of the French school has

always been a machine whose lateral stability is neutral so that it will not be thrown about by the wind.

The Curtiss type military tractor tested by us in a manner identical with that described in this paper, was found at high speed to have resistance derivatives of the same order of magnitude as the Clark tractor, except that a large rudder and deep rectangular body make N_r twice as large for the Curtiss, and there being no high fin surface L_r for the Curtiss is small. As would be expected the spiral motion is slightly unstable, tending to double itself in 28 seconds. The "Dutch roll" is very stable, having a period of 5.25 seconds and damping to half amplitude in 1.77 seconds. The machine in flight at high speed should then have the characteristics of the Blériot and be steady and easily controlled. This is, in fact, the general reputation of this type of aeroplane.

At low speed, matters are not so favorable. We have no data for the Blériot at slow speed, but the Clark model is seen to become spirally unstable to such an extent that an accidental deviation doubles itself in 7.2 seconds.

The "Dutch roll" for the Clark model remains stable at low speed, but is somewhat less strongly damped than at high speed. The period is 12 seconds damped to half amplitude in 6 seconds. This motion should be not uncomfortable.

The Curtiss, at low speed, due to falling off of N_v and marked increase in L_r, becomes spirally stable. The spiral motion is a subsidence damped 50 per cent in 3.3 seconds. The wings had no dihedral angle. A separate test[1] made on a single wing without body or tails showed a small rolling moment for an oblique wind indicating a small and positive L_r. At large angles of incidence this effect was considerably magnified. The decrease in N_r (or in the weather helm) at large angles of incidence cannot be laid to the straight wings. Tests on a wing alone show a small negative N_r which is not changed at large angles of incidence.

The increase in L_r and decrease in N_v for the Curtiss aeroplane, favorable to stability of the spiral motion, are unfavorable to stability in the "Dutch roll." Furthermore, N_p increases from zero at high speed to $+38$ at the low speed, and L_p decreases from -314 to -78. These changes are very unfavorable and, as we should expect, the "Dutch roll" for the Curtiss is unstable. The natural period is about 5.7 seconds and any initial amplitude is doubled in 7.66 seconds.

[1] Smithsonian Misc. Coll., Vol. 62, No. 4. "Experiments on a Dihedral Angle Wing," J. C. Hunsaker and D. W. Douglas.

The motion is a swaying of the aeroplane of increasing amplitude and intensity. However, we must always point out that an alert pilot with powerful controls can check the natural motion of the aeroplane before it has became violent and so maintain his equilibrium.

The increase in N_p at low speed or rather large angle of incidence is due to the steeper drift curve for a wing at large angles. As the aeroplane rolls, the downward moving wing has its drift relatively more increased as the normal flight attitude requires a larger angle of incidence.

The drop in L_p is due to the less steep lift curve at high angles of incidence. As the aeroplane rolls, the increase in angle of incidence of the downward moving wing gives very little increase in lift on that wing if the wing be already near its angle of maximum lift. We might imagine an aeroplane flying at an angle of incidence giving the maximum lift. Any increase in incidence can produce no additional lift. In most aeroplane wings, an increase in incidence beyond the optimum angle causes the wing to lift less at the same air speed. Now if the aeroplane in such an attitude roll, the increased angle of incidence of the downward moving wing gives no more lift on that wing and hence the rolling is unresisted. The damping of the roll will be zero, or even negative. In the Curtiss aeroplane, the low speed chosen required an incidence of 15°.5, very near the "burble point," or angle of maximum lift for the wings. The small value -78 of L_p appears to be one of the principal causes of the instability. In the Clark model, the wing loading is smaller and an equal speed about 44 miles per hour is obtained for an incidence of only 6°, giving $L_p = -319$. The lowest speed of the Clark model is taken as about 37 miles per hour where an incidence of but 12° is needed. L_p at this angle is -224.

It appears that lateral dynamical stability is incompatible with a high wing loading which requires a large angle at landing speed. The analysis of longitudinal stability led to a similar conclusion.

If we turn to practical aviation we observe that aeroplanes which are noted for their steadiness at low speeds are the light Antoinette, Farman, and the various German Taubes derived from the Etrich. All these aeroplanes have large wing area and light loading, probably between 3 and 4 pounds lift per square foot. The light loading enables these aeroplanes to gain a safe low speed without having the angle of incidence near the angle of maximum lift.

In the Clark model the loading is about 3.55 pounds per square foot, while it is 5.2 in the Curtiss type discussed. More recently the

Curtiss has been given greater wing area in order to reduce the loading. It should be stated that the comparison is not quite fair, since the total weight of the Clark aeroplane was taken as 1,600 pounds which includes only half the full 5.6 hours' gasolene supply. However, the advantage of light wing loading is more clearly brought out by the marked difference in weight per square foot wing area.

The following table summarizes all the information available and may be used to make further comparisons if desired:

	Clark Tractor[2]			Curtiss Tractor[2]		Blériot Monoplane[3]
Wing area........	464.0	.....	.....	384.0	.....	244.0
Mean span........	40.2	.....	.....	36.0	.....	.....
Mean chord	5.77	.....	.....	5.3	.....	.....
Mean gap.........	6.37	.....	.....	5.3	.....	.....
Area, fixed tail....	16.1	.....	.....	23.0	.....	.....
Area, elevators....	16.0	.....	.....	19.0	.....	.....
Area, rudder......	9.35	.....	.....	7.8	.....	.....
Length, body......	24.5	.....	.....	26.0	.....	.....
Weight, lbs.......	1600	.....	.....	1800	.....	1800
Rise of wing......	1°.63	.....	.....	0°	.....	1°.8
Lbs. per sq. ft.....	3.55	.....	.....	5.2	.....	7.38
Angle of incidence	0°	6°.0	12°.0	1°.0	15°.5	6°.0
V, miles, hour....	76.9	44.6	36.9	78.9	43.6	65.0
U, ft.-seconds.....	−112.5	− 65.3	− 54.0	−115.5	− 63.8	− 95.4
m	50.0	.....	.....	56.0	.....	56.0
K_A, feet.........	5.2	.....	.....	6.06	.....	5.0
K_C, feet.........	6.975	.....	.....	8.4	.....	6.0
Y_v...............	− .204	− .0878	− .106	− .248	− .09	− .108
L_v...............	+ 3.06	+ 3.44	+ 1.91	+ .844	+ 2.7	+ .70
N_v...............	− .449	− .351	− .53	− .894	− .45	− .44
Y_p...............	0	0	0	0	0	0
L_p...............	−631.0	−319.0	−224.0	−314.0	− 78.0	−167.0
N_p	0	+ 33.5	+ 57.0	0	+ 37.7	+ 24.0
Y_r	0	0	0	0	0	0
L_r...............	+ 77 0	+132.5	+160.0	+ 55.2	+101.0	+ 54.0
N_r	− 39.4	− 26.0	− 38.9	− 27.0	− 30.4	− 31.0
A_2...............	1310.0	1310.0	1310.0	2590.0	2590.0	900.0
B_2...............	31800.0	16350.0	12090.0	23800.0	6860.0	6780.0
C_2...............	32700.0	5910.0	1630.0	18000.0	209.0	5580.0
D_2	41780.0	5490.0	3490.0	34600.0	5590.0	6640.0
E_2...............	2770.0	1386.0	−335.0	−855.0	1175.0	− 68.0
Routh's discr.....	37×10^{12}	12×10^{10}	4×10^{9}	9×10^{12}	-7×10^{10}	21.5×10^{10}
Spiral Motion						
Damp 50% in, sec.	10.4	2.7	.....	.....	3.3	.....
Double in, sec.....	.....	.....	7.2[1]	28.0[1]	.....	68.0[1]
Rolling						
Damp 50% in, sec.	.03	.06	.076	.08	.26	.10
"Dutch Roll"						
Period, sec........	5.9	10.7	12.0	5.24	5.7	6.5
Damp 50% in, sec.	1.4	1.3	5.95	1.77	.....	1.65
Double in, sec.....	.....	.....	.....	.....	7.66[1]	.....

[1] Unstable.
[2] Tested at Mass. Institute of Technology, Boston.
[3] Tested at Teddington, England.